FIFTH INTERNATIONAL CONGRESS ON SOUND AND VIBRATION
DECEMBER 15-18, 1997
ADELAIDE, SOUTH AUSTRALIA

ANALYSIS OF COUPLED VIBRATION FOR THE ELASTICALLY SUPPORTED BEAM AND SPRING-MASS SYSTEM

Yun S. Ryu[*1], Chong D. Choi[*1], Hee B. Cho[*1]

[*1] : Advanced Research Lab., Hyundai Motor Company, Korea

ABSTRACT

The coupled vibration of a wheel-railway track system has been considered as that of a moving mass on a beam. In this paper, an analytical model is proposed to analyze the coupled vibration when a wheel travels on a railway track.

The railway track supported by sleepers is considered as a beam on Winkler's foundations, and the wheel traveling on railway track at constant speed is considered as a moving mass. Hertz's contact stiffness is assumed between the wheel and railway track.

Numerical results are compared with experimental ones to verify the validity of the numerical method. The numerical method is found to be efficient to analyze this system. Based on the numerical simulation, the appropriate analysis range of the beam model and the characteristics of coupled vibration are discussed.

1. INTRODUCTION

When a vehicle runs on railway track or steel bridge, the coupled vibration is occurred between the vehicle and steel bridge. This vibration problems can be treated as a coupled vibration between moving mass and beam, and the vibration system is very complicated(1). Because the causes of this coupled vibration are very complicated, traveling conditions are not always constant, and the characteristics of this vibration system continuously varies with contact point, the frequency analysis may not give a good result.

Recently, the modeling of the coupled vibration between vehicle and railway track is discussed, especially the effective length of railway track. It is proposed that railway track is modeled to an infinite beam and the analysis range must be determined by the effective static deflection(2).

In the case when railway track is modeled to an infinite beam, it is difficult to consider the frequency of the vibration system, in detail(3). By the experimental results obtained when vehicle runs on the railway track, it can be thought that the vibration of railway track must not be ignored. Therefore, it should be important to discuss the analysis range of railway track, especially length of finite beam.

In this paper, the railway track supported by sleepers is considered as a beam on Winkler's foundations, and the wheel traveling on it at constant speed is considered as a moving mass. Numerical results are compared with experimental ones to verify the validity of the numerical approaches.

2. ANALYTICAL MODEL AND GOVERNING EQUATIONS OF MOTION

2.1 ANALYTICAL MODEL

An analytical model of wheel-railway track system is shown in Fig.1. The railway track supported by sleepers is modeled to elastically supported finite beam. This beam has constant sectional area and the gravitational effect is ignored. Its bending stiffness, sectional area, mass per unit length, and frictional damping factor are $\mathbf{EI}$, $\mathbf{A}$, $\rho\mathbf{A}$, and $\mathbf{c}$, relatively. The supporting part's frictional damping factor and stiffness are $\mathbf{c_s}$ and $\mathbf{k_s}$. The wheel is modeled to moving mass $\mathbf{m}$ which runs at constant speed $\mathbf{V}$ on the elastically supported finite beam. The mass of boggie is included to this mass $\mathbf{m}$. Between wheel and railway track, Hertzian contact stiffness $\mathbf{k_w}$ and frictional damping factor $\mathbf{c_w}$ are considered. The Hertzian contact stiffness $\mathbf{k_w}$ is defined within the linear range as follows(4).

$$\mathbf{k_w} = \left(\frac{2}{\eta}\right)\left(2\mathbf{E}^{*2}\mathbf{R_e}\mathbf{P_0}\right)^{\frac{1}{3}} \tag{1}$$

where, $\mathbf{E}^{*2} = \mathbf{E}^2 / (1-\nu^2)$, $\mathbf{E}$ is Young's modulus, $\mathbf{P_0}$ is static load at contact point, $\mathbf{R_e}$ is effective radius of gyration at contact point, η is non-dimensional factor determined by $\mathbf{R_e}$, and ν is Poisson's ratio.

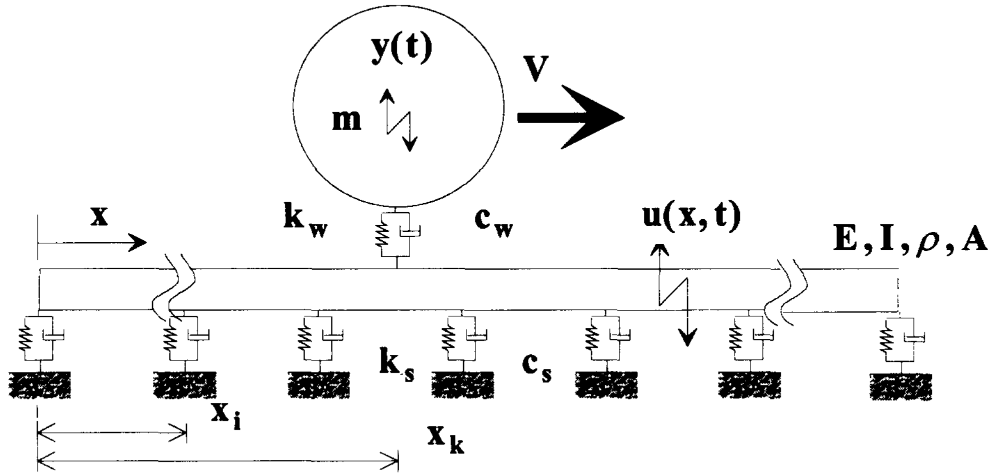

Fig. 1 The Analytical Model and Coordinate System

2.2 GOVERNING EQUATIONS OF MOTION

On the analytical model as shown in Fig.1, the deflection of beam $u(x,t)$ is defined from the static unloaded state, and the displacement of moving mass $y(t)$ is defined from the static balanced state. Where, x is the position of beam in axial direction and t is time. The equations of coupled vibration are shown as followings in the relations of power equilibrium between contact spring-moving mass and elastically supported finite beam(5).

$$EI\frac{\partial^4 u(x,t)}{\partial x^4} + \rho A\frac{\partial^2 u(x,t)}{\partial t^2} + c\frac{\partial u(x,t)}{\partial t} = G - P - M \qquad (2)$$

$$m\frac{d^2 y(t)}{dt^2} + c_w\frac{dy(t)}{dt} + k_w y(t) = Q \qquad (3)$$

Where, $$P = \sum_{i=1}^{n}\left\{c_s\frac{\partial u(x,t)}{\partial t} + k_s u(x,t)\right\}\delta(x - x_i) \qquad (4)$$

$$G = mg \qquad (5)$$

$$M = m\frac{d^2 y(t)}{dt^2}\delta(x - x_k) \qquad (6)$$

$$Q = \left\{c_w\frac{du(x,t)}{dt} + k_w u(x,t)\right\}\delta(x - x_k) \qquad (7)$$

$\mathbf{n}$, $\mathbf{x}_i$, $\mathbf{x}_k$ mean the number of sleepers, position of sleepers, and position on contact between wheel and railway track, each other.

Ignoring the effects of shear deflection and rotary inertia, the boundary condition is assumed to elastically supported both-ends, which is Euler's beam. Only lateral vibration of beam is considered.

Following matrices can be obtained from eq. (2) and (3).

$$[\mathbf{M}]\{\ddot{\mathbf{u}}\}+[\mathbf{C}]\{\dot{\mathbf{u}}\}+[\mathbf{K}]\{\mathbf{u}\}=\{\mathbf{G}-\mathbf{P}-\mathbf{M}\} \tag{8}$$

$$\mathbf{P}=\sum_{i=1}^{n}\langle[\mathbf{C}_s]\{\dot{\mathbf{u}}\}+[\mathbf{K}_s]\{\mathbf{u}\}\rangle\delta(\mathbf{x}-\mathbf{x}_i) \tag{9}$$

Where, $[\mathbf{M}]$, $[\mathbf{C}]$, $[\mathbf{K}]$ are mass matrix of railway track, damping and stiffness matrices. $[\mathbf{C}_s]$, $[\mathbf{K}_s]$ are damping and stiffness matrices of sleepers. From eq. (8) and (3) the equation of coupled vibration is obtained as follows.

$$\begin{bmatrix}[\mathbf{M}] & \\ & \mathbf{m}\end{bmatrix}\begin{Bmatrix}\{\ddot{\mathbf{u}}\}\\ \ddot{\mathbf{y}}\end{Bmatrix}+\begin{bmatrix}[\mathbf{C}] & \\ & \mathbf{c}_w\end{bmatrix}\begin{Bmatrix}\{\dot{\mathbf{u}}\}\\ \dot{\mathbf{y}}\end{Bmatrix}+\begin{bmatrix}[\mathbf{K}] & \\ & \mathbf{k}_w\end{bmatrix}\begin{Bmatrix}\{\mathbf{u}\}\\ \mathbf{y}\end{Bmatrix}=\begin{Bmatrix}\mathbf{G}-\mathbf{P}-\mathbf{M}\\ \mathbf{Q}\end{Bmatrix} \tag{10}$$

Through the numerical simulation, eq. (10) is solved by Duhamel's integration and Runge-kutta method.

3. NUMERICAL CALCULATIONS AND EXPERIMENTS

On the lateral vibration of railway track occurred when vehicle runs on it, the numerically simulated results are compared with the experimental ones, in order to verify the validity of the analytical modeling. The parameters used in the numerical simulations are similar to principal dimensions of the objective vehicle and railway track which are measured. The principal dimensions used to calculations are shown in Table 1.

The experimental results are shown in Fig. 2. In Fig. 2-1, the lateral acceleration of sleeper is shown during vehicle runs on railway track. Fig. 2-2 shows the lateral displacement of sleeper. From the above figures, the lateral vibration of sleeper becomes very dominant in the moment that vehicle passes just upon the sleeper. Fig. 2-2 shows the phenomenon that dual wheels pass the sleeper in order. By comparing Fig. 2-1 and Fig. 2-2, the maximum displacement of sleeper can occur when the wheel meets the sleeper. It should be noted that the maximum displacement could generate the maximum acceleration of the sleeper.

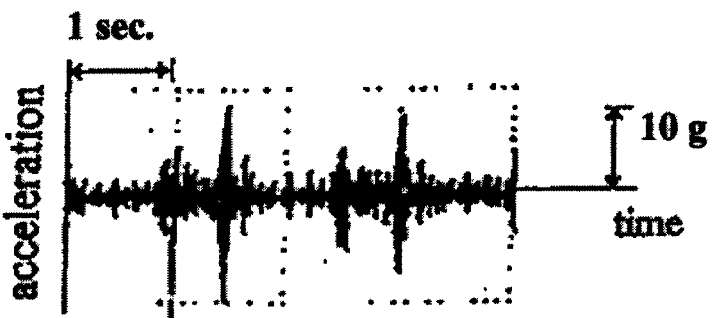

Fig. 2-1 Acceleration of Railway Track

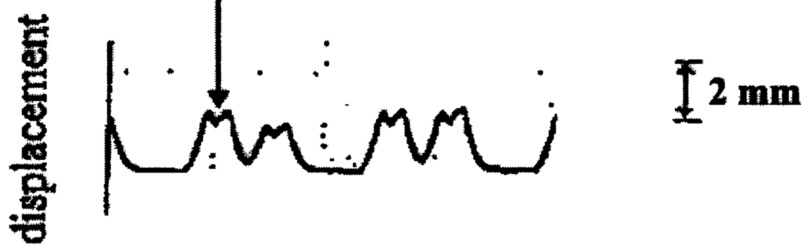

Fig. 2-2 Displacement of Railway Track

Fig. 2 The Vibration Responses of Sleeper (Measured)

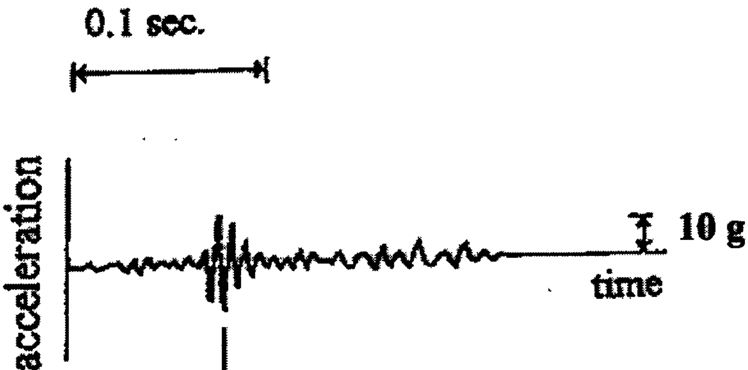

Fig. 3-1 Acceleration of Sleeper

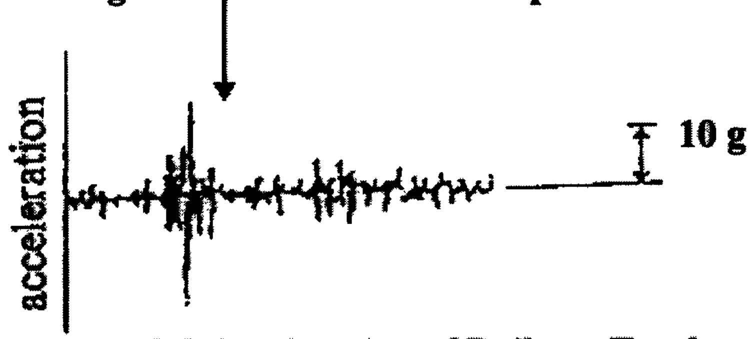

Fig. 3-2 Acceleration of Railway Track

Fig. 3 The Vibration Responses of Sleeper and Railway Track (Measured)

Table 1 The Principal Data for Calculation

EI	Flexural Rigidity of a Railway Track	4.03 MNm^2
$\mathbf{m_r}$	Mass of Railway Track	50.47 kg/m
A	Sectional Area of Railway Track	6.4×10^{-3} m^2
I	Moment of Inertia of Railway Track	1.96×10^{-5} m^4
l	Distance between Sleepers	0.625 m
$\mathbf{k_s}$	Stiffness of Sleeper	0.68 MN/m
$\mathbf{c_s}$	Damping Coefficient of Sleeper	5 kNsec./m
$\mathbf{k_w}$	Stiffness of Hertzian Contact Spring	2000 MN/m
$\mathbf{c_w}$	Damping Coefficient of Contact Spring	51 kNsec./m
m	Effective Mass of a Wheel	349 kg
V	Traveling Velocity	37 km/h

Fig.3 shows the lateral acceleration of sleeper and railway track system. From Fig. 3, just when the wheel meets sleeper, maximum vibration response will occur, but in the case of railway track, the maximum vibration response occurs before the wheel arrives on sleeper.

Fig. 4 shows the result of numerical simulation using the analytical model as shown in Fig. 1. The length of beam is 15 m, and the calculated lateral acceleration of beam just on the support is shown at the middle of a span (between supports). The dotted line in Fig. 4 means the time when moving mass passes on the elastic support. Comparing the experimental results as shown in Fig. 3-2, the maximum vibration amplitude of beam occurs when the moving mass is on between the middle of a span and forward elastic support, commonly.

In Fig. 5, the experimental result is shown, which is obtained when the vehicle runs on commercial line, and traveling speed is 37 km/h. It can be thought that the governing frequency components of railway track are contact frequency and sleeper passing frequency. This trend shows a good agreement between the calculated and measured result. Contact frequency caused by the contact stiffness between wheel and railway track, during vehicle running, can be varied on contact positions, that is, time varying frequency system. Sleeper passing frequency is caused by the different stiffness of railway track and sleeper, that is, the effect of an elastically supports. Generally, sleeper passing frequency is lower than contact one.

So many agreements can be found between the result of calculation and experiment, but the differences can, also. Because single wheel is considered in numerical simulation, but the measured results are obtained by multi-wheel vehicle, the difference of results may occur between numerical simulation and

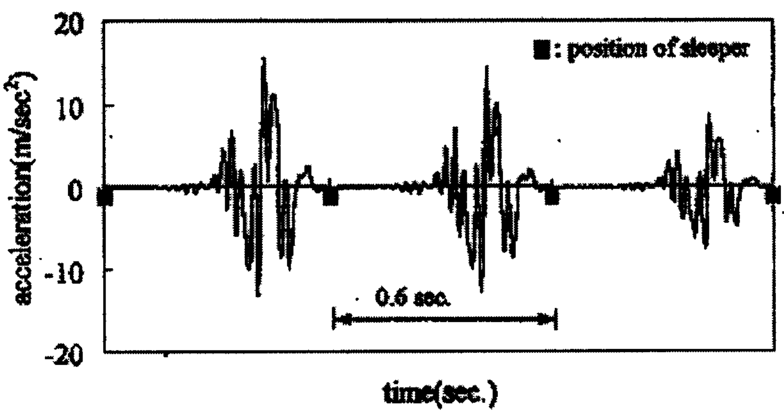

Fig. 4 The Acceleration of Railway Track for Single Wheel (Calculated)

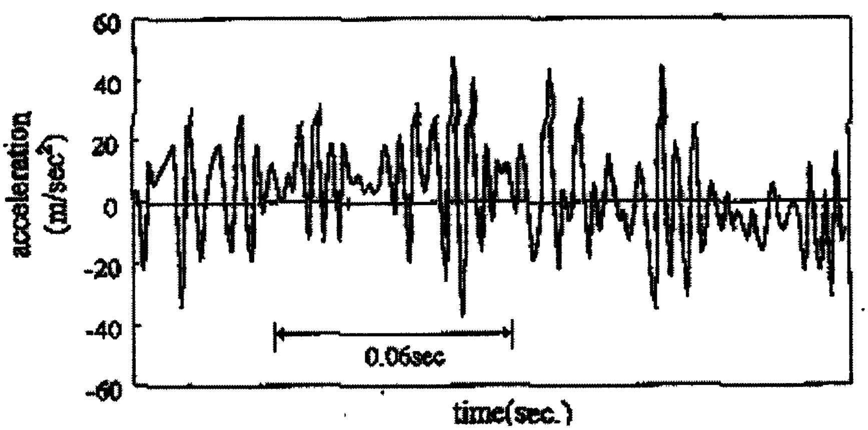

Fig. 5 The Acceleration of Railway Track for Multi-Wheels (Measured)

experimental one. Furthermore, the dynamic effects during vehicle traveling cannot be considered in numerical simulation. Therefore, it should be noted that it is effective to use the proposed analytical model in obtaining the characteristics of coupled vibration of running vehicle and railway track system.

4. DISCUSSIONS

As an example of numerical simulation for the analytical model, the vibration response of railway track is calculated using eq.(10) during vehicle running. On vehicle traveling the responses of (a) upon the sleeper, (b) middle of the sleeper , and (c) upon the forward sleeper are shown in Fig. 6. The dotted lines in Fig. 6 are corresponding time to the positions of sleepers as shown the above marked.

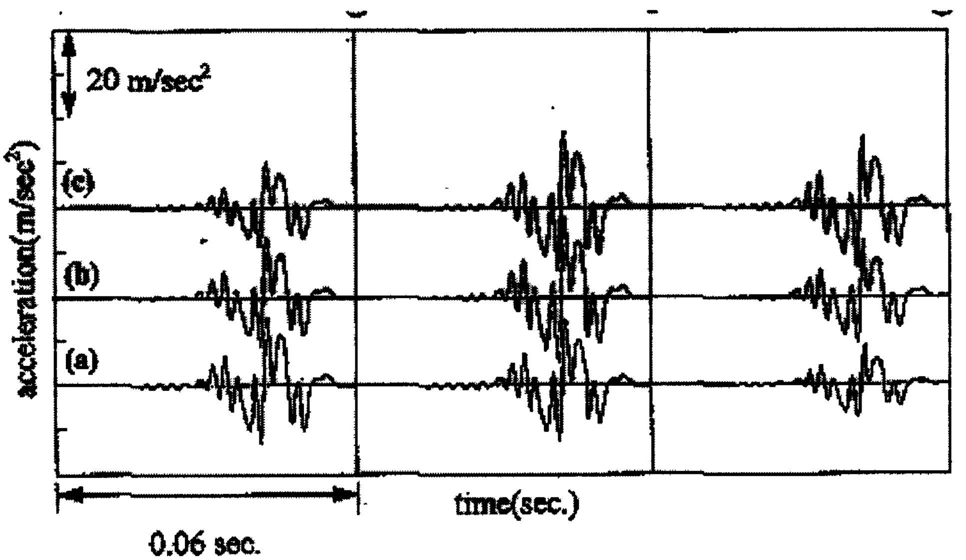

Fig. 6 The Vibration of Railway Track (Calculated)

It can be shown that the maximum amplitude of a span occurs after wheel passing in the middle of span. In addition, the maximum responses at each points simultaneously occur. The experimental results (ref. Fig. 2) may be explained from this numerical simulation using the proposed analytical model.

5. CONCLUSION

On the vehicle-railway track system, it should be thought that the more effective model is needed to analyze the coupled vibration. So, including the inertia effects of vehicle the analytical model is proposed to analyze the coupled vibration of vehicle-railway track system.

By comparing the numerical results of analytical model and the experimental ones, it should be noted that the vibration of railway track becomes dominant after vehicle passed the middle of span (between 2 sleepers). This phenomenon is independent to measuring points.

REFERENCES

1. Dahlberg, T., Vehicle-Bridge Interaction, J. of Veh. Sys., 13 (1984), 187-206
2. Clark, R.A., et al, An Investigation into the Dynamic Effects of Railway Vehicle Running on Corrugated Rail, J. IMechE, 24-2 (1982), 65-76
3. Duffy, D.G., The Response of an Infinite Railroad Track to a Moving Vibrating Mass, Trans. ASME, 57 (1990), 66-73
4. Roak, R.J. and Young W.C., Formulas for Stress and Strain, 5th ed., (1975), 517, McGraw-Hill.
5. Ryu, Y.S., et al, A Coupled Vibration Analysis Model of a Wheel-Railway Track System with Consideration of the Contact Stiffness, J. of JSME 62, no.603 (1996), 25-30