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VARIOUS METHODS FOR NONLINEAR NOISE & VIBRATION SIGNAL PROCESSING

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Nonlinear vibration signals are more well-defined and established than nonlinear noise signals which most works are dealing with electronic noise and the unwanted noise in signals.In this paper,we will consider nonlinear industrial and machinery noise,in particular non-Gaussian noise and chaotic signals and nonlinear vibration signals. First we consider chaotic signals. Here two types of fractal functions are used to represent them:the Weierstrass function and the radial basis function. These functions have to be subjected to computation of their fractal dimension. The Hausdorff definition of fractal dimension is used. Next the technique of higher order statistics is used to process nonlinear noise and vibration signals. The bispectrum is used which is a nonlinear generalisation of the spectral approach to linear time series analysis. Here we review the method of estimation of bispectrum and study possible applications to non-Gaussian signals,such as chaos. We find that the estimated bispectrum could be used to distinguish between nonlinear deterministic stable systems and nonlinear deterministic chaotic systems. We also consider the properties of one nonlinear model,namely bilinear model and study their application to nonlinear noise and vibration signal processing.

INTRODUCTION

Most of the noise and vibration signals in nature are nonlinear in nature.Conventionally the Fourier analysis using the superposition principle assumes that the signals are linear in nature.This results in the loss of phase informations contained in the individual signals.Sofar the the noise and vibration instrumentiontation industry is based on the Fourier analysis and the resulting autocorrelation function,crosscorrelation function and power spectrum.During the past few years there has been the beginning of the treatment on the nonlinear nature of noise and vibration signals.With the rapid increase of computation power and also within economical reach,the incorporation of the nonlinearity of noise and vibration signals becomes a commercial reality.In this paper we consider nonlinear industrial and machinery noise,in particular non-Gaussian noise and chaotic signals and nonlinear vibration signals.

CHAOTIC SIGNALS

In nonlinear noise and vibration systems, a frequently encountered signal is the chaotic signal. A wellknown chaotic noise signal is aerodynamic noise which is due to turbulence. In this paper we will limit the scope of signal processing only to spectrum analysis. For linear signals,Fourier transform is used to compute the autocorrelation function and hence the power spectrum. For nonlinear signals,Fourier series are not applicable and other time series representations are necessary. For chaotic signals, which are fractal in nature,we look for time series with fractal properties. We will test on two functions:(a)Weierstrass non-differentiable function,(b) radial basis function.

A. Weierstrass non-differentiable function

Fourier series involves a linear progression of frequencies. The Weierstrass function on the other hand,involves a geometric progression:

$$V_{MW}(t)=\sum_{n=-\infty}^{\infty} A_n R^{nH} \sin(2\pi r^{-n} t+\Phi_n) \qquad (1)$$

where A_n is a Gaussian random varriable with the same variance for all n,n is a random phase uniformly distributed on (0, 2π),$R_n = 1/f_n$,f_n = discrete frequencies and H=2-D where D=fractal dimension for the case of fractional Brownian Motion (fBm). The fractal nature of the Weierstrass function means it is self-similar and nowhere differentiable.

The usual procedure is spectrum analysis.This is to calculate the power spectrum density (PSD) using Fourier transform and correlation function. This is correct only for linear noise and vibration signals. For chaotic signals which are also fractal and nonstationary in nature,we will first represent the time series by a fractal function instead of the usual Fourier series and compute its PSD using the Wigner-Ville theorem which is applicable to nonstationary signals. We call the resulting power spectrum “fractum” to differentiate it from the power spectrum of stationary linear signal.

The first step is to test whether the signal is chaotic in nature.This can be done by computing Lyapunov exponent.

Lyapunov exponent can be defined as:

$$\lambda(x_o)=\ell im_{n-\infty} \frac{1}{n} \ell n \left| \frac{df^{n(x)}}{dx} \right|_{x_o} \Big| \qquad (2)$$

where $f(x_n)= x_{n+1}$ = one dimensional map. A positive Lyapunov exponent will confirm that the signal is chaotic. A chaotic signal will have fractal characteristics. Hence the next step is to compute its fractal dimension. We choose the Hausdorff definition of fractal dimension which gives

$$D= \frac{\ell nN}{\ell n(1/r)} \qquad (3)$$

where N= number of self-similar parts and r= size of ruler.

The next step is to compute the PSD using the Wigner-Ville theorem. Before that we have to calculate the covariance function which is the point autocorrelation function defined by

$$R_W(\tau)=\int_{-\infty}^{\infty} V(t)\,V(t+\tau)dt \qquad (4)$$

where $V=V_{MW}(t)$ for our case.

The Wigner-Ville spectrum of a nonstationary process $f(t)$ with covariance function $R_f(t,s)$ is given by

$$W_f(t,\omega)=\int_{-\infty}^{\infty} R_f(t+\frac{\tau}{2},t-\frac{\tau}{2})\,e^{-j\omega\tau}d\tau \qquad (5)$$

The required PSD,the Wigner-Ville spectrum will be obtained by substituting (1) and (4) into (5).We call this resulting spectrum,the"fractum"because it represents the characteristics of a fractal signal without using the linear representation of Fourier series,

$$W_f(t,\omega)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} A_n R^{nH}\sin[2\pi r^{-n}(t+\frac{\tau}{2})+\Phi_n]\sum_{n=-\infty}^{\infty} A_n R^{nH}\sin[2\pi r^{-n}(t-\tau/2)]\,e^{-j\omega\tau}d\tau \qquad (6)$$

B. Radial basis function

Radial basis functions (RBF) can be used for extrapolation as well as interpolation and are attractive for nonlinear modeling such as chaotic modeling. We require that the interpolation be exact at the known data points. Then

$$y_i=\sum_{j=1}^{m} \lambda_j\,\phi(|\underline{x}_i-\underline{x}_j|) \qquad (7)$$

for i=1,....,m,where ϕ is the radial basis function.Writing the matrix Φ with element

$$\Phi_{ij}=\phi(|\underline{x}_i-\underline{x}_j|) \qquad (8)$$

we can rewrite (7) as in linear equations in m unknowns:

$$\underline{\Phi}\,\underline{\lambda}=y \qquad (9)$$

where y and λ are the vectors with elements y_i and λ_i ,i=1,..,m.Everything is known except $\lambda_1 ....\lambda_m$.Solving (9) therefore, determines f completely where f is the radial basis approximations to g and is defined by

$$f(\underline{x})=\sum_{i=1}^{m} \lambda_i\,\phi(|\underline{x}-\underline{x}_i|) \qquad (10)$$

and suppose that from experiment,values $y_1 ...y_m$ of y have been found at $x_1 ...x_m$ Then we have $y_i=g(x_i)$ for i=....,m.

Computationally,the significant part of the problem is that of solving the linear equations (9) for λ .The size of the matrix Φ is the number m of data points and so the computational effort which is of order ...., may be large.Fortunately,this calculation is only performed once for a particular set of data points and Φ.The work involved to interpolate for any given point is then considerably less,of order m.

For our purposes,we shall assume that the number of data points required is sufficiently small (up to a few hundred with current workstations) that numerical and computational difficulties do not nominate the problem.

The next step is to introduce fractal and chaotic characteristics into radial basis function.To start with, the chaotic signal usually takes the form of a time series.To construct a dynamical model from a time series,we apply the phase space reconstruction technique to the chaotic data sequence. The general technique of this approach is to generate several different scalar signals $v_k(t)$ from the original v(t) in such a way as to reconstruct an m-dimensional space where,under some conditions,we can obtain a good representations of the attractor of the dynamical system.

The easiest and most popular way to do that is to use time delays.We write

$$v_k(t)=v(t+(k-1)\tau),k=1,..,m \quad (11)$$

where τ is the time delay.In this manner,an m dimensional signal is generated,which can be represented by the vector

$$\underline{x}(t)=(x_1(t),x_2(t),...,x_m(t)) \quad (12)$$

Note that,on varying the set of variables which can be constructed from x(t),we get in principle the same geometric information. If d is the dimension of manifold containing the attractor,Takens showed that m=2d is sufficient to embed the attractor by the Whitney embedding theorem.

Next we try to determine the possibility of a nonlinear model for chaotic noise and vibration signals by using a more rigorous analysis,i.e.fractal dimension.Measuring dynamical invariants such as fractal dimension and Lyapunov exponent has been a widely used approach in detecting chaos. The correlation dimension definition of fractal dimension is used :

$$d_c = \lim_{r\to o} \frac{\ell n[c(r)]}{\ell nr} \quad (13)$$

where c (r) = cumulative correlation. The determination of the correlation dimension is usually found by plotting c(r) versus r on a ℓn-ℓn graph for different values of the embedding dimension,m.

Having measured the correlation dimension we next assume that the dynamics can be written as a map in the form

$$v(t+\tau)= F(v(t)) \quad (14)$$

where the current state is v(t) and v(t+τ) is a future state.Since the only new component in vector $\underline{v}(t+\tau)$ is the point $\underline{v}(t+\tau)$,the dynamical system(14) is equivalent to the problem of prediction of v(t+τ) from the vector $\underline{v}(t)$ .That is

$$v(t+\tau) =\phi[v(t)] \quad (15)$$

To reconstruct the dynamical system (14) we need to produce $\hat{v}(t+\tau)$ of $v(t+\tau)$. In other words,we need to approximate the mapping ϕ by an approximation $\hat{\phi}$ and we use the radial basis function as an approximation for ϕ.Hence the radial basis function is determined within the framework of a phase space reconstruction for chaotic time series.

To use the Wigner-Ville theorem,we have to calculate the 2 point autocorrelation function for the radial basis approximation in (10),bearing in mind that the x's are functions of time.The Wigner-Ville theorem will give us the fractum.

HIGHER ORDER STATISTICS

The technique of higher order statistics (HOS) is used here to process nonlinear noise and vibration signals.The bispectrum is used.It is the simplest form of the higher order spectrum.It is a nonlinear generalisation of the spectral approach to linear time series analysis.Here we review the method of estimation of bispectrum and study possible applications to non-Gaussian signals such as chaotic signals.

The bispectrum of a signal is defined as the Fourier transform of its third-order correlation function.That is,

$$B(f_1,f_2)=\sum_m\sum_n C_{xxx}(m,n)\exp\{-2\pi i f_1 n-2\pi i f_2 m\} \quad (16a)$$

where $C_{xxx}=E[X(t+n)X(t+m)X(t)]$ (16b)

The bispectrum of the stationary process {X(t)} can be consistently estimated using a sample{X(o),X(1),.....,X(N-1)}as follows[2]. Let

$$F_x(j/N,k/N)=N^{-1} A_N(j/N)\, A_N(k/N)\, A_N^*[(j+k)/N] \quad (17)$$

where j and k are integers and

$$A_N(j/N)=\sum_{t=o}^{N-1} X(t)\exp\left(\frac{-i2\pi jt}{N}\right) \quad (18)$$

Here $A_N(o)$ is set to zero,this is equivalent to subtracting out the sample mean of {X(t)}. $F_x(j/N,k/N)$ is an estimator of the bispectrum of {X(t)} at frequency pair (j/N,k/N).However,it must be smoothed,i.e. averaged over adjacent frequency pairs,in order to obtain the consistent estimator $\hat{B}_\chi(g_m,g_n)$:

$$\hat{B}_\chi(g_m,g_n)=M^{-2}\sum_{j=(m-1)M}^{nM-1}\;\sum_{k=(n-1)M}^{nM-1} F_\chi\left(\frac{j}{k},\frac{k}{N}\right) \quad (19)$$

where $g_j=(2j-1)M/(2N)$ (20)

Now, $\hat{B}_\chi(g_m,g_n)$ is the average value of F(j/N,k/N) over a square, a M^2 points where the centres of the squares are defined by the lattce L=[(2m-1)M/2,(2n-1)M/2:m= 1,...,n,and m ≤ N/2M-N/2+3/4] in the principal domain

This averaging procedure is precisely analogous to smoothing the periodogram to obtain a consistent estimator of the spectrum.As in the averaging procedure, smoothing reduces the sampling variance of the cost of increasing the finite sample bias. It can be shown that consistency requires that M be an integer exceeding $N^{\frac{1}{2}}$ and

$$\gamma_{m,n}=\frac{\hat{B}_x(g_m,g_n)}{[N/M^2]^{\frac{1}{2}}\,\hat{s}_x(g_m)\hat{s}_x(g_n)\hat{s}_x(g_n+g_m)]^{\frac{1}{2}}} \quad (21)$$

where $\hat{S}_x$ is the usual (smoothed) estimator of the power spectrum of {X(t)}

The bispectrum is being computed for each record of aerodynamic noise according to (19) and smoothed over adjacent frequency pairs with a square smoothing window five samples on aside.The bispectrum will be normalized by the smoothed averaged power spectrum according to (21).

COMPUTATION OF BISPECTRUM FOR CHAOTIC AERODYNAMIC NOISE

In order to carry out the computation of the bispectrum for the chaotic aerodynamic noise,we have to represent the chaotic noise by Logistic Map,a wellknown chaotic model.Let the series be generated from the model x(t+1) = ax(t)[1-x(t)],(t= 1,2,...500). Interesting trajectories can be observed when 3<a>4.Unstable trajectories (or chaos) occur when a ≥ 3.5. In order to observe this,the bispectrum[3] has been calculated for various values of a using the product windows with the truncation point M=32. The modules of the bispectrum | f(ω,ω) | for several values of a from 3.0 to 3.9 are plotted in Fig1.For values of a, 3 ≤ a ≤ 3.4, the values of the modulus of the bispectrum are very,very small. The values are smaller than 10^{-7} (see Figs 2 and 3) Though a ridge is found along the line $\omega_1+\omega_2=\pi$ in the bispectra,they are not significant because their values are very small.When a=3.5,these are three dominant peaks:one at the frequency corresponding to the period-four units.This is when period doubling starts.The bispectral values obtained when a> 3.5 are several times larger than the corresponding values when a< 3.5.It is also informative to compare those values with the values obtained for a< 3.4.These are several times higher.As the values of a increases,several ridges can be found in the modulus of the bispectrum,a phenomenon observed consistently in chaotic models.

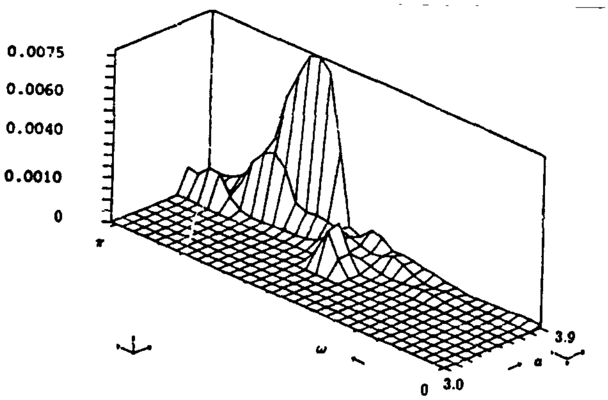

FIGURE 1 Plot of $|f(\omega,\omega)|$ against a of the logistic map. (from T.S.Rao[3])

EST.MODULUS OF BISPECTRUM

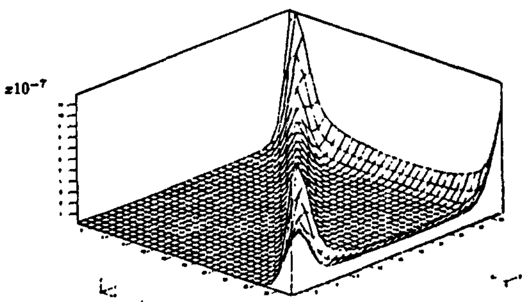

FIGURE 2 Bispectrum of sample (logistic) with a = 3.0. (from T.S.Rao[3])

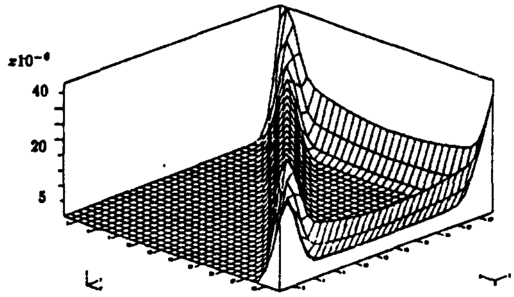

FIGURE 3 Bispectrum of sample (logistic) with a = 3.1. (from T.S.Rao[3])

APPLICATION OF BILINEAR MODEL TO NONLINEAR NOISE AND VIBRATION SIGNAL PROCESSING

If we find that the noise or vibration signal is nonlinear,it is important to see whether we can find a finite-parameter nonlinear model to describe the series.One such model is a bilinear model[3] ,whose analytic properties have been extensively studied.Let x (t) satisfy the difference equation

$$x(t)+\sum_{j=1}^{p} a_j\, x(t-j) = e(t) + \sum_{j=1}^{q} b_j\, e(t-j) + \sum_{i=1}^{P}\sum_{j=1}^{Q} a_{ij}\, x(t-i)e(t-j) \qquad (22)$$

The above model is called the bilinear model and is denoted by (p,q,P,Q).It is linear in x(t),e(t) but not jointly.For p=P,q=o,Q=1,Subba Rao[3] has shown that one can write an equivalent state-space form and then it is easy to evaluate the moments of the process x(t).The solution of the equation can be written in the form of Volterra series.It is observed that when the coefficients of the nonlinear part of the model tend towards the nonstationary region, the trajectories generated by these bilinear models may produce behaviour similar to chaotic models.

To apply the bilinear models to nonlinear noise and vibration signal processing,we have to consider certain special cases of the bilinear model.One is the Exponential Auto-regressive model (EAM) introduced by Ozaki[4] in an attempt to construct time series models which reproduce certain features of nonlinear random vibration theory.

Nonlinear random vibration are typically described by second-order differential equations of the form

$$\ddot{x}(t)+f\{\dot{x}(t)\}+g\{x(t)\}=y(t) \qquad (23)$$

where f{.},the damping force and g{.},the restoring force are nonlinear functions and y(t) is a stochastic driving force input.
Two examples are :
(i) Duffing's equation

$$\ddot{x}(t)+c\dot{x}(t)+ax(t)+b\{x(t)\}^3=y(t) \qquad (24)$$

(ii) Van der POL's equation

$$\ddot{x}(t)+f\{\dot{x}(t)\}+ax(t)=y(t) \qquad (25)$$

The three important nonlinear features of nonlinear systems are :
(i) Amplitude-dependent frequency
(ii) Jump phenomena
(iii) Limit cycles

In order to construct time series models (in discrete time) which reproduce the effect of amplitude-dependent frequency, Ozaki[4] started by taking,an AR(2) model of the form

$$X_t - a_1 X_{t-1} - a_2 X_{t-2} = e_t \qquad (26)$$

and then allowed the coefficients a_1 ,a_2 to depend on X_{t-1}.Specifically,he proposed that the coefficients be made exponential functionals of X_{t-1}^2 i.e. take the form

$$a_1 = \phi_1 + \pi_1 \exp(-\gamma X_{t-1}^2) \qquad (27)$$

$$a_2 = \phi_2 + \pi_2 \exp(-\gamma X_{t-1}^2) \qquad (28)$$

With these values of a_1 and a_2,(26) is called a second-order exponential auto-regressive model.

If,we ignore the fact that a_1 and a_2 are functions of X_{t-1} and think of (26) as a linear model,then its resonant frequency will occur at the minimum of

$$|1 - a_1 e^{-i\omega} - a_2 e^{-2i\omega}|$$

and hence will change with the magnitude of $|X_{t-1}|$.In effect,we are assuming that locally X_t behaves as if it were generated by a linear AR(2) model with the coefficients a_1, a_2,frozen at the value which they attained at time t.

Note that,for large $|X_{t-1}|$, $a_1 \sim \phi_1$,$a_2 \sim \phi_2$ and for small $|X_{t-1}|$, $a_1 \sim \phi_1 + \pi_1$, $a_2 \sim \phi_2 + \pi_2$

Thus,the exponential AR model behaves rather like the threshold AR model,but here the coefficients change smoothly between the two extreme values.

In addition to generating "amplitude-dependent frequency"effects,the exponential AR model can also give rise to jump phenomena and limit cycle behaviours.It should be noted,however ,that the class of exponential AR models is not unique in these respects: threshold autoregressive models are also capable of generating amplitude-dependent frequency,jump phenomena,and limit cycles.

General Model

The second-order model (26) can be readily extended to a general-order model. Then,a kth-order exponential AR model is given by,

$$X_t = (\phi_1 + \pi_1 e^{-\gamma X_{t-1}^2})X_{t-1} + \ldots +$$

$$(\phi_k + \pi_k e^{-\gamma X_{t-1}^2})X_{t-k} + e_t \qquad (29)$$

Ozaki[4] has shown that necessary conditions for the existence of a limit cycle for (29) are :

(i) all the roots of

$$z^k - \phi_1 z^{k-1} \ldots - \phi_k = 0$$

lie inside the unit cycle,$|z| = 1$;and

(ii) some of the roots of

$$z^k - (\phi_1 + \pi_1)z^{k-1} \ldots (\phi_k + \pi_k) = o$$

lie outside the unit circle.

A sufficient condition for the existence of a limit cycle is then

(iii) $$\frac{1 - \sum_i \phi_i}{\sum_i \pi_i} > 1 \text{ or} < 0$$

The last condition is required to prevent the occurrence of a stable singular point.

CONCLUSIONS

With the increase in computing power and speed,nonlinear noise and vibration signal processing is no longer an academic exercise.In particular,the bispectrum computation appears to have commercial value.Real time bispectral analyses of machinery noise is capable to detect machine failure under operating conditions when conventional power spectral analysis could not distinguish the defects.The phase preservation in higher order statistical analysis also yields additional information on noise and vibration sources.

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