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SOUND WAVE SCATTERING FROM AN AIR FILLED SHELL IN A LAYER OF LIQUID

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A new numerical modelling method for sound wave scattering by elastic objects in an acoustic waveguide is proposed. The method is applied to the case of sound propagation and scattering in a plane 2-dimensional layer of liquid 80 metres deep containing a cylindrical air filled shell of finite wall thickness and an outer shell radius of 8 metres. The incident wave is considered to be the lowest order propagating mode of the waveguide. The reflection coefficient is calculated in the frequency range between 5 and 300 Hz for several values of wall thickness and distance between the shell and the waveguide bottom. Pictorial output shows that the amount of acoustic energy reflected strongly depends upon all variables. Maxima and minima in the reflection coefficient associated with cut-on frequencies of the waveguide modes and structural resonances of the shell are identified. The calculations show that the conventional definition of the target strength is inappropriate.

1. INTRODUCTION.

This paper addresses the problem of sound wave scattering by elastic objects in waveguide systems. It contains a further extension of a method which combines the advantages of integral equations and eigenfunction methods. This method allows reduction of the boundary value problem to a system of integral-functional equations in terms of sources of scattered waves on the surface of the object. Solutions of the diffraction problem previously obtained by means of this method for the cases of sound scattering by one[1] and many[2] homogeneous parallel cylinders in a plane waveguide proved that the method allows calculation of the scattering matrix for reflected and transmitted waves with high accuracy. This paper presents a solution for the problem of sound wave diffraction by an air filled elastic shell in a plane waveguide. Results of numerical experiment are shown for several values of the shell thickness and its location in the waveguide.

2. STATEMENT OF THE PROBLEM.

Let us consider a planar waveguide filled with compressible perfect liquid of density, ρ, and sound speed, c, which contains an air filled elastic shell with outer radius, R_1, and inner radius R_2. The elastic material of the shell is described by density, ρ_s, and Lame coefficients, λ, μ. The gas inside the shell is characterised by density, ρ_g, and sound speed, c_g. A cross-section of the waveguide is shown in Fig.1.

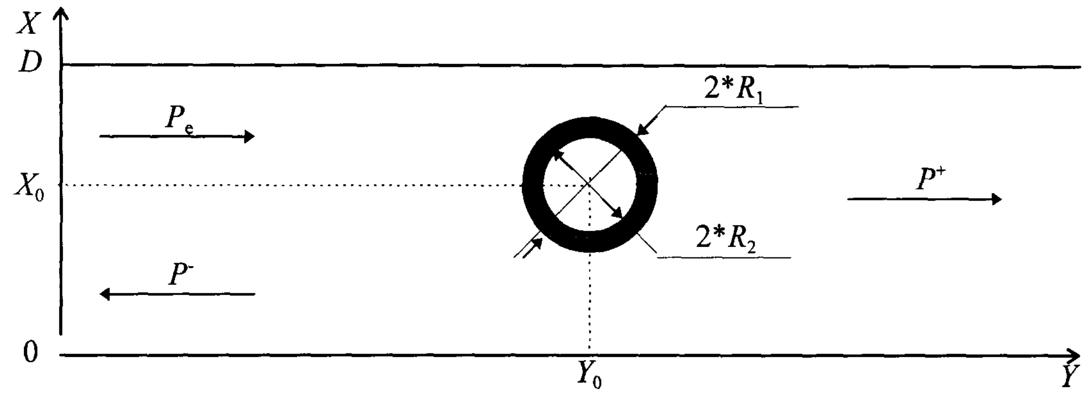

Fig. 1.

The incident wave P_e is emitted by a source located at $Y = -\infty$. The amplitude and phase of the incident wave are considered known and constant. The purpose is to calculate the amplitude and phase of the reflected and transmitted waves, P^- and P^+. Harmonic temporal dependence $\exp(-i\omega t)$ will be assumed. In the analysis below all variables having dimension of length are normalised on D/π.

The acoustic field in the waveguide will be described by a pressure field $P(x, y)$ that can be imagined as a sum of normal waveguide modes $P_n(x, y)$:

$$P(x, y) = P_e(x, y) + \sum_{n=0}^{\infty} P_n(x, y) \tag{1}$$

Elastic waves inside the wall of the shell can be described by the scalar potential $F(r, \varphi)$ and the z-component of the vector potential $\Phi(r, \varphi)$, so that the displacement vector $\Delta\boldsymbol{r}$ in the elastic material of the shell can be expressed as

$$\Delta\boldsymbol{r}(r, \varphi) = \nabla F(r, \varphi) + \nabla \times \Phi(r, \varphi) \tag{2}$$

where r and φ are polar coordinates with the origin in the centre of the shell, and z is the axial coordinate.

the acoustic field in the gas filled interior of the shell is described by the pressure field $P_g(r, \varphi)$.

The functions $P(x, y)$, $F(r, \varphi)$, $\Phi(r, \varphi)$ and $P_g(r, \varphi)$ are the functions to be determined. They must satisfy the following five conditions given by equations (3) to (7).

1. Helmholtz equations

$$(\Delta + k^2)P(x, y) = 0 \tag{3}$$

in the region $r > R_1$, $0 < x < \pi$, $-\infty < y < \infty$,

$$(\Delta + k_g^2)P_g(x, y) = 0$$

in the region $r < R_2, 0 \le \varphi \le 2\pi$,

$$(\Delta + k_1^2)F(x, y) = 0 \qquad (4)$$
$$(\Delta + k_t^2)\Phi(x, y) = 0$$

in the region $R_2 \le r \le R_1$, $0 \le \varphi \le 2\pi$, where Δ is the Laplace operator, $k = \omega/c$, $k_g = kc/c_g$, $k_1 = kc/c_1$, $k_t = kc/c_t$. Velocities of longitudinal and transversal waves c_1 and c_t in the elastic material are expressed as $c_1 = [(\lambda + 2\mu)/\rho]^{1/2}$, $c_t = [\mu/\rho]^{1/2}$.

2. Boundary conditions on the waveguide walls

$$[\alpha_0 + \beta_0 \frac{\partial}{\partial x}]P_{x=0} = 0 \qquad (5)$$

$$[\alpha_1 + \beta_1 \frac{\partial}{\partial x}]P_{x=\pi} = 0.$$

3. Conditions on the external shell boundary joining the acoustic pressure in the liquid $P(x, y)$ with the elastic waves in the shell. They correspond to:

- the absence of a tangential component of the stress tensor (the liquid is perfect),

$$\sigma_{r\varphi}|_{r=R_1} = \mu\left[\frac{\pi}{D}\right]^2\left[2\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial F}{\partial \varphi}\right) - k_t^2\Phi - 2\frac{\partial^2\Phi}{\partial r^2}\right]_{r=R_1} = 0 \qquad (6.1)$$

- the continuity of the normal component of the stress tensor,

$$\sigma_{rr}|_{r=R_1} = 2\mu\left[\frac{\pi}{D}\right]^2\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial F}{\partial \varphi}\right) - \frac{\lambda}{2\mu}k_t^2 F + \frac{\partial^2 F}{\partial r^2}\right]_{r=R_1} = -P|_{r=R_1} \qquad (6.2)$$

- the continuity of the normal component of the displacement vector,

$$u_r|_{r=R_1} = \left[\frac{\pi}{D}\right]^2\left[\frac{\partial F}{\partial r} + \frac{1}{r}\frac{\partial \Phi}{\partial \varphi}\right]_{r=R_1} = \left[\frac{1}{\rho(kc)^2}\frac{\partial P}{\partial r}\right]_{r=R_1} \qquad (6.3)$$

4. Conditions on the internal boundary between the shell and the gas have the same physical meaning and can be written as follows:

$$\mu\left[\frac{\pi}{D}\right]^2\left[2\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial F}{\partial \varphi}\right) - k_t^2\Phi - 2\frac{\partial^2\Phi}{\partial r^2}\right]_{r=R_2} = 0 \qquad (7.1)$$

$$2\mu\left[\frac{\pi}{D}\right]^2\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial F}{\partial \varphi}\right) - \frac{\lambda}{2\mu}k_t^2 F + \frac{\partial^2 F}{\partial r^2}\right]_{r=R_2} = -P_g|_{r=R_2} \qquad (7.2)$$

$$\left[\frac{\pi}{D}\right]^2\left[\frac{\partial F}{\partial r} + \frac{1}{r}\frac{\partial \Phi}{\partial \varphi}\right]_{r=R_2} = \left[\frac{1}{\rho(kc)^2}\frac{\partial P_g}{\partial r}\right]_{r=R_2} \qquad (7.3)$$

5. Absence of the sources of the scattered field at infinite distance from the shell.

3. METHOD OF SOLUTION.

The method is discussed in detail in articles by Belov at al.[1,2]. A solution of the boundary value problem is found in terms of a known field, $P_e(x,y)$. The unknown functions $P(x, y)$, $F(r, \varphi)$, $\Phi(r, \varphi)$ and $P_g(r, \varphi)$ can be represented as follows:

A) Pressure field in the liquid $P(x,y)$ is a sum of the incident field, $P_e(x,y)$ and the field scattered by the shell, $P_s(x,y)$:

$$P(x, y) = P_e(x, y) + P_s(x, y). \qquad (8)$$

The incident field may be modelled as a series of complete eigenfunctions of the boundary value problem determined by equations (3), (5):

$$P_e(x,y)=\sum_{n=0}^{\infty}A_n\exp(ig_n y)\cdot\sin(\eta_n x-\theta_n)=\sum_{m=-\infty}^{\infty}a_m J_m(kr)\exp(im\varphi) \tag{9}$$

where $g_n=(k_n^2-\eta_n^2)^{1/2}$, g_n and η_n are correspondingly longitudinal and transversal wavenumbers. The spectrum of eigenvalues η_n is determined by the dispersion equation,

$$\tan(\eta_n\pi)=\frac{\xi_1-\xi_0}{\xi_0\xi_1+\eta_n^2}\eta_n \tag{10}$$

$$\theta_n=\tan^{-1}\left(\frac{\eta_n}{\xi_0}\right),\quad \xi_0=\frac{\alpha_0}{\beta_0},\quad \xi_1=\frac{\alpha_1}{\beta_1}$$

The scattered field $P_S(x,y)$ can be found as a potential of a simple layer:

$$P_s(x,y)=\int_L G(x,y;x_0,y_0)\mu^0(x_0,y_0)dl_0 \tag{11}$$

In equation (11) $\mu^0(x_0,y_0)$ is an unknown function describing the field source distribution on the surface of the shell, (x_0,y_0) are the coordinates of the current integration point over the line L which is a circle of radius R_1 centred at the point $x=X_0$, $y=Y_0$, $dl_0=dx_0dy_0$, and (x,y) are the coordinates of an observation point.

The Green's function $G(x,y;x_0,y_0)$ is the field radiated by a point source in the waveguide and has the following form:

$$G(x,y;x_0,y_0)=\sum_{n=0}^{\infty}G_n(x,y;x_0,y_0) \tag{12}$$

$$G_n=\frac{i}{\gamma_n g_n}\sin(\eta_n x-\theta_n)\sin(\eta_n x_0-\theta_n)\begin{cases}\exp(ig_n(y-y_0)), & y\ge y_0\\ \exp(ig_n(y_0-y)), & y\le y_0\end{cases}$$

$$\gamma_n=\pi+\frac{\xi_1-\xi_0}{(\xi_0-\eta_n^2)(\xi_1-\eta_n^2)}$$

The pressure field $P_S(x,y)$, given by equation (11), satisfies the Helmholtz equation (3), the radiation conditions at $y\to\pm\infty$ and the boundary conditions (5).

The finiteness of the field $P_s(x,y)$ ensures that the function $\mu^0(x_0,y_0)$, describing the distribution of sources can be represented as a Fourier series,

$$\mu^0(\varphi)=\sum_{p=-\infty}^{\infty}b_p\exp(ip\varphi);\quad b_p=\frac{1}{2\pi}\int_0^{2\pi}\mu^0(\varphi)\exp(ip\varphi)d\varphi \tag{13}$$

B) The potentials $F(r,\varphi)$ and $\Phi(r,\varphi)$ in the shell will be found as a series of eigenfunctions of a cylinder,

$$F(r,\varphi)=\sum_{m=-\infty}^{\infty}(A_m^J J_m(k_l r)+A_m^Y Y_m(k_l r))\exp(im\varphi) \tag{14.1}$$

$$\Phi(r,\varphi)=\sum_{m=-\infty}^{\infty}(B_m^J J_m(k_t r)+B_m^Y Y_m(k_t r))\exp(im\varphi) \tag{14.2}$$

J_m and Y_m are Bessel functions of the first and second kind.

C) The representation of the field $P_g(x,y)$ in the gas filled interior of the shell has the following form,

$$P_g(r,\varphi) = \sum_{m=-\infty}^{\infty} C_m J_m(k_g r)\exp(im\varphi) \tag{15}$$

It is possible to reduce the problem to a system of linear equations with respect to Fourier coefficients b_p by substituting formulae (10), (11), (13) (14) and (15) into boundary conditions (6) and (7) on the outer and inner surfaces of the shell:

$$b_m + \sum_{p=-\infty}^{\infty} b_p M_{pm} = P_{pm} a_m \tag{16}$$

where $-\infty < m < \infty$, and

$$M_{pm} = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{2\pi}\left[L(\varphi,\varphi_0) - z_p K(\varphi,\varphi_0)\right]\exp(i(p\varphi_0 - m\varphi)d\varphi_0 d\varphi \tag{17}$$

$$K(\varphi,\varphi_0) = \left[G(x,y;x_0,y_0)\right]_{r=R_1}; \qquad L(\varphi,\varphi_0) = \left[\frac{\partial}{\partial r}G(x,y;x_0,y_0)\right]_{r=R_1}$$

$$x = X_0 + R_1 \sin\varphi; \quad y = Y_0 + R_1\cos\varphi$$

$$P_{pm} = -\left[kJ'_m(kR_1) + z_p J_p(kR_1)\right]\delta_{pm}$$

The impedance z_m on the outer shell boundary describes the relationships between pressure and its radial derivative on the boundary for every cylindrical mode and can be expressed as,

$$z_m = \frac{R_1\rho(kc)^2}{\mu}\frac{\left[A_m^J imJ_m(k_l R_1) + A_m^Y imY_m(k_l R_1) + B_m^J k_l R_1 J'_m(k_l R_1) + B_m^Y k_l R_1 Y'_m(k_l R_1)\right]}{\left[iA_m^J\phi_{1m}^J(k_l R_1) + iA_m^Y\phi_{1m}^Y(k_l R_1) + B_m^J\phi_{3m}^J(k_l R_1) + B_m^Y\phi_{3m}^Y(k_l R_1)\right]} \tag{18}$$

where the complex amplitudes $A_m^J, A_m^Y, B_m^J, B_m^Y$ describe elastic waves inside the shell (formulae (14)) and are defined by the following system of linear equations:

$$\begin{Vmatrix} \phi_{2m}^J(k_l R_1) & \phi_{2m}^Y(k_l R_1) & -i\phi_{1m}^J(k_l R_1) & -i\phi_{1m}^Y(k_l R_1) \\ \phi_{2m}^J(k_l R_2) & \phi_{2m}^Y(k_l R_2) & -i\phi_{1m}^J(k_l R_2) & -i\phi_{1m}^Y(k_l R_2) \\ imJ_m(k_l R_2) & imY_m(k_l R_2) & R_2 k_l J'_m(k_l R_2) & R_2 k_l Y'_m(k_l R_2) \\ i\phi_{1m}^J(k_l R_2) & i\phi_{1m}^Y(k_l R_2) & \phi_{3m}^J(k_l R_2) & \phi_{3m}^Y(k_l R_2) \end{Vmatrix} \cdot \begin{pmatrix} A_m^J \\ A_m^Y \\ B_m^J \\ B_m^Y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \dfrac{k_g R_2 J'_m(k_g R_2)}{\rho_g (k_g c_g)^2} \\ -R_2^2 J_m(k_g R_2) \end{pmatrix} \tag{19}$$

Auxiliary functions $\phi_m(x)$ are defined by the formulae,

$$\phi_{1m}^J(x) = 2m(xJ'_m(x) - J_m(x)); \qquad \phi_{1m}^Y(x) = 2m(xY'_m(x) - Y_m(x))$$

$$\phi_{2m}^J(x) = x^2(J_m(x) + 2J''_m(x)); \qquad \phi_{2m}^Y(x) = x^2(Y_m(x) + 2Y''_m(x))$$

$$\phi_{3m}^J(x) = 2x^2(J''_m(x) - \frac{\lambda}{2\mu}J_m(x)); \quad \phi_{3m}^Y(x) = 2x^2(Y''_m(x) - \frac{\lambda}{2\mu}Y_m(x))$$

Solution of the system (16) permits one to calculate the acoustic field $P_S(x,y)$ scattered by the elastic shell in the waveguide by formula (11) and, consequently, other parameters of the scattered field like reflection coefficient, modal structure of the scattered field etc. The matrix $\boldsymbol{M}$ of the system (16) is generally infinite. However, it is found that over an extensive range of input parameter values the absolute value of the matrix elements, M_{pm}, approaches 0 as the distance from the centre of the matrix increases. For example, in the case of scattering by a homogeneous cylinder considered in the article of Belov at al.[2] the maximum matrix index m_{max} equals 5 when the non-dimensional wavenumber $K = 2D/\lambda$ is in the interval $0.5 \le K \le 12.5$. In the case considered here m_{max} is 13 in the wavenumber range $0.5 \le K \le 32$. The matrix elements M_{pm} are negligible when $|m|$, $|p| > m_{max}$.

4. NUMERICAL EXPERIMENT.

The solution of the problem of sound wave scattering by an elastic shell in an acoustic waveguide is found for a waveguide with pressure release upper boundary and perfectly rigid bottom, e.g. for $\alpha_0 = \beta_1 = 0, \quad \alpha_1 = \beta_0 \neq 0$. The expression for the incident field takes the following form:

$$P_e(x.y) = \sum_{n=0}^{\infty} A_n \cos[(n+0.5)x]\exp(ig_n y) \tag{20}$$

where longitudinal wavenumbers $g_n = \left[k^2 - (n+0.5)^2\right]^{1/2}$.

The following parameter values were used in calculations:

- Waveguide depth $D = 80$ m;
- Parameters of the incident wave: $A_0 = 1, A_n = 0, n > 0$;
- Parameters of the liquid: $\rho = 1000$ kg/m^3, $c = 1493$ m/s;
- Parameters of the shell: $R_1 = 8$ m, $\rho_s = 7700$ m/s, $\lambda = 1.11 \times 10^{10}$ Pa, $\mu = 8 \times 10^{10}$ Pa;
- Parameters of the gas: $c_g = 330$ m/s, $\rho_g = 1.29$ kg/m^3;
- The frequency range: from 5 Hz to 300 Hz, corresponding to a non-dimensional wavenumber, K, range from 0.5 to 32 in steps $\Delta K = 0.1$.

The reflection coefficient R_c can be defined as $R_c = S_e / S_r$, where S_e is the total amount of acoustic energy in the incident wave moving through the vertical cross-section of the waveguide in unit time, and S_r is the amount of energy reflected in the backward direction from the shell. For the waveguide being considered R_c can be expressed as

$$R_c = \frac{\sum_{n=0}^{\infty} |A_n^r|^2 \operatorname{Re} g_n}{|A_0|^2 \operatorname{Re} g_0} \tag{21}$$

where A_0 is the coefficient in the first term of the series (20), and A_n^r are coefficients in a similar series for the reflected wave $P^-(x,y)$. It is necessary to note that only lower order modes of order $n < k - 0.5$ contribute to the energy flow in the waveguide. Higher order modes exponentially damp with increasing distance from the source and do not transmit acoustic energy.

Calculated dependencies of the reflection coefficient on the frequency of the incident wave for different values of the shell thickness $d = R_1 - R_2$ are depicted in Fig. 2. In the case of $d =$ 15 cm the dependencies are shown for 3 values of the height H above the bottom to the centre of the cylinder. Points on the graphs marked by symbols correspond to the resonance frequencies of the waveguide $k_n = n + 0.5$.

Analysis of the pictures shows the following:

I. Substantial changing of the reflected signal near the resonance frequencies k_n can be explained by a sharp increase of the density of acoustic energy because of multiple reflections at the waveguide walls. The pictures show that the resonance frequencies of the waveguide can be associated with both maxima and minima of the reflection coefficient, depending on the frequency, f. This fact can be explained by differences in phase between the shell oscillations and the acoustic wave scattered by the shell, which is reflected from the waveguide boundaries and returned back to the shell. The changes of phase are occuring at frequencies that are associated with resonances of the internal shell oscillations.

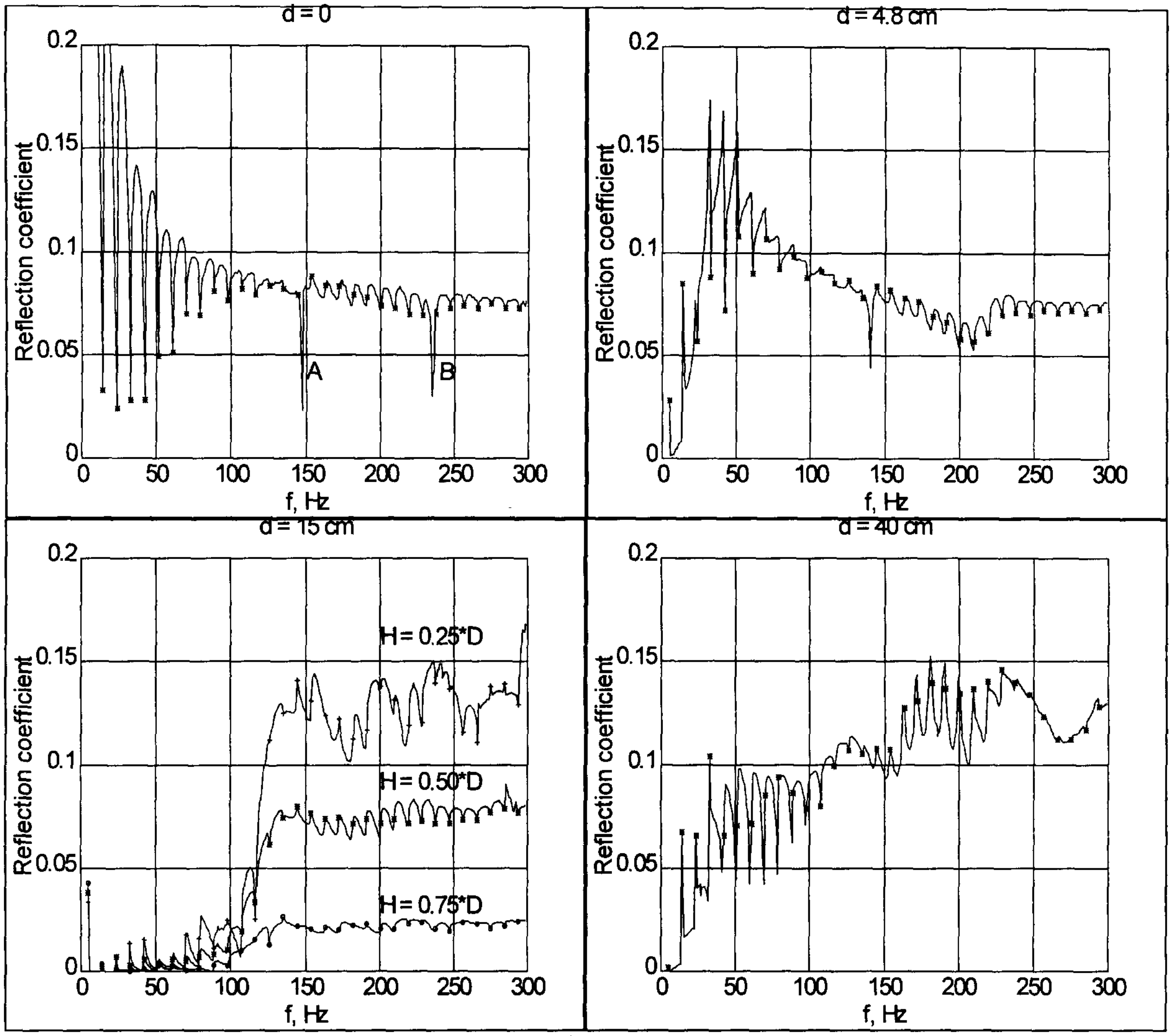

Fig. 2.

II. Two separate frequency ranges with different behaviour of the reflection coefficient can be identified at d = 0, 4.8 and 15 cm. In the cases of zero thickness ($d = 0$) and thin shell ($d = 4.8$ cm) in the frequency range below approximately 100 Hz the changes of K_r are very sharp. At these frequencies the shell diameter is less than half wavelength and reflection can be caused only by monopole oscillations of the shell. Near critical frequencies $f_n = k_n c/2\pi$ the acoustic pressure amplitude is high, which leads to effective excitation of the radial oscillations of the shell and, in turn, to the change of reflected signal. In the range above 100 Hz geometrical reflection plays a significant role and variations of the reflected signal are not so sharp.

When the shell is thick ($d = 15$ cm), reflection below 100 Hz is very small, because the stiffness of the elastic material of the wall is sufficient to prevent excitation of monopole oscillations of the gas in the shell. Geometrical reflection at $f > 100$ Hz is substantial in this case as well.

III.At zero wall thickness the reflection is extremely high at the very beginning of the frequency range near the resonance frequency of the waveguide mode of order $n = 0$. At such frequencies the wave vector of the incident wave is nearly vertical and interaction between the shell and the incident wave at a very sharp angle leads to almost total reflection. When the shell thickness is not zero total reflection does not exist because of energy transmission through the elastic shell.

IV. Two minima in the reflection coefficient indicated by A and B in the first picture ($d = 0$) at $f \approx 150$ and 230 Hz are very interesting features associated with the complex resonance oscillations of the shell itself. The first of them ($f = 150$ Hz) exists at $0 \le d \le 15$ cm while the other one nearly disappears at d = 4.8 cm. Obviously, resonance excitation of higher order modes in the gas filled interior of the shell plays a significant role in these phenomena.

V. The third picture (d = 15 cm) shows that the reflection coefficient in the waveguide strongly depends on the vertical location of the reflecting object. For the current waveguide parameters the shell reflects more energy when it is located close to the bottom which can be explained by the spatial configuration of the incident wave that has maximum pressure amplitude at the bottom and zero pressure amplitude at the upper boundary. The proximity of the rigid bottom also causes more substantial variation of the reflection coefficient due to a more efficient interaction between the shell and the boundary.

VI. When the shell is quite thick ($d = 40$ cm) multimode oscillations in the elastic material lead to a complex frequency dependence of the reflection coefficient.

5. TARGET STRENGTH AND ITS APPLICATION TO WAVEGUIDE SYSTEMS.

One of the parameters that is most often used in measuring efficiency of reflection of a plane acoustic wave by elastic objects in the ocean is the target strength TS, that may be defined as

$$TS = 10 \log \frac{I_r}{I_i}$$

where I_r is the intensity of the reflected wave referenced to 1 m distance from the acoustic centre of the target, and I_i is the incident wave intensity.[3]

However, as the above analysis shows, the use of TS is not appropriate for a shallow ocean when the acoustic field is clearly dependent upon the modal structure. First, the reflection depends not only on the geometrical shape of the object, but also on the location of the object in the waveguide and the resonance properties of the object. Second, the rate of amplitude decrease with increasing distance from the target, differs for different waveguide modes. Third, in the immediate proximity of the object the higher exponentially damping waveguide modes with imaginary g_n give significant contributions to the total acoustic field. And fourth, the reflection will depend on the location of the source, because the modal composition of the incident wave described by the Green's function (12) depends on the source coordinates.

For all these reasons the target strength does not characterise fully the reflective properties of the object inside the waveguide. In this case the object can be described by the scattering matrix containing coefficients of transformation of the modes in the incident wave to the modes in the reflected wave. The reflection coefficient similar to the one calculated above can also be used to describe overall reflection in the waveguide.

6. SUMMARY.

The solution of the problem of sound wave scattering by an elastic gas filled shell in a plane waveguide reveals the advantages of the method used which allows the calculation of all characteristics of the acoustic field in the waveguide with very high accuracy. The amplitude and phase of the acoustic waves can be found throughout the waveguide: in the liquid, the elastic shell, and the gas filled interior of the shell. Calculated dependencies of the reflection coefficient on the frequency of the incident wave are strongly affected by resonance properties of the waveguide and internal structural resonances of the shell. Dependence of the reflection coefficient on the vertical location of the shell also is important. It is shown that the use of the conventional definition of the target strength is not appropriate in waveguide systems.

7. ACKNOWLEDGMENTS.

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