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ACTIVE CONTROL OF FLEXURAL VIBRATIONS USING A HYBRID MODE/WAVE APPROACH

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1.0 INTRODUCTION

Vibrations can be described in terms of modes and in terms of waves. Active vibration control schemes can be based on either of these two approaches. Both modal and wave-based active control methods have advantages and disadvantages, and they each have applications where they are most appropriate. This paper describes an application of a hybrid active vibration control approach involving both modal and wave based control, the aim being to ameliorate the disadvantages of each scheme alone. The specific example of control of a simply-supported beam is discussed.

In a modal approach, the vibration of a structure is described in terms of its global modes (e.g., natural frequencies and damping factors), which are then controlled normally using feedback methods [1]. Knowledge of the properties of the whole structure is required to accurately model these modes. Difficulties arise at higher frequencies, where higher order modes of vibration contribute to the response. These modes are sensitive to the detailed properties of the structure, which are normally not known to sufficient accuracy. Serious problems of robustness then arise, not to mention difficulties with controller complexity.

The propagation and transmission of waves through a structure depend on the local properties of the structure, in contrast to the global modal description. At higher frequencies (especially when the structure is large compared to the wavelength), therefore, wave based descriptions become more appealing. In wave approaches, in simple terms, the vibrations produced by waves incident on the control location are measured, and the control forces then act to either cancel the transmission of incoming waves or to absorb their energy. Wave-based methods are predominantly feedforward [2], the incoming disturbance being sensed and fed forward to the control location, although feedback methods exist [3, 4]. The main disadvantage of wave based methods is that in general they only control the local flow of vibrational energy and not the structure's global behaviour.

In this paper a hybrid approach is suggested. Modal control is applied to the lowest few modes of the structure. It is known that the unmodelled and uncontrolled modes (primarily, the next few modes) deteriorate the performance of modal control. Therefore wave-based control is applied to add significant damping, especially in a frequency range containing these next few modes. The design of wave control is performed first - the numerical example in this paper involves a simple collocated feedback system - and this then modifies the equation of motion of the structure. Originally undamped, the structure-plus-wave-controller is now non-self-adjoint. The modified equations are used to design modal control. In the next two sections feedback wave control and modal control are briefly reviewed. Then the hybrid approach is described and applied to the control of a simply supported beam.

2.0 WAVE BASED FEEDBACK ACTIVE VIBRATION CONTROL

Wave based active vibration control methods generally aim at either maximising the energy absorbed by the controller or isolating the vibrations (i.e., stopping energy transmission from one part of the structure to another). Most wave based active control systems are feedforward [2]. In the example system considered below, however, collocated feedback control [3,4] is applied to control flexural vibrations in a beam, as shown in Figure 1. In this w denotes the sensor measurement, F the control force and $H_w(\omega)$ the transfer function of the controller. The control force can be written in terms of the beam's deflection as

$$F = -H_w(\omega)\, w. \qquad (1)$$

The control is dynamically identical to an attached spring with a dynamic translational stiffness $H_w(\omega)$, which may be frequency dependent and complex. A wave is then transmitted and reflected at the control location with the transmission and reflection coefficients t and r being given by [5, 6]

$$t = 1 + i\mu;\quad r = i\mu;\quad t_N = \mu;\quad r_N = \mu, \qquad (2)$$

where

$$\mu = \frac{H(\omega)}{\omega^{3/2} - (1+i)H(\omega)}, \qquad H(\omega) = \frac{H_w(\omega)}{4\sqrt[4]{m^3 EI}} \qquad (3)$$

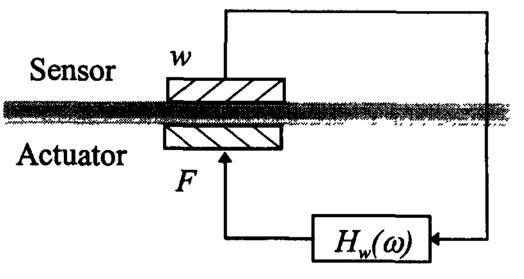

Figure 1. Collocated feedback control.

In the hybrid control system described below, velocity feedback is adopted, i.e.,

$$H_w(\omega) = i\omega c. \qquad (4)$$

The objective of the wave control here is to absorb as much of the incident energy as possible, so as to damp the uncontrolled modes. The optimal control gain c_{opt} can be found by assuming a wave is incident on one side of the control location and then by tuning the control gain c so as to maximise the absorbed incoming energy, in other words to minimise $|r|^2 + |t|^2$. In this case the optimal control gain is found to be

$$c_{opt} = \sqrt{\omega/2}\,. \qquad (5)$$

The corresponding energy radiated away from the controller is about 58.6% of the total incoming energy.

The optimal wave velocity feedback control gain is frequency dependent. It is also noncausal. In a digital implementation, a causal controller is found which has a frequency response that is an approximation to the ideal, for example by truncating the noncausal part of the ideal controller. An alternative approach, and the one that is adopted in the example below, is to tune the controller so that it is frequency independent and optimal at a certain frequency ω_d. The control gain is then a constant given by

$$c_{opt} = \sqrt{\omega_d/2}\,. \tag{6}$$

3.0 MODE CONTROL

Vibration control can be achieved by controlling the modes of vibration. In the absence of damping, the equation of motion of a continuous system can be written as [1]

$$Lw(x,t) + m(x)\ddot{w}(x,t) = f(x,t)\,. \tag{7}$$

where $w(x,t)$ is the displacement, L a stiffness operator, $m(x)$ the mass density and $f(x,t)$ the external forces, which are the sum of the control force and any external disturbance. If $\phi_i(x)$ and $q_i(t)$ (i=1,2,…,∞) are the mass-normalised mode shapes and modal coordinates of the system respectively, then the response can be expressed as a sum of modal components

$$w(x,t) = \sum_{i=1}^{\infty} \phi_i(x) q_i(t)\,. \tag{8}$$

Generally only the first n modes are considered, the infinite sum truncated and the response written as

$$w(x,t) = \sum_{i=1}^{n} \phi_i(x) q_i(t) = \Phi^T(x)\mathbf{q}(t), \tag{9}$$

where Φ and $\mathbf{q}$ are mode shape and modal coordinate vectors. Substituting equation (9) into equation (7), multiplying both sides by $\phi_i(x)$, integrating over the structure, and noting the orthonormality properties of the mode shapes, the equations of motion become

$$\ddot{q}_i(t) + \omega_i^2 q_i(t) = f_i, \quad f_i = \int_0^L f(x,t)\phi_i(x)dx, \qquad i=1,2,\ldots,n. \tag{10}$$

Since distributed control is rarely realisable, it is now assumed that the control comprises r point actuators located at x_j, j=1,2,…,r. The control force $f(x,t)$ is then

$$f(x,t) = \sum_{j=1}^{r} F_j(t)\delta(x - x_j)\,, \tag{11}$$

where $F_j(t)$ is the amplitude of the j'th control force and $\delta(x-x_j)$ is the Dirac delta function. The modal control forces are

$$f_i(t) = \sum_{j=1}^{r} \phi_i(x_j) F_j(t), \qquad i=1,2,\dots,n. \tag{12}$$

From equation (10) it can be seen that, in the absence of damping, the modal equations are decoupled. The control design is relatively straightforward, and a number of approaches can be used [1].

4.0 HYBRID ACTIVE VIBRATION CONTROL

In this hybrid approach, the design of the wave control is performed first and modal control is then designed based on the modified equation of motion of the structure. In the presence of the wave velocity feedback control, equation (7) can be rewritten as

$$Lw(x,t) + m(x)\ddot{w}(x,t) = f(x,t) + f_c(x,t), \quad f_c(x,t) = -c_{opt}\,\dot{w}(x,t)\delta(x-x_c) \tag{13}$$

where $f_c(x,t)$ is the optimised wave velocity feedback control force applied at point x_c. In terms of the modes of the uncontrolled system

$$f_c(x,t) = -c_{opt}\left(\sum_{j=1}^{n} \phi_j(x)\dot{q}_j(t)\right)\delta(x-x_c). \tag{14}$$

The modal equations of motion now become

$$\ddot{q}_i(t) + \omega_i^2 q_i(t) = f_i + f_{ci}; \quad f_{ci}(x,t) = -c_{opt}\sum_{j=1}^{n} \phi_j(x_c)\phi_i(x_c)\dot{q}_i(t). \tag{15}$$

The new equations of motion of the system are therefore

$$\ddot{q}_i(t) + c_{opt}\sum_{j=1}^{n} \phi_j(x_c)\phi_i(x_c)\dot{q}_i(t) + \omega_i^2 q_i(t) = f_i. \tag{16}$$

The new modes of vibration of the structure can now determined by an eigenanalysis of equation (16) and the modal controller designed on the basis of these modes. Note, however, that the wave feedback controller couples the equations of motion, when written in terms of the undamped, uncontrolled structural modes. Furthermore, these equations are now non-self-adjoint. These factors complicate the control design process [1].

In this paper, state-space methods are used to design the control. First, equation (16) is written in matrix form as

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{f}, \tag{17}$$

where

$$\mathbf{M} = \mathbf{I}; \quad (C)_{ij} = c_{opt}\phi_i(x_c)\phi_j(x_c); \quad \mathbf{K} = diag(\omega_i^2);$$
$$\mathbf{q} = [q_1, q_2, \ldots, q_n]^T; \quad \mathbf{f} = [f_1, f_2, \ldots, f_n]^T. \tag{18}$$

Introducing the state vector $\mathbf{X}(t) = [\mathbf{q}^T(t) : \dot{\mathbf{q}}^T(t)]^T$, equation (17) can be rewritten in state space form as

$$\dot{\mathbf{X}}(t) = \mathbf{A}\mathbf{X}(t) + \mathbf{B}\mathbf{f}(t). \tag{19}$$

where the coefficient matrices

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1} \end{bmatrix}. \tag{20}$$

The characteristic matrix **A** is nonsymmetric, and the system is normally non-self-adjoint because of the added discrete wave controller. Both the left and right eigenproblems must therefore be solved, these giving matrices **V** and **U** of left and right eigenvectors and a diagonal matrix of eigenvalues Λ, which satisfy biorthonormality relations [1]

$$\mathbf{V}^{\mathrm{T}}\mathbf{A}\mathbf{U} = \Lambda, \quad \mathbf{V}^{\mathrm{T}}\mathbf{U} = \mathrm{I}. \tag{21}$$

Substituting $\mathbf{X}(t) = \mathbf{U}\mathbf{Z}(t)$ into equation (19), premultiplying by $\mathbf{V}^{\mathrm{T}}$ and noting the biorthonormality properties, gives

$$\dot{\mathbf{Z}}(t) = \Lambda\mathbf{Z}(t) + \mathbf{V}^T\mathbf{B}\mathbf{f}(t). \tag{22}$$

Modal control design in the Z-domain can now proceed along a number of lines [1, 7], depending on the desired control structure (e.g., single- or multi-input or single- or multi-output). Schemes include pole allocation, optimal control etc. The design consists of finding a matrix **G** of gains such that $\mathbf{f}(t) = \mathbf{G}\mathbf{Z}(t)$ and therefore

$$\dot{\mathbf{Z}}(t) = \Lambda\mathbf{Z}(t) + \left[\mathbf{V}^T\mathbf{B}\right]\mathbf{G}\mathbf{Z}(t). \tag{23}$$

The control gain matrix **G** is normally designed so that the eigenstructure of the controlled system has some desired properties. In the numerical example below, multi-input, multi-output, coupled mode control is adopted.

5.0 NUMERICAL EXAMPLE

In this section the example of hybrid vibration control of a simply supported beam is considered and some numerical results presented. The material is brass with Young's modulus E=100.6 GPa and mass density ρ=8530 kg/m^3. The length, width and thickness are 400 mm, 40 mm and 2 mm respectively. For a simply supported beam, the natural frequencies ω_i and mass-normalised mode shapes $\phi_i(x)$ are given by

$$\omega_i = \sqrt{\frac{EI}{\rho A}}\left(\frac{i\pi}{L}\right)^2 \; ; \; \phi_i(x) = \sqrt{\frac{2}{\rho AL}}\sin\left(\frac{i\pi}{L}x\right); \; i=1,2,\ldots,\infty, \tag{24}$$

where I is the second moment of area, A the cross sectional area, L the length of the beam and x is measured from one end of the beam.

Coupled mode control is to be designed for the first two modes of vibration, while the wave velocity feedback controller is tuned to be optimal at the third natural frequency. The location of the disturbance input is $0.1L$ and the response found at $0.45L$.

Figure 2 shows the response without control and with wave control only. Without control clear, sharp resonances can be observed. The wave control is tuned to be optimal at ω_3 and is located at $(5/6)L$. The velocity feedback is seen to add damping to the structure, the resonances in general being significantly less sharp. Note that the amount of damping varies from mode to mode, and that the wave control is ineffective at frequencies around the sixth natural frequency. This arises because the collocated, point, displacement/force system used responds equally to waves travelling in both directions along the beam. The right-going wave is reflected from the boundary at $x=L$, producing a left-going wave. At certain frequencies these two waves interface destructively at the control location, and hence can be neither detected nor controlled by this particular wave controller - in modal terms, the control is applied at (or around) a node of a global mode (here, $x=(5/6)L$ is a node of the sixth mode). These difficulties can be reduced by applying control close to a boundary, one side effect then being that its effect on the lowest modes is generally reduced. The sensitivity to control location can be removed altogether by using more sophisticated wave control, which can respond or excite both wave components (e.g., using two point measurement and excitation, or by measuring both displacement and rotation).

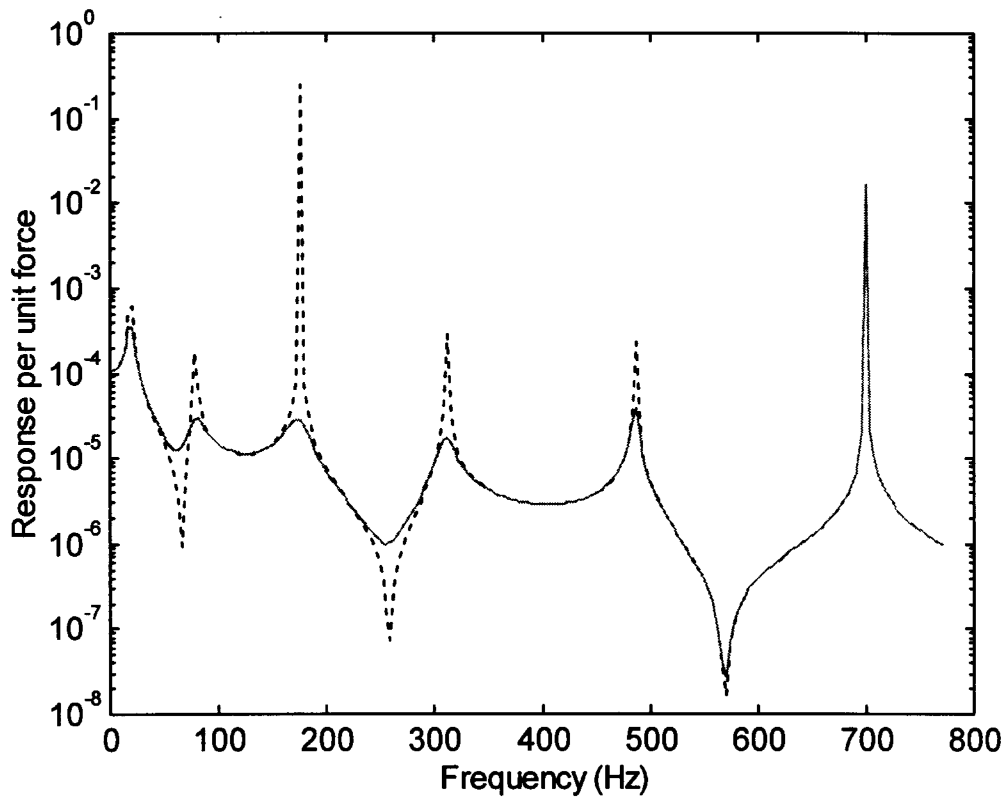

Figure 2. Frequency responses of the beam before (···) and after (—) wave control.

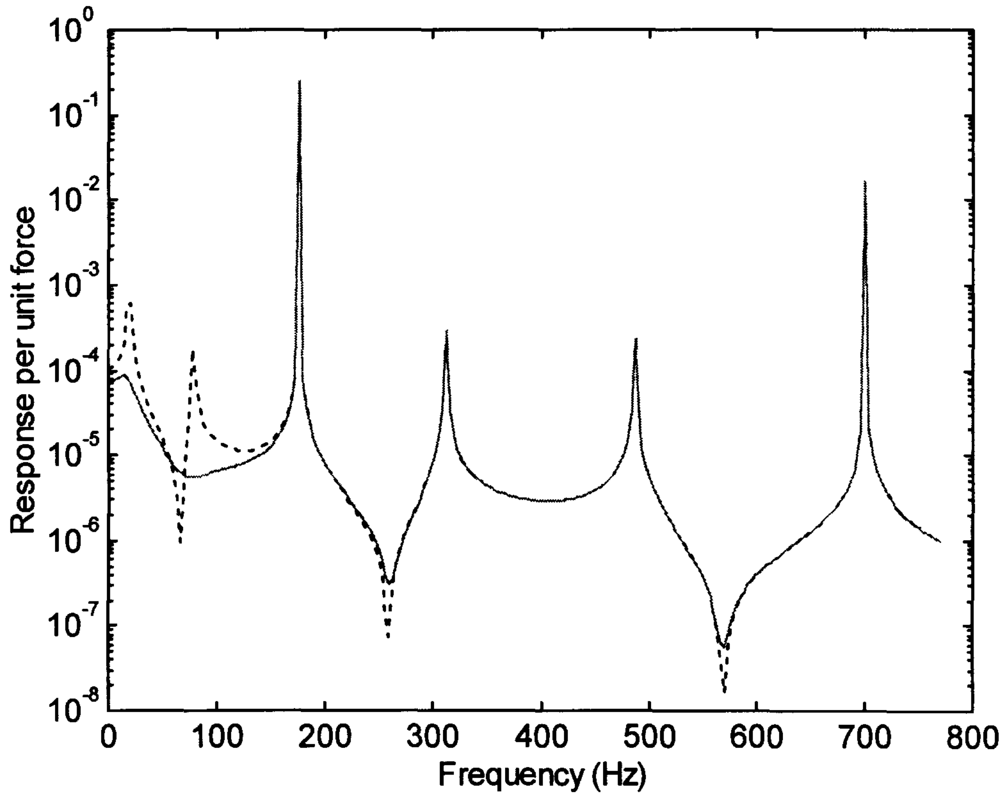

Figure 3. Frequency responses of the beam before (···) and after (—) modal control.

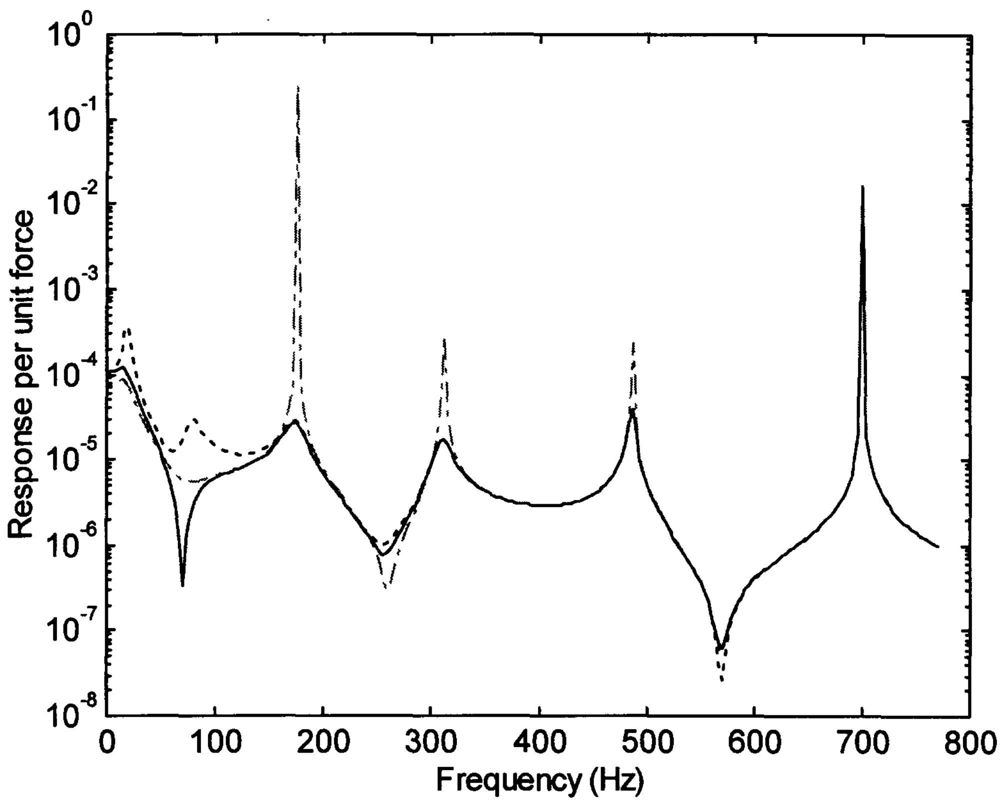

Figure 4. Frequency responses of the beam after wave (···), modal (-.-.) and hybrid (—) control

Figure 3 shows the responses before and after modal control. Here two actuators have been chosen for the modal control implementation, they being located at $(1/6)L$ and $(5/12)L$. In multi-input, multi-output, coupled mode control, it is assumed that all the $\mathbf{Z}$ states are measured or estimated, typically by measuring or estimating the modal states $\mathbf{X}$ and then forming $\mathbf{Z} = \mathbf{V}^T\mathbf{X}$. There exist various ways of extracting modal states from sensor measurements [8] for the modal control design (e.g., modal filtering), but this issue is not addressed here. Since, in this example, two control actuators are used, the gain matrix $\mathbf{G}$ is a two-row matrix. The first two eigenvalues of the system are required to have damping factors of 0.5. From Figure 3 it can be seen that the lowest two modes are clearly well controlled, while the remainder are virtually unaffected. Sharp resonances associated with these modes still, therefore, exist.

Finally Figure 4 shows the response with hybrid control. The control gains for the modal controller are designed using the same approach as that described above, but now on the basis of the modes of the non-self-adjoint system. It is seen that at low frequencies the presence of wave control somewhat deteriorates the performance, but substantially improves it at higher frequencies. At higher frequencies still (around the sixth natural frequency) the same problem regarding wave controller location are seen to exist.

6.0 CONCLUDING REMARKS

This paper briefly described a hybrid approach to active vibration control and its application to control of a simply supported beam. Control designed on the basis of local wave propagation is used to absorb energy from the structure (add damping). This results in a new, non-self-adjoint equation of motion for the structure, from which a modal control scheme is designed. The hybrid control reduces the effects of the unmodelled modes and uncertainty, and improves robustness at the cost of (relatively small) deterioration in performance at low frequencies. The additional single-input, single-output collocated wave feedback control increases the controller complexity very little, compared to including additional modes and modal actuators in the system.

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