FIFTH INTERNATIONAL CONGRESS ON SOUND AND VIBRATION
DECEMBER 15-18, 1997
ADELAIDE, SOUTH AUSTRALIA

EVALUATION OF THE EFFECT OF DAMPING TREATMENT USING APPROXIMATED VIBRATION MODE SHAPES

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ABSTRACT

Levels of structural vibration reduced by damping treatment can be evaluated by consumption of vibration energy derived from distributions of structural intensity and strain energy. However, it is difficult to estimate accurately structural intensity and strain energy in the structure with damping treatment. In this paper, therefore, we propose a method to estimate more accurately structural intensity and strain energy using the mode shapes which are approximated by superposition of the natural mode shapes of the structure without damping treatment. First, structural intensity and strain energy of the equivalent system of a beam with damping treatment and a damper are calculated by this method, and its accuracy is examined. Then, the effect of damping treatment are discussed based on the distributions of the structural intensity and the strain energy. Finally, validity of this method is examined.

1. INTRODUCTION

To get maximum reduction of levels of structural vibration and noise, damping treatment should be applied to a structure with consideration of the mode shapes of it. Since the reduction of the levels of structural vibration and noise depend on the mode shapes of the structure. It is necessary to design optimal layout of damping material for reducing vibration and noise of the structure. Furthermore, if the effect of damping treatment on the structure can be evaluated, further reduction of the levels of vibration and noise can be achieved by improving the layout of damping materials. The reduction of the levels of vibration and noise can be evaluated by the consumption of vibration energy derived from the distribution of structural intensity. And, if loss factors of the structure and the damping material are known, consumption of vibration energy can be estimated using the strain energy and loss factor. However, it is difficult to estimate accurately structural intensity and strain energy of structures with damping treatment as will be shown in sections 3.2 and 3.3. In this paper, therefore, we propose a method for estimating more accurately the structural intensity and the strain energy.

In this method, distributions of structural intensity and strain energy are estimated using the approximated mode shapes which are derived by the superposition of the natural mode shapes of the structure without damping treatment. First, structural intensity and strain energy of the equivalent system of a beam with damping treatment and a damper are calculated by this method, and its accuracy is examined. Then, effect of damping treatment are discussed based on the distributions of the structural intensity and the strain energy. Finally, validity of this method is examined.

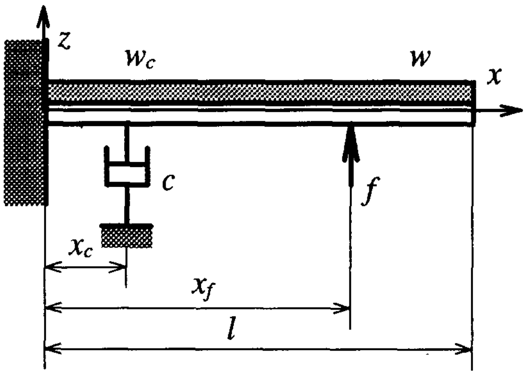


Fig. 1 A cantilever beam with damping treatment and a damper.

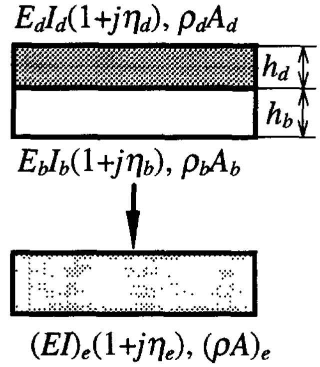


Fig. 2 A beam with damping treatment and an equivalent system.

2. THEORY

2.1 The Equivalent System and the Equation of Motion

For the purpose of basic examine of the method proposed in this paper to estimate the distributions of structural intensity and strain energy of the structure with damping treatment, we use a cantilever beam with damping treatment and a damper as shown in Fig.1. Where $f(t)$ is unit point force, c is viscous damping factor of the damper, l is length of the beam, x_c is position of the damper connected to the beam, x_f is position of the exciting point. The beam with damping treatment can replace an equivalent homogeneous beam as shown Fig.2. Where b is width, h is thickness, ρ is density, η is loss factor and E is Young's modulus, and suffix b indicates the beam, suffix d the damping material and suffix e the equivalent system. Flexural rigidity $(EI)_e$, mass per unit length $(\rho A)_e$ and loss factor η_e of the equivalent system are given by following equations[1]:

$$(EI)_e = \frac{1+e_2\left(4h_2+6h_2^2+4h_2^3\right)+e_2^2h_2^4}{1+e_2h_2}E_bI_b, \tag{1}$$

$$(\rho A)_e = (1+m_2h_2)\rho_bA_b, \tag{2}$$

$$\eta_e = \frac{e_2h_2\left(3+6h_2+4h_2^2+2e_2h_2^3+e_2^2h_2^4\right)}{(1+e_2h_2)\left\{1+e_2\left(4h_2+6h_2^2+4h_2^3\right)+e_2^2h_2^4\right\}}\eta_d, \tag{3}$$

where

$$e_2 = \frac{E_d}{E_b},\quad h_2 = \frac{h_d}{h_b},\quad \eta_b \ll \eta_d. \tag{4}$$

For complex modulus damping of the equivalent system, the Euler-Bernoulli equation of motion becomes following three equations[1]:

$$(\rho A)_e\frac{\partial^2 w}{\partial t^2}+(EI)_e(1+j\eta_e)\frac{\partial^4 w}{\partial x^4} = \delta(x-x_c)f_c+\delta\left(x-x_f\right)fe^{j\omega t}, \tag{5}$$

$$f_c = -c\frac{dw_c}{dt}, \tag{6}$$

$$w_c(t) = w(t,x_c), \tag{7}$$

with the boundary conditions

$$w(t,0) = \frac{\partial w(t,0)}{\partial x} = 0, \qquad \frac{\partial^2 w(t,l)}{\partial x^2} = \frac{\partial^3 w(t,l)}{\partial x^3} = 0, \tag{8}$$

where w is displacement of the beam, w_c is displacement of the beam at the point connected with the damper, f_c is internal force between the beam and the damper, and $\delta(x)$ is Dirac delta function.

2.2 Approximate Function for Vibration Mode Shape

Measured mode shape $W(x,f)$ of the structure with damping treatment is approximated by superposition of several natural mode shapes of it without damping treatment, as following equation:

$$W(x,f) \cong \sum_{i=1}^{n} a_i(f)\phi_i(x), \tag{9}$$

where $W(x,f)$ is complex mode shape at the frequency f, the function $\phi_i(x)$ is the ith normal function, and $a_i(f)$ is weight coefficient. The weight coefficient $a_i(f)$ is given by the following equation:

$$a_i(f) = \int_0^l W(x,f)\phi_i(x)dx. \tag{10}$$

The approximated mode shapes obtained from Eqs.(9) and (10) are continual functions. The rotation, moment and shear force, therefore, can be obtained as continual functions. Similarly, structural intensity and strain energy can be obtained as continual functions. If the mode shape $W(x,f)$ is given as discrete measured values, Eq.(10) should be integrated numerically. The optimal number of modes to superpose depends on the interval of measured points.

3. NUMERICAL STUDY

The Structural intensity and the strain energy of the beam and the damping sheet will be calculated to examine the validity of this method. The properties of the beam and the damping sheet are given as shown in Table 1, and viscous damping factor of the damper c is 1.0 Ns/m, position of the damper x_c is 0.1m and position of the exciting point x_f is 0.3 m. The natural circular frequencies of the beam without damping treatment are shown in Table 2.

Table 1 The properties of the beam and the damping sheet.

	Length l [mm]	Height h [mm]	Density ρ [kg/m^3]	Young' modulus E [GPa]	Loss factor η
Beam	400	3	7800	206	0
Damping sheet	400	3	1500	0.6	1

Table 2 The natural circular frequencies of the beam without damping treatment.

Mode No.	1	2	3	4	5	6
ω_n [rad/s]	130.1	815.2	2283	4473	7394	11050
Mode No.	7	8	9	10	11	12
ω_n [rad/s]	15430	20540	26380	32950	40250	48290

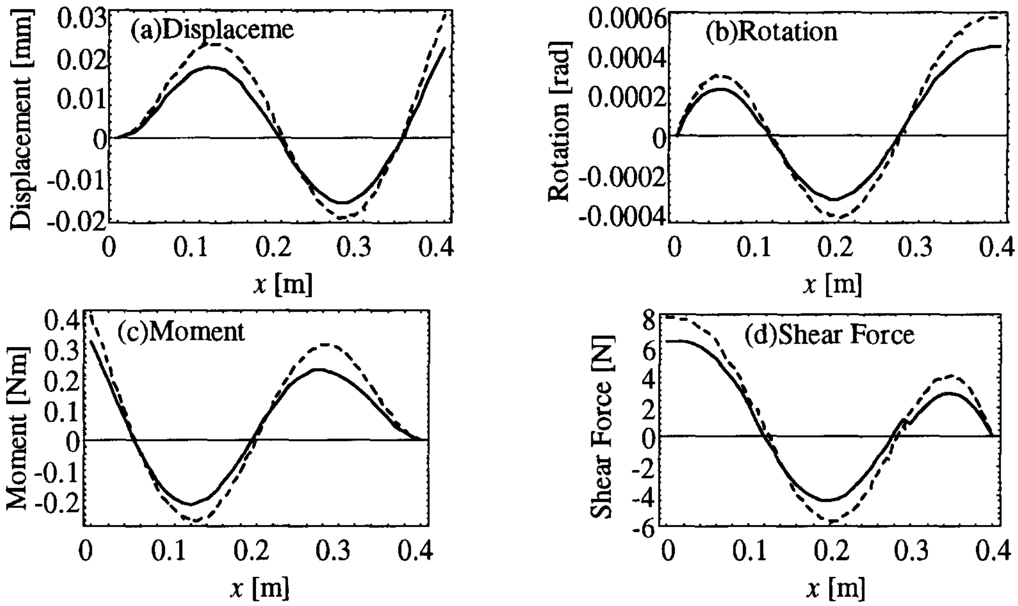
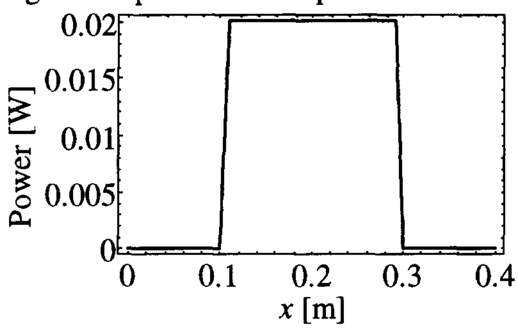
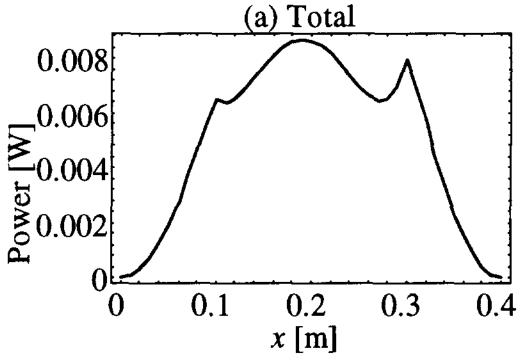


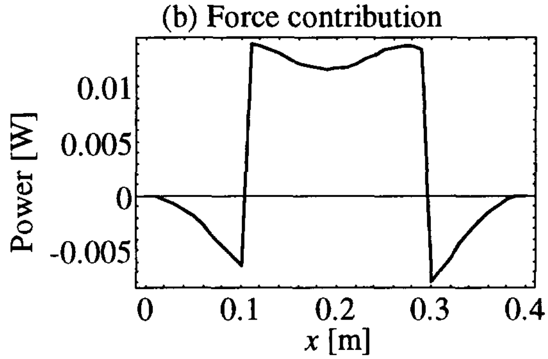
Fig.3 Complex mode shape of the beam with η_e=0 and ω=2300 rad/s.



(a) Total

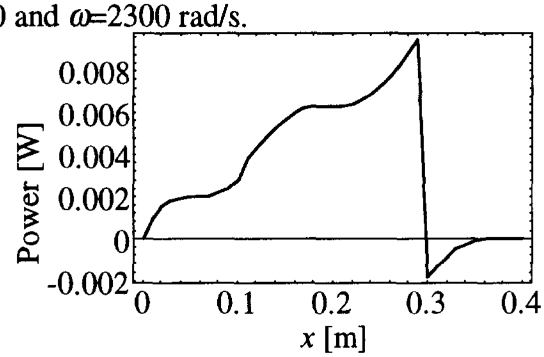


(b) Force contribution

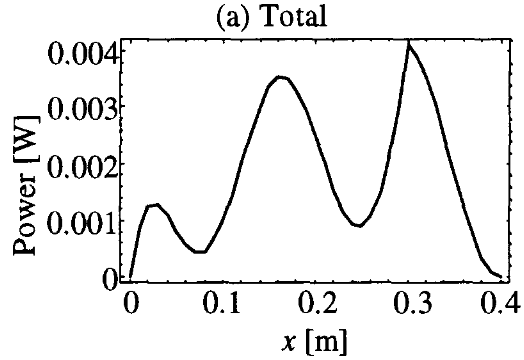


(c) Moment contribution

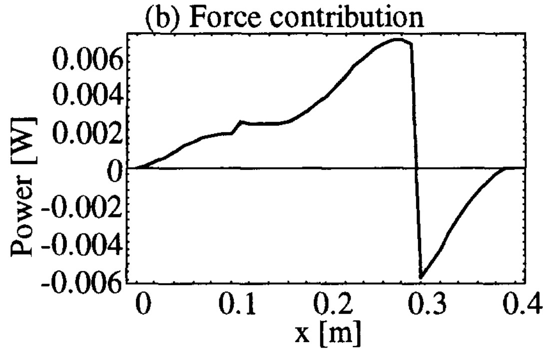
Fig.4 The vibration power of the beam without damping treatment, where η_e=0.



(a) Total



(b) Force contribution



(c) Moment contribution

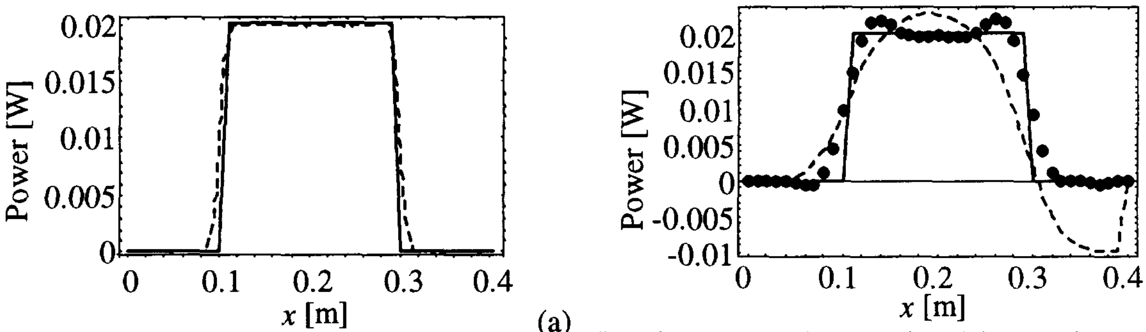
Fig. 5 The vibration power of the beam with damping treatment, where η_e=0.035.

3.1 Structural Intensity

Figures 3(a), 3(b),3(c), and 3(d) show displacement, rotation, moment, and shear force of the beam with η_e=0 and ω=2300 rad/s, respectively, where the solid line stands for the real part and the broken line for imaginary part. Figures 4(a), 4(b), and 4(c) show the total power obtained from the estimated structural intensity, force contribution, and moment contribution, respectively. It is evident from Fig. 4(a) that the entire vibration energy supplied by exciting force to the beam without damping treatment is consumed in the damper. Similarly, Figs.5 (a),5(b), and 5(c) show the power of the beam with damping treatment, where η_e=0.035 and ω=2200 rad/s. It can be seen from Fig.5 that reduction of the vibration energy corresponds with the mode shapes of the beam.

3.2 Simulation of Measuring Structural Intensity

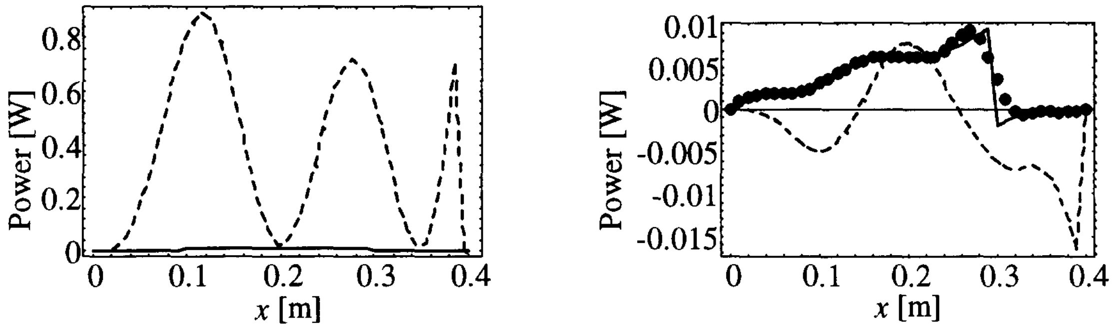
To discuss the accuracy of the structural intensity, complex mode shapes of the beam are calculated with intervals of $dx=l/40$=10 mm. Figures 6(a) and 6(b) show the power obtained from the structural intensity estimated using the data of complex mode shapes of the beam without damping treatment, where η_e=0 and ω=2300 rad/s. In the Fig.6(a), the broken line stands for the power estimated by 4-transducer array method[2,3] and the solid line for the theoretical curve. In the Fig.6(b), the dots stand for the power estimated by the method proposed in this paper, the broken line for the power estimated by 2-transducer array method[2,3] and the solid line for the theoretical curve. From Figs. 6(a) and 6(b), the following results became clear for the beam without damping treatment. The result estimated by 4-transducer array method corresponds almost with theoretical that. The error in the estimated by 2-transducer array method is large in the near-fields. The error in the estimation by the proposed



(a) 4-transducer array method

(b) The proposed method and 2-transducer array method

Fig.6 Estimation of the structural intensity with η_e=0 and ω=2300 rad/s,.



(a) 4-transducer array method

(b) The proposed method and 2-transducer array method

Fig.7 Estimation of the structural intensity with η_e=0.035 and ω=2200 rad/s.

method is large in the same near-fields of the exciting point and the connecting point to the damper. However, the error of estimation by the proposed method is less than that by 2-transducer array method.

Similarly, Figs. 7(a) and 7(b) show the estimations of structural intensity for the beam with damping treatment, where η_e=0.035 and ω=2200 rad/s. From Figs. 7(a) and 7(b), the following results became clear for the beam with damping treatment. The error in the estimation by 4-transducer array method is large. The error in the estimation by 2-transducer array method has increased. The accuracy of estimation by the proposed method is good, whether with or without damping treatment. It can be said from above results that the proposed method is valid for measurement of the structural intensity of the beam with damping treatment.

Figure 8 shows the estimations of the power when the number of superposed modes was changed, where η_e=0.035 and ω=2200 rad/s. Fig.8 indicates that the results are approximating to the theoretical curve with increasing of the number of superposed modes.

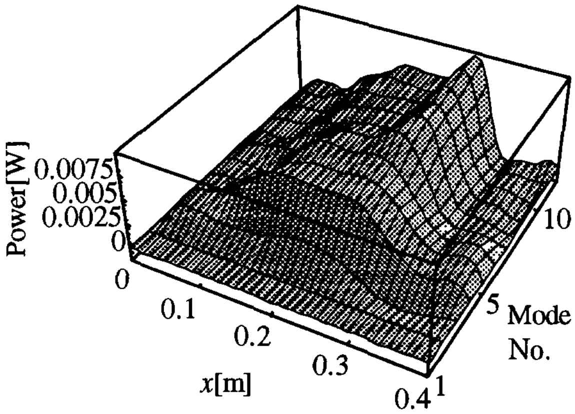
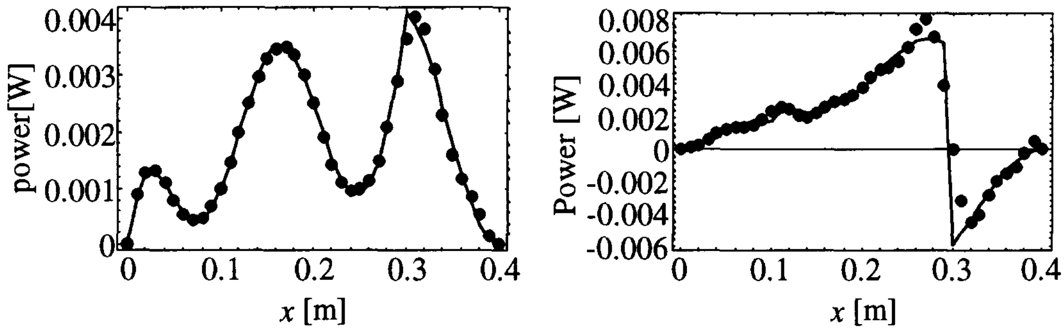


Fig. 8 The structural intensity for number of mode shapes superposed .



(a) Moment contribution (b) Force contribution

Fig. 9 The estimations of moment contribution and force one, where η_e=0.035.

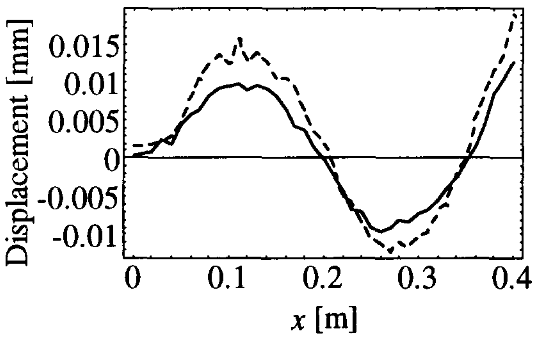


Fig. 10 Sample of displacement of the beam included 5% noise.

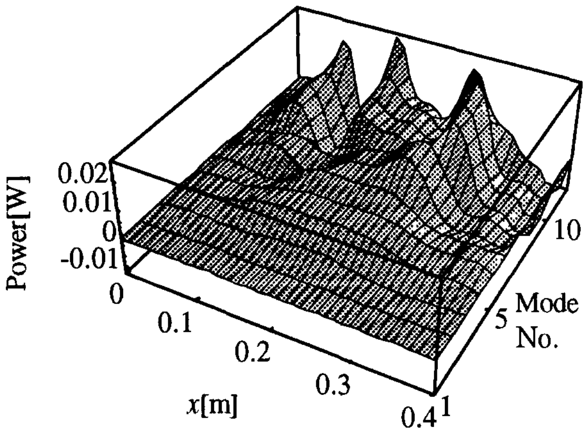


Fig.11. Estimations of the structural intensity with η_e=0.035 and ω=2200 rad/s.

Figures 9(a) and 9(b) show moment contribution and force contribution, where the dots stand for the contribution estimated by the proposed method and the solid line for theoretical curve. It can be said from Figs. 9(a) and 9(b) that the proposed method can estimate moment and force contributions of the structural intensity.

3.3 Influence of Error on the Estimation of Structural Intensity

In order to test the influence of error on the estimation of structural intensity, we use the data of complex mode shapes including errors whose standard deviation σ are 1% and 5% of the maximum amplitude of the mode shapes. Figure 10 shows a sample of the displacement including error with σ=5%, where the solid line stands for the real part and the broken line for the imaginary part. Figure 11 shows the estimation of the power when the number of superposed modes was changed, where σ=1%, η_e=0.035 and ω=2200 rad/s. As shown in Fig.11, the estimations of the structural intensity are scattered by superposition up to the fifth mode. It can be considered from Fig.11 that optimal number of mode is n=5. Figure 12(a) shows the structural intensity estimated from the data including errors with σ=1%, n=5, η_e=0.035 and ω=2200 rad/s, where the dots stand for the proposed method, the broken line for 2-transducer array method and the solid line for the theoretical curve. Figure 12(b) shows similarly the estimation with σ=5% and n=5. It is seen from Figs. 12(a) and 12(b) that The error of the estimation by 2-transducer array method increases, and the error by the proposed method is small, except near-field of exciting point. As a result, It can be said that the proposed method is possible to estimate the structural intensity for the measurement data including errors, except near-field of exciting point.

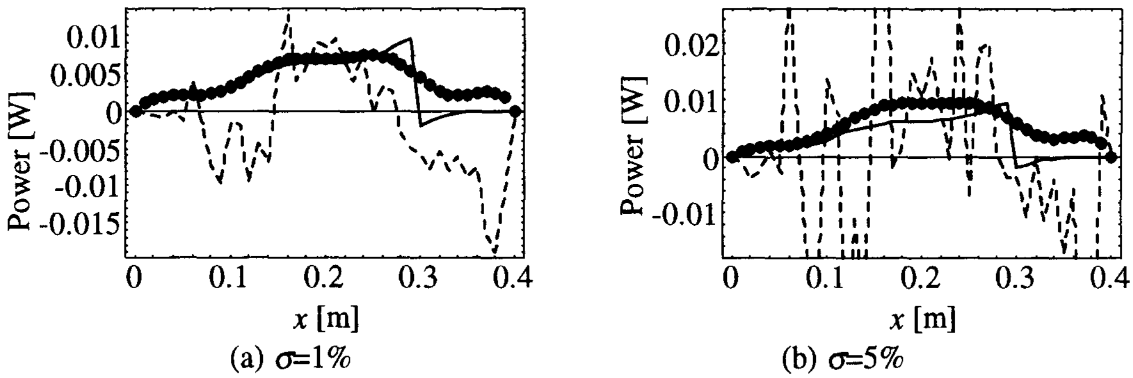


Fig.12 The vibration power estimated from the data including errors with η_e=0.035.

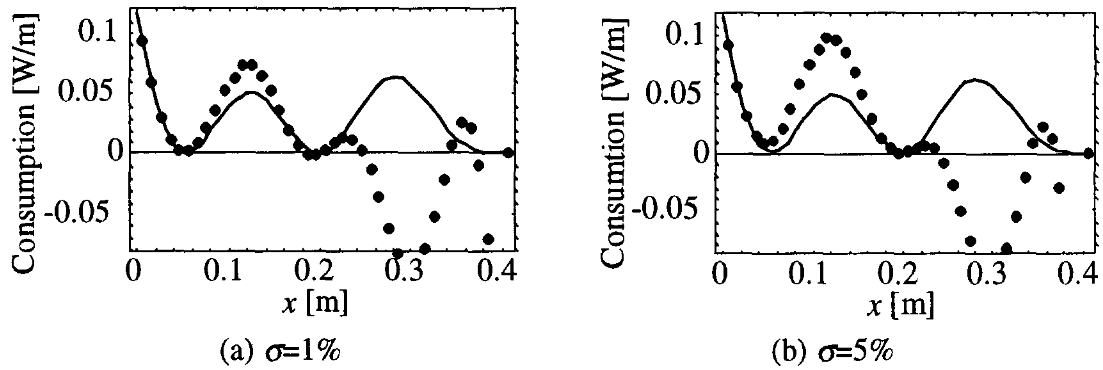
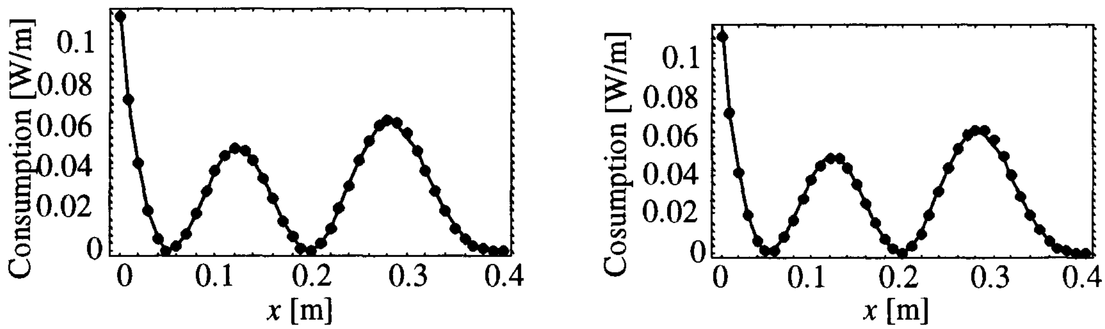


Fig.13 Estimation of consumption of the vibration power per unit length.

3.4 Estimation of the Effect of Damping Treatment

Figures 13(a) and 13(b) show estimation of the consumption of the vibration power per unit length with σ=0% and 5%, respectively, where the solid line stands for the theoretical curve and the dots for the estimations, which are obtained by differentiating the estimation of structural intensity by the proposed method. It can be considered from Figs. 13(a) and 13(b) that the consumption of the vibration energy can be derive by the structural intensity estimated by the proposed method for the measurement data including errors.

If loss factors of the structure and the damping material are known, consumption of the vibration energy can be estimated using the strain energy and loss factor[4]. Figures 14(a) and 14(b) show the consumption of the vibration power estimated by the strain energy and the loss factor, where the solid line strands for the theoretical curve and the dots for the estimation. It is obtained from Figs. 14(a) and 14(b) that the consumption of the vibration power can be derive accurately by the strain energy estimated by the proposed method.



(a) σ=1% (b) σ=5%

Fig.14 The consumption of the vibration energy estimated by the strain energy and the loss factor.

4. CONCLUSION

The results of the present study are summarized as follows:

(1) By the method proposed in this paper, it is possible to estimate accurately the level of structural intensity propagating in the beam with damping treatment.

(2) This method is valid for the measurement data including errors.

(3) The consumption of vibration energy can be estimated using the structural intensity estimated by the proposed method.

(4) The consumption of vibration energy can be estimated using the strain energy estimated by the proposed method.

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