ABSTRACT
This paper describes the dynamic behaviour of a roller chain drive. A mechanical model of the chain and all other parts of the drive is presented. For a detailed analysis of the forces and the contact configuration each link, sprocket, shaft, and guide is treated as a separate body. All these bodies are specified with their exact contour. In order to verify the model a test bench has been built where all major excitations and replies of the system are measured. A method to measure chain forces is shown. A comparison between measurement and simulation for some results is given.

INTRODUCTION
In combustion engines either roller chain drives, belt drives or geared drives are used to set the cam shaft and accessory aggregates going. The characteristic features of roller chains are high solidity, stiffness, temperature stability and a long working life, while belts are known to break unscheduled. Every chain drive can consist of several single chain drives, which again consists of sprockets, pulleys, one-sided and two-sided guides, a tension device and chain strands.

Because of the large relative velocity not well designed drives suffer from high wear and ground noise. Moreover it affects the cyclic irregularity of the crank shaft and cam shaft, which again influences the gas exchange of the engine. GROESEL [7] and other authors studied the variation in speed and acceleration of a sprocket due to the polygonal effect. The basic work on natural frequencies of a chain span has been done by BINDER [1] and RACHNER [11] modeling the span as a series of rigid bodies. Among others FRITZER [3] researched the behaviour of the strands, modeled as a uniform string.

NAKANISHI and SHABANA [9] developed a very detailed chain model for large-scale tracked vehicles, regarding links, sprockets and dampers as separate bodies.

Seyfferth (1992), Pfeiffer and Glocker (1993) did the basic work on multi-body systems
with variant structure. They appointed a valid contact configuration and calculated the contact forces in this case. The impact equations for such systems are derived in Pfeiffer (1984) and Glocker and Pfeiffer (1995).

In this investigation all links are modeled as separate rigid bodies connected by revolute joints. Contacts to the sprockets are described by nonlinear force laws, regarding the exact tooth contour.

The model properties of all components of a roller chain drive are shown and the method to measure chain forces is displayed. The simulation results for a BMW 318 combustion engine will be compared with the corresponding measurement.

MODEL PROPERTIES

Excitations
The major excitations from the engine are the time dependent rotational speed of the crank shaft and the variable torque of the cam shaft. Moreover the discrete nature of chains enforces an extraordinary excitation at a high frequency, the so called polygonal frequency, that influences the vibration, noise spectrum and the wear of the engine. Every time a link gets into contact with a sprocket, a pulley, or a guide an impact occur which stimulate all eigenfrequencies of the chain drive.

Roller Chain
A roller chain consists of links with two pins and others with two bushings. They differ in their mass, their moment of inertia and their pitch. The nonlinear force element (Fig. 2) is used to connect two links together. The elasticity of the link is added to the elasticity of the force element. Therefore the link can be treated as a rigid body. The motion of a chain link is given by the translational vector $r_L$ and the rotational vector $\varphi_L$. For planar motion it is sufficient to consider two translatory degrees of freedom and on rotational degree of freedom. The vector of minimal coordinates is given as

$$q_L^T = (x_L, y_L, \varphi_L).$$

Figure 1: Kinematic of a Link

Figure 2: Force Element and Characteristic of Damping
Chain-Shaft
The item chain-shaft enfolds all shafts with a sprocket. It optionally consists of an elastic body, eccentricities like cams, plain bearings and sprockets. Depending on the frequency range the rigid degrees of freedom of the shaft have to be supplemented by the elastic DOF's according to Ritz:

\[ u_{el} = u^T(z)q_u(t) \]  
\[ u_{el} = v^T(z)q_v(t) \]  
\[ u_{el} = \varphi^T(z)q_\varphi(t) \]

The vector of the coordinates of the shaft can be written as:

\[ q^T_R = (q_u^T \ q_v^T \ q_\varphi^T) \]

Every sprocket is assigned to a chain-shaft. Chain-shafts which connect several single chain drives possess two or more sprockets. The exact tooth contact is considered, in order to give a detailed description of the contact configuration. Figure (3) illustrates the typical toothing of a sprocket with the contact areas tooth profile and seating curve. The contact contour is defined by circles. Using a toothing fixed coordinate system the centers of these circles can be easily determined.

\[ r_K(s) = \begin{pmatrix} x(s) \\ y(s) \end{pmatrix} \]

For chain shafts plain bearings can be defined. Forces on the shafts are calculated according to GLIENICKE [4] and SOMEYA [13]. The translatory degrees of freedom of a chain-shaft are determined by the number of bearings per shaft. If no bearings are given \( q_u \) and \( q_v \) of equation (4) vanish.

Chain Guide
One sided and double sided guides are applied to reduce the vibrations of the chain strands. Furthermore a tension guide is seated at the slack side, to give the chain drive a definite initial stress. The contour of a guide is composed of circles and lines. It is again described by the parameter \( s \). At the connection point the value of the first derivation of the two functions must be the same, to prevent unsteady velocities of the link. Curvatures of lines and circles differ from each other. Therefore contact forces are discontinuous at this point.

Figure 3: Toothing of a Sprocket

Figure 4: Model Properties of a Tension Guide
**Tension Device**

Tension devices applied in combustion engines usually consist of a spring, several oil chambers, valves, a liner and a husk. Due to their plurality every kind of tensioner has to be modeled separate. This should be done very accurately because of the great influence of the tensioner to the dynamic of the whole system chain drive.

The tension device of the BMW 318 has got only one pressure chamber and one spring. It can be specified as a force element consisting of a spring and a damper.

**Contact Model**

If a link comes in contact with a sprocket the meeting point is defined by the contour of the sprocket and by a contour circle with the diameter of a chain roller seated in the reference point $H_L$ (Fig.1). Modeling the contact between a link and a guide the link plate is the contact partner. Corresponding to the contact model above, a contour circle with the diameter of the plate width is used.

Experience shows that links can loose the contact. Therefore the model has to comply the following requests:

- All contacts are characterized by unilateral constraints, allowing a link to lift off the contour.
- An algorithm, describing the complementarity conditions of the contacts, is used to make the mutual dependence of the contact configuration apparent.

**MECHANICAL MODEL**

A detailed description of the mechanical model is described at FRITZ [2]. The following describes only the basic ideas. Starting with the NEWTON-EULER equation

$$J_T^T(\dot{p} - F^e) + J_R^T(L - M^e) = 0$$

the equation of motion for every body has to be calculated and transformed into the following complexion:

$$M\ddot{q} = h + \sum J^T F + W\lambda$$

$m$ represents the mass matrix, $q$ the minimal coordinates. The right hand side of the equation includes all forces and torques in different notations.

- Vector $h$ denotes forces and moments already formulated in the configuration space.
- $J^T F$ transpose forces and moments from the constraint to the configuration space.
- $W\lambda$ is the result of the segmentation of $F = n\lambda \Rightarrow J^T F = J^T n\lambda = w\lambda$. For several forces the amounts and vectors get combined to the vector $\lambda$ and the matrix $W$.

A valid contact configuration has to be determined by evaluating the position and it’s derivations for all bodies. The complementarity for unilateral constrains is given according to GLOCKER [6]:

$$\ddot{g}_{n,i} \geq 0 \quad ; \quad \lambda_i \geq 0 \quad ; \quad \ddot{g}_{n,i}\lambda_i = 0$$
MEASUREMENT METHODS

In the following the measurement of chain forces is described. For the description of the test rig as well as other measurements made see KELL [8].

Figure (6) shows the configuration of sensors and telemetry. The antenna has the same geometry like the chain line of the drive. Therefore the distance between the transmitter and the antenna is constant. The antenna is connected to a receiver, which also supplies the energy for the signal transmitter by frequency modulation. The transmitter is glued on a L-shaped link. The link in front of the transmitter bears four strain gauges aligned to a full bridge. At the outer side of the link the stresses resulting from bending and load add up. At the inner side load and bending compensate each other.

Therefore the best sensitivity of the full bridge can be reached by using strain gauges along the link orientation at the inner side of the link-plates and such across the link orientation at the outer side.

To insure a faultless transmission of the signal the necessary energy used at the moving parts must be much smaller than the energy supplied by the antenna. Therefore the following facts have to be kept in mind:

- The antenna must be exactly tuned on the environment of the test rig. This ensures that a maximum of energy will be transmitted to the signal transmitter. This can be obtained by parallel and serial switched condensers.

- The impedance of the strain gauges must be as large as possible (≥ 2000Ω) to minimize the current in the bridge.

- The full bridge must be balanced very exactly (approximately 0.1 mV). Otherwise the receiver will be out of the frequency range.

- All electrical components have to be NP0 Components to reach temperature stability.

Due to the size of the transmitter it is not possible to check the supplied voltage for the full bridge (5.9 V). Hence if the transmission is troubled the signal may runaway within a short spot.

RESULTS

A numerical simulation was performed on the chain drive shown in Figure (7) The model has got 320 degrees of freedom (106×3 links, 1 guide, 1 sprocket). The crank shaft is excited by a time dependent rotation speed, resulting from measurements. Hence this sprocket as well as the fixed guide on the tight strand has no degrees of freedom.

Verification

Figure (5) gives an exemplary comparison of measuring (left) and simulation (right). The first curve shows the rotational irregularity of the cam shaft the second the displacement of the tension guide in the time and frequency domain.

The frequency domain of the cam shaft irregularity shows a very good analogy for the important frequencies and their magnitudes. The BMW 318 is a four-cylinder-four-stroke combustion engine. Hence the second order of the crankshaft revolution and it’s multiples are the main frequencies.
The frequency domain of the displacement of the guide is again close to the measurement. Here again the second order is the main frequency. The time domain of the signal is somewhat different to the measurement, which is related to a different phase. Because every link is modeled as a separate body slightly wrong parameters of the damping characteristic or the mass summarize to a phase displacement in the positions.

**Chain Forces**

Figure (7) shows the graph of the forces in a link during one and a quarter withlow. The link passes four areas during one circulation. Zone (I) starts at the beginning of the tight strand. The respective magnitudes are dominated by changes in the highly dynamic torque of the cam shaft which oscillates with the second order of the crankshaft revolution. The link leaves the tight strand and enters the crank shaft (II). The magnitudes decreases rapidly until the loose strand is reached. Scope (III) is dominated by little changing forces at a low level. The force in this area mainly depends on the pre-load and the dynamic of the tensioner. After passing the cam shaft (IV) the link enters again the tight strand.

The measured and simulated chain forces show the same global manner of the magnitudes (7). In zone (III) are two essential differences between measurement and simulation transparent. The simulation shows the same maximum forces but smaller minimum forces and in the simulation the polygonal frequency is much more dominant than in the measurement. The loose strand is dominated by the tension device. The parameters of the model of the tension device have been determined without the measurement of the chain forces. A simulation with a different pre-load of the tensioner, should reach better results.
CONCLUSION

The presented model of a roller chain drive includes all items with their exact contour. Due to the size of the model many parameters have to be determined which is sometimes difficult. The measurement of chain forces gives already a good insight on the dynamic of the chain drive. However it’s difficult to get a stable transmission of the signal. Moreover a control of the supply voltage for the full bridge is not yet performed.

The verification of the model shows a good agreement between measurement and simulation. The chain forces shows differences while the link passes the loose strand, which can be solved by a better coordination of the parameters of the tension device.
Acknowledgment
The research work presented in this paper is supported by a contract with the FVV
(Forschungsvereinigung Verbrennungsmotoren FVV-Nr. 605660, Aif-Nr. 9172)

References

    1956.

    of the 1995 Design Engineering Technical Conference, Boston, USA, Sept 17-21 1995,

    Reihe 11, Nr. 176, VDI-Verlag, Düsseldorf, 1992.

    maschinen und deren Einfluß auf das Schwingungsverhalten eines einfachen Ro-
    tors, Forschungsbericht 2-211/12, Forschungsvereinigung Verbrennungskraftmaschi-


    Fortschrittberichte, Reihe 18, Nr. 182, VDI-Verlag, Düsseldorf, 1995.


    of the 2nd European Nonlinear Oscillation Conference, ENOC ’96, Prag, Sept. 9-13
    1996, S. 91-94


[10] T. Nakada, H. Tonosaki: Study of the excitation mechanism of half-order vi-
    brations in an inline 4-cylinder internal combustion engine, Nippon Kikai Gakkai
    Ronbunshu, C Hen/Transactions of the Japan Society of Mechanical Engineers, Part
    C v 60 n 577 Sep 1994. p2977-2983.

