FIFTH INTERNATIONAL CONGRESS ON SOUND AND VIBRATION
DECEMBER 15-18, 1997
ADELAIDE, SOUTH AUSTRALIA

Experimental Determination of the Complex Moduli of Hereditary-Elastic Materials Used as Isolators

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Summary: An investigation aimed at determining the complex moduli for a number of polymeric materials is discussed. The complex moduli are determined over a particular frequency range on the basis of transmissibility measured on mass loaded cubic specimens subjected to controlled base harmonic excitation. The mathematical models of the response of the specimens are reviewed and a finite element based correction factor is introduced to allow the use of one dimensional theory to three dimensional test samples. Relationships between the measured transmissibility and the calculated moduli presented. Consideration is given to the manner in which the experimental data may be fitted and to the errors that are involved.

1. Background

The function of an isolator is to reduce the magnitude of motion transmitted from a vibrating foundation to an item of equipment, or to reduce the magnitude of the force transmitted from the equipment to its foundation[1,2]. Polymeric isolators are commonly used in such systems in order to minimize vibration levels and to reduce the effect of transmitted vibration[3]. In this type of isolator, both the load-supporting functions and the energy-dissipating functions are performed by the same element. Such polymers are often modeled as hereditary-elastic materials i.e. materials whose constitutive relationship can be expressed in the form (for one dimension deformation)[4,5,6]

$$\sigma + A_1 \frac{d\sigma}{dt} + \cdots + A_N \frac{d^N \sigma}{dt^N} = B_0 e + B_1 \frac{de}{dt} + \cdots + B_N \frac{d^N e}{dt^N} \tag{1}$$

or in the equivalent form

$$\sigma(t) = E[e(t) - \int_0^t \Gamma(t-\tau) e(\tau) d\tau] \tag{2}$$

where $\Gamma(t-\tau) = \sum_{n=1}^{N} a_n \exp[-\alpha_n (t-\tau)]$ is known as the relaxation kernel for the material and a_n, α_n and N must be determined experimentally. If the response of the specimen is harmonic such that $\varepsilon(t) = \varepsilon\, e^{i\omega t}$, then $\sigma(t) = E^*(i\omega)\varepsilon\, e^{i\omega t}$, where $E^*(i\omega) = E_1(\omega) + iE_2(\omega)$ is called the complex Young's modulus, and can be expressed in the form

$$E^*(i\omega) = E(1 - \sum_{n=1}^{N} \frac{a_n}{\alpha_n + i\omega}) \tag{3}$$

Relations (1), (2) and (3) are equivalent representations that may or may not represent adequately the constitutive characteristics of the material under consideration in the frequency range of interest. More accurate representations may be found[7], but the form assumed in this paper has the advantage of leading to closed form solutions for the free vibration response of such materials[8,9].

In the more general case involving three dimensional deformation of an isotropic hereditary-elastic material the specification of two complex elastic constants is required. The results presented will be confined to those pertaining to Young's modulus.

2. Determination of Complex Modulus

To determine the complex Young's modulus of such polymeric isolators, longitudinal vibrating viscoelastic rod theory is often applied[10-14]. The equation governing the axial displacement of a thin rod subjected to harmonic excitation of frequency ω is the one dimensional wave equation and the transmissibility for such a rod with a mass M at the other end is given by[3]

$$T_m(i\omega) = [\cos(n^* l) - \eta (n^* l)\sin(n^* l)]^{-1} \tag{4}$$

where $n^* = \omega(\frac{\rho}{E^*})^{\frac{1}{2}} = \frac{\omega}{c^*}$, c^*=complex velocity of wave propogation; ρ = mass density, l = rod length and $\eta = M/(\rho A L)$ is the ratio of M to the mass of the rod.

The above equation describes the behaviour of a system in which the only important variable is the deformation in the direction of the axis of the rod. If the lateral dimensions of the rod are significant the above equation will be in error.

According to this theory the transfer function is given by Eq.(4). Its magnitude is usually called the transmissibility function. If its discrete values are measured, the values of complex Young's modulus can be obtained through an iterative calculation of the following equation

$$\beta^2 = (\eta \frac{\sin\beta}{\beta})^{-1}(\cos\beta - \frac{1}{T_m(i\omega_j)}) \tag{5}$$

where $\beta = n^* l$. For $|\beta| << 1$, Eq.(4) and (5) can be simplified as

$$T_m(i\omega_j) = (1 - \eta\beta^2)^{-1} \quad \text{and} \quad \beta^2 = \eta^{-1}(1 - T_m^{-1}(i\omega_j)) \tag{6}$$

It should be noted that the transfer function is always 1 when ω is zero, independent of the value of the complex modulus, so it is not possible to get good estimates of the complex modulus in the frequency range near zero.

The above theory may be applied to tests conducted on long thin rods and such testing could be expected to result in accurate values for the complex Young's modulus.

If the specimens being tested do not satisfy the assumption of one dimensional deformation, a correction factor γ_s has been proposed to account for such cases[3,15]., that is, E^* in Eq. (4) is replaced by $E^*\gamma_s$, where $\gamma_s = 1 + \mu S^2$, S is the so-called shape factor and μ is a constant.

An approximate rod theory given by Love can also be employed to describe the vibration of internally damped rods that have significant lateral dimensions. A dynamic correction factor is derived as[3,16] $\gamma_d = 1 - (\beta \nu r)^2 L^{-2}$, where ν is Poisson's ratio and r is the radius of gyration of an elementary section of the rod about the x axis.

It turns out that, by using finite element theory, one dimensional theory can also be usefully employed for specimens of uniform cross section whose geometry is clearly not one dimensional. As an example consider the test sample shown in Fig.1 which consists of a cubic test specimen supporting a rigid mass subjected to harmonic base excitation. Assuming that Poisson's ratio is constant and known, the following equation, which describes the deformation of the specimen and the supported mass in the vertical direction can be developed (See Fig.2)

$$\begin{bmatrix} \mathbf{M}_{tt} & \mathbf{M}_{tc} & 0 & 0 & 0 \\ \mathbf{M}_{ct} & \mathbf{M}_{cc}^{t}+\mathbf{M}_{vv}^{t} & \mathbf{M}_{cv}^{t} & 0 & 0 \\ 0 & \mathbf{M}_{vc}^{t} & \mathbf{M}_{vv} & \mathbf{M}_{vc}^{b} & 0 \\ 0 & 0 & \mathbf{M}_{vc}^{b} & \mathbf{M}_{cc}^{b}+\mathbf{M}_{vv}^{b} & \mathbf{M}_{cb} \\ 0 & 0 & 0 & \mathbf{M}_{bc} & \mathbf{M}_{bb} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{x}}_{t} \\ \ddot{\mathbf{x}}_{tc} \\ \ddot{\mathbf{x}}_{v} \\ \ddot{\mathbf{x}}_{bc} \\ \ddot{\mathbf{x}}_{b} \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{tt} & \mathbf{K}_{tc} & 0 & 0 & 0 \\ \mathbf{K}_{ct} & \mathbf{K}_{cc}^{t} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{K}_{cc}^{b} & \mathbf{K}_{cb} \\ 0 & 0 & 0 & \mathbf{K}_{bc} & \mathbf{K}_{bb} \end{bmatrix} \begin{Bmatrix} \mathbf{x}_{t} \\ \mathbf{x}_{tc} \\ \mathbf{x}_{v} \\ \mathbf{x}_{bc} \\ \mathbf{x}_{b} \end{Bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{K}_{vv}^{t} & \mathbf{K}_{cv}^{t} & 0 & 0 \\ 0 & \mathbf{K}_{vc}^{t} & \mathbf{K}_{vv} & \mathbf{K}_{vc}^{b} & 0 \\ 0 & 0 & \mathbf{K}_{cv}^{b} & \mathbf{K}_{vv}^{b} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} E(t) * \begin{Bmatrix} \mathbf{x}_{t} \\ \mathbf{x}_{tc} \\ \mathbf{x}_{v} \\ \mathbf{x}_{bc} \\ \mathbf{x}_{b} \end{Bmatrix} = \begin{Bmatrix} \tilde{\mathbf{f}}_{t} \\ 0 \\ 0 \\ 0 \\ \tilde{\mathbf{f}}_{b} \end{Bmatrix} \quad (7)$$

where the subscripts or superscripts t, b, v and c denotes the top mass, the base mass, the viscoelastic material domain and the connection boundary respectively, and the vectors $\mathbf{x}_t$, $\mathbf{x}_b$, $\mathbf{x}_v$, $\mathbf{x}_{tc}$ and $\mathbf{x}_{bc}$ are vectors representing the displacements of the top mass, bottom mass, isolator, connection points between top mass and isolator, and connection points between bottom mass and isolator respectively. It is assumed that each finite element node has three translational degrees of freedom. Bold notation is used to represent a matrix or a vector.

If the stiffness of the top and bottom masses is sufficiently high, compared with the viscoelastic material domain, they may be treated as rigid masses, and there exists the following transformation:

$$\mathbf{x}_t = [\mathbf{e} \quad \mathbf{e} \quad \cdots \quad \mathbf{e}]^T u_t \qquad \mathbf{x}_{tc} = [\mathbf{e} \quad \mathbf{e} \quad \cdots \quad \mathbf{e}]^T u_t$$

where $\mathbf{e} = [1 \quad 0 \quad 0]$, $j=t$ or b and u_t, u_b are the top and the base axial displacement respectively. After the transformation, its form in the Laplace domain with zero initial conditions becomes

$$(s^2 \begin{bmatrix} M_{tt} & \mathbf{M}_{tv} & \mathbf{0} \\ \mathbf{M}_{vt} & \mathbf{M}_{vv} & \mathbf{M}_{vb} \\ \mathbf{0} & \mathbf{M}_{bv} & M_{bb} \end{bmatrix} + \begin{bmatrix} K_{tt} & \mathbf{K}_{tv} & \mathbf{0} \\ \mathbf{K}_{vt} & \mathbf{K}_{vv} & \mathbf{K}_{vb} \\ \mathbf{0} & \mathbf{K}_{bv} & K_{bb} \end{bmatrix} E^*(s)) \begin{Bmatrix} U_t \\ \mathbf{X}_v \\ U_b \end{Bmatrix} = \begin{Bmatrix} F_t \\ \mathbf{0} \\ F_b \end{Bmatrix} \quad (8)$$

where U_t, $\mathbf{X}_v$, U_b, F_t and F_b are the Laplace transformation of u_t, $\mathbf{x}_v$, u_b, f_t and f_b respectively. When only base motion excitation is considered, the corresponding elastic eigenvalue problem of this equation yields a diagonal eigenvalue matrix $\mathbf{\Omega}$, in which the jth diagonal element is ω_j^2, and an eigenvector matrix $\mathbf{\Phi}$. Using the orthogonality relationships, we can solve Eq.(8) to obtain:

$$\begin{Bmatrix} U_t \\ \mathbf{X}_v \end{Bmatrix} = -\mathbf{\Phi}(s^2\mathbf{I} + \mathbf{\Omega} E^*(s))^{-1}\mathbf{\Phi}^T \begin{Bmatrix} 0 \\ s^2\mathbf{M}_{vb} + \mathbf{K}_{vb}E^*(s) \end{Bmatrix} U_b \quad (9)$$

The displacement transfer function is therefore:

$$F(i\omega) = \frac{U_t(i\omega)}{U_b(i\omega)} = \sum_{l=2}^{N_f-1} \sum_{j=1}^{N_f} \frac{\varphi_{j1}\varphi_{jl}}{-\omega^2 + \omega_j^2 E^*(i\omega)} (-\omega^2 \mathbf{M}_{vb} + \mathbf{K}_{vb} E^*(i\omega))_l \quad (10)$$

where N_f is the order of Eq.(8). The characteristic equation of the FEM formulation and simple rod theory can be written respectively as

$$g_{fem}(s) = \prod_{n=1}^{N_f} (s^2 + \omega_n^2 E^*(s)) \qquad g_{rod}(s) = \prod_{n=1}^{\infty} (s^2 + \beta_n^2 \frac{\gamma A E^*(s)}{A\rho L^2})$$

where β_n are the eigenvalues of the rod. Comparing these expressions, a dynamic correction factor can be defined as follows:

$$\gamma_{dn} = \omega_n^2 (\beta_n^2 \frac{A}{A\rho L^2})^{-1} \quad (11)$$

If the mass ratio is large, a transformation, so called static condensation[17], can be introduced to reduce the finite element equations to a single degree of freedom system:

$$\mathbf{x}_v = -\mathbf{K}_{vv}^{-1}(\mathbf{K}_{tv}u_t + \mathbf{K}_{bv}u_b)$$

resulting in the following equation

$$\begin{bmatrix} M_t + \frac{1}{3} M_v & \frac{1}{6} M_v \\ \frac{1}{6} M_v & M_b + \frac{1}{3} M_v \end{bmatrix} \begin{Bmatrix} \ddot{u}_t \\ \ddot{u}_b \end{Bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} CE(t) * \begin{Bmatrix} u_t \\ u_b \end{Bmatrix} = \begin{Bmatrix} f_t \\ f_b \end{Bmatrix} \tag{12}$$

where $CE(t)$ is the equivalent axial stiffness and M_{tt}, M_{bb} and M_v are the top mass, base mass and mass of viscoelastic domain respectively. This equation is similar to that obtained for a massless rod connecting two masses at its top and base. If the viscoelastic body is treated as a massless rod, a static correction factor γ can be defined as: $\gamma = CL/A$. Generally, γ is a function of the geometry of the isolator and Poisson's ratio ν, which can be readily calculated using the finite element method. All that is required is to construct a finite element elastic model of the isolator, assume $E = 1$, and apply a unit force distributed on its top with its base fixed. Assuming that the isolator has appropriate symmetry one can then write $C = u_t^{-1}$.

3. Experimental Procedure

A diagrammatic representation of the experimental set up is shown in Fig.1. The sample under test is approximately cubic and two steel plates are glued to each end of the sample. The lower one is attached to an electrodynamic shaker and the upper one, which has mass M, is free to vibrate. Sinusoidal excitation is provided and the transfer function is measured by means of accelerometers mounted on each plate. A FFT analyzer is used to calculate the transfer function or transmissibility function.

4. The Form of Complex Modulus and Fitting Method

Different representations of the complex modulus $E^*(i\omega)$ have been proposed[4,5,6]. The form Eq.(3) corresponding to Eq.(2) will be used. The coefficients a_n and α_n are determined by the viscoelasticity of the material. To get these coefficients, one way is to use the the Least Squares Curve Fitting method. For convenience, Eq.(3) can be changed to the form of rational fraction polynomials corresponding to Eq.(1)

$$E^*(\omega) = (\sum_{n=0}^{N} B_n (i\omega)^n) / (1 + \sum_{n=1}^{N} A_n (i\omega)^n) \tag{13}$$

When values of the complex moduli are obtained at discrete points, there exist errors for every frequency point ω_j. The square summation of the errors at each frequency point can be weighted by an independent factor w_j and the summation can be taken as the objective function of a least squares minimization method

$$\Sigma = \sum_{j=1}^{N_j} W_j(i\omega_j) \| E^*(i\omega_j)(1 + \sum_{n=1}^{N} A_n (i\omega_j)^n) - \sum_{n=0}^{N} B_n (i\omega_j)^n \|^2 \tag{14}$$

The weight functions $W_j(i\omega) = w_j / (1 + \sum_{n=1}^{N} A_n (i\omega)^n)$ may be set as constants by selecting initial values of A_n. The iteration method can be used to get a set of more accurate values.

When N is large, the coefficient matrix of the sovling equation will be poorly conditioned. To improve the fitting precision, a weighted complex orthogonal expansion method is suggested to fit the function. Two sets of weighted complex orthogonal functions are selected to describe the complex modulus as follows

$$E^*(i\omega) = (\sum_{n=0}^{N} c_n p_n (i\omega)) / (\sum_{n=0}^{N} d_n q_n (i\omega)) \tag{15}$$

The weighted square summation of the errors is

$$\Sigma = \sum_{j=-N_j}^{N_j} W_j(i\omega_j) \| E^*(i\omega_j)(\sum_{n=0}^{N} d_n q_n (i\omega_j)) - (\sum_{n=0}^{N} c_n p_n (i\omega_j)) \|^2$$

where $W_j(i\omega_j)$ are the weight functions: $W_j(i\omega) = w_j / \sum_{n=0}^{N} d_n q_n(i\omega)$ and $E^*(i\omega_{-j}) = E^*(-i\omega_j)$.

If $p_n(i\omega)$ and $q_n(i\omega)$ are orthogonal functions, they must satisfy the following relationships

$$\sum_{j=-N_j}^{N_j} \bar{f}_m(i\omega_j) f_n(i\omega_j) \| W_j(i\omega_j) \|^2 \sigma_j^2 = \begin{cases} 1 & if\ m = n \\ 0 & if\ m \neq n \end{cases}$$

in which $\bar{f}_m(i\omega_j)$ is the conjugate function of $f_m(i\omega_j)$ and

$$\sigma_j^2 = \begin{cases} 1 & if\ f_n(i\omega) = p_n(i\omega) \\ \| E^*(i\omega_j) \|^2 & if\ f_n(i\omega) = q_n(i\omega) \end{cases}$$

An iteration method is needed to solve this problem and can be expected to provide a set of more accurate values. In the first step $W_j(i\omega_j)$ may be taken as 1 or all d_n can be set as 1 as initial values.

5. Error Estimation

When using the transfer function to calculate the complex Young's modulus, the main error source will be from the error in measuring the transfer function. It is necessary to know the sensitivity of the calculated modulus to these measurement errors. From Eq.(4) there exists a small variation $\delta\beta$ caused by a small $\delta T_m(i\omega)$, where

$$\delta T_m(i\omega) = [\cos(\beta + \delta\beta) - \eta(\beta + \delta\beta)\sin(\beta + \delta\beta)]^{-1} - T_m(i\omega)$$

Also it may be shown that

$$\delta E^*(i\omega) / E^*(i\omega) = -(2\beta + \delta\beta)\delta\beta / (\beta + \delta\beta)^2 \qquad (16)$$

and $\delta\beta$ can be solved from the following equation

$$(\eta \cos\beta + \frac{1}{2T_m(i\omega)})(\delta\beta)^2 + (\sin\beta + \eta\beta\cos\beta + \eta\sin\beta)\delta\beta - \frac{\delta T_m(i\omega)}{T_m(i\omega)} \frac{1}{T_m(i\omega) + \delta T_m(i\omega)} = 0$$

For a SDOF system, when the transfer function has a variation $\delta T_m(i\omega)$, then from Eq.(6) the sensitivity of E^* to the measured transmissibility T_m is given by

$$\delta E^*(i\omega) / E^*(i\omega) = -\delta T_m(i\omega) / T_m(i\omega) / (\delta T_m(i\omega) + T_m(i\omega) - 1) \qquad (17)$$

6. Transmissibility Dependence on the Mass Ratio

To compare the damping character of different viscoelastic materials used as isolators, they must have the same resonant frequency. One way to achieve this is to change the top mass. The relation between resonant frequency ω_r and top mass M can be derived by differentiating the transmissibility function. If ω_r is the resonant frequency, then

$$\frac{\partial \| T_m(\omega) \|}{\partial \omega} \Big|_{\omega=\omega_r} = 0 \qquad (18)$$

From Eq. (4)

$$|T_m(i\omega)|^2 = [(\cos\beta_1 ch\beta_2 - \eta\beta_1 \sin\beta_1 ch\beta_2 + \eta\beta_2 \cos\beta_1 sh\beta_2)^2 + (\sin\beta_1 ch\beta_2 + \eta\beta_1 \cos\beta_1 sh\beta_2 + \eta\beta_2 \sin\beta_1 ch\beta_2)^2]^{-1} \qquad (19)$$

Substituting Eq.(19) in to Eq.(18), gives

$$\eta = \frac{-\alpha_2 + \sqrt{\alpha_2^2 - 4\alpha_1\alpha_3}}{2\alpha_1} \qquad (20)$$

in which

$\alpha_1 = A_{12}A'_{12} + A_{22}A'_{22}$ $\quad \alpha_2 = A_{11}A'_{12} + A_{12}A'_{11} + A_{21}A'_{22} + A_{22}A'_{21}$ $\quad \alpha_3 = A_{11}A'_{11} + A_{21}A'_{21}$

$A_{11} = \cos\beta_1 ch\beta_2$ $\quad A_{12} = -\beta_1 \sin\beta_1 ch\beta_2 + \beta_2 \cos\beta_1 sh\beta_2$

$A_{21} = \sin\beta_1 sh\beta_2 \qquad A_{22} = \beta_1\cos\beta_1 sh\beta_2 + \beta_2\sin\beta_1 ch\beta_2$

$$\beta_1 = \left(\frac{A\rho}{A\gamma(E_1^2+E_2^2)}\right)^{\frac{1}{2}}\omega L\left(\frac{E_1+\sqrt{E_1^2+E_2^2}}{2}\right)^{\frac{1}{2}} \qquad \beta_2 = -\left(\frac{A\rho}{A\gamma(E_1^2+E_2^2)}\right)^{\frac{1}{2}}\omega L\left(\frac{-E_1+\sqrt{E_1^2+E_2^2}}{2}\right)^{\frac{1}{2}}$$

The relation between the top mass and the resonant frequency for SDOF case is

$$M = \frac{\gamma A}{\omega_r^2 L}\cdot\frac{2\omega_r E_1(\omega_r)(E_1(\omega_r)E_1'(\omega_r)+E_2(\omega_r)E_2'(\omega_r)) - (E_1^2(\omega_r)+E_2^2(\omega_r))(\omega_r E_1'(\omega_r)+2E_1(\omega_r))}{(E_1(\omega_r)E_1'(\omega_r)+E_2(\omega_r)E_2'(\omega_r))\omega_r - 2(E_1^2(\omega_r)+E_2^2(\omega_r))}$$

7. Results and Conclusions

The materials under test are Isodamp EAR C-1002, Chloroprene and Deproteinized Natural Rubber, whose acronyms are EAR, CR and NBR respectively. Firstly, the influence of mass ratio η on the eigenvalues β are considered. Table 1 lists the mass ratios of the samples that were tested. Fig.3 presents the first four eigenvalues for different mass ratios η. It is obvious that when η becomes very large, the lowest of eigenvalue will tend to zero, and meanwhile the other eigenvalues will be the same as those of a rod fixed at its top and bottom. A large ratio of the second eigenvalue to the first one implies the influence of the second or higher modes on the frequency range of the first mode will be very small and means the single degree of freedom idealization is appropriate in this range. Fig.4-6 compare the modal displacements of the specimen calculated using rod and finite element theory. The influence of Poisson's ratio on the transverse displacements, which gives rise to the correction factor, can clearly be seen. Fig.7 and 8 show how the correction factors vary with mass ratio for different modes and different values of Poisson's ratio. All the finite element analytical results are calculated by MSC/NASTRAN. The error estimation in Fig.9 shows that the least error during identification of complex Young's modulus by means of Eq.(5) occurs near the resonant frequency. The error estimation from the SDOF system is similar to that from the rod theory at frequency range sufficiently below the second resonant frequency of the rod. All the second resonant frequencies of the tested samples are larger than 1500 Hz, thus the single degree of freedom idealization can be used in the frequency range considered.

As mentioned above, the formulation of complex Young's modulus in terms of Equation (3) can be determined by measuring the transfer function; calculating the corresponding discrete complex Young's modulus; and using the weighted complex orthogonal expansion method to curve fit the data. The materials tested were CR, EAR and NBR, and typical transfer functions are showed in Fig.10. The fitting results are obtained as follows

CR: $$E^*(i\omega) = 1.9808\times10^7\left(1-\frac{32.721}{i\omega+296.93}-\frac{2524.6}{i\omega+5132.2}\right)$$

EAR: $$E^*(i\omega) = 9.5905\times10^7\left(1-\frac{38.047}{i\omega+353.82}-\frac{5524.4}{i\omega+6491.0}\right)$$

NBR: $$E^*(i\omega) = 2.7102\times10^7\left(1-\frac{28.340}{i\omega+215.80}-\frac{3807.1}{i\omega+5984.7}\right)$$

in which Poisson's ratios were taken as 0.499, 0.32 and 0.499 respectively. A typical comparison between the measured data and fitting curves of EAR is shown in Fig.11. As may be noted, the fit at low frequencies where the moduli are changing most rapidly is worst. When the expression for the complex Young's modulus has approximately been fitted, it is easy to calculate and compare their transmissibility. Fig.12 compares the maximum transmissibilities for CR, EAR and NBR, and shows that there is close comparison between the calculations and the measured results. EAR always has better transmissibility characteristic than the others.

Acknowledgment

Funds for this study were provided by Defense Research Establishment, Atlantic, Canadian Department of National Defense and by the National Science and Engineering Research Council of Canada.

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Table 1 Mass of Materials

	η (Mass1)	η (Mass2)	η (Mass3)	η (Mass4)	ρ (Kg/m^3)	ρAL (Kg)
CR	3.71	21.75	58.14	102.77	1.39×10^3	0.0333
EAR	3.87	22.71	60.69	106.39	1.34×10^3	0.0319
NBR	5.04	29.56	79.02	139.69	1.12×10^3	0.0245

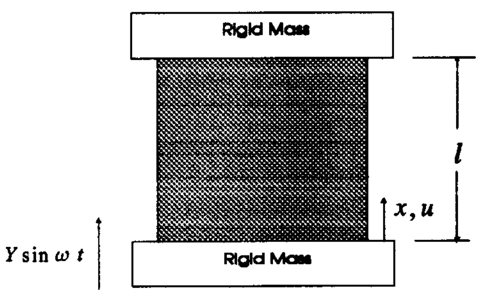

Fig.1 A test sample

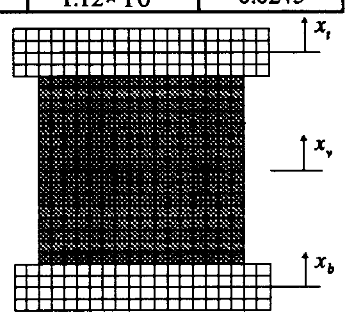

Fig.2 Finite element model

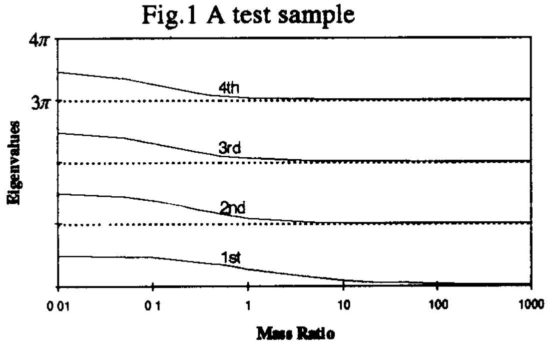

Fig.3 Variation of eigenvalues β_n with mass ratio η

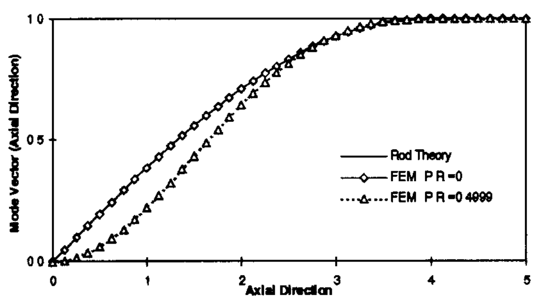

Fig.4 The first order mode: η = 100 (Axial vector)

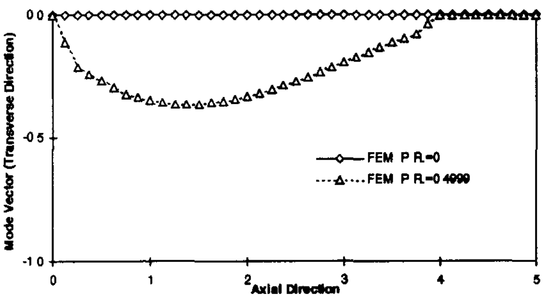

Fig.5 The first order mode, $\eta = 100$ (Transverse Vector)

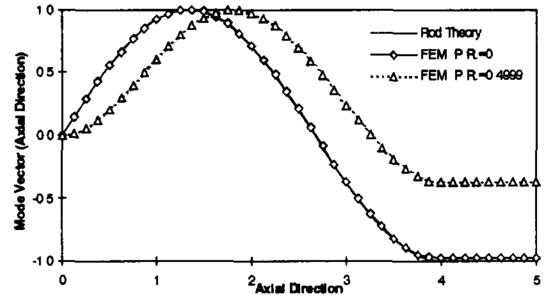

Fig.6 The second order mode: $\eta = 100$ (Axial vector)

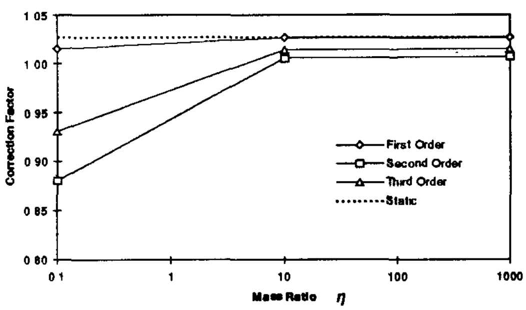

Fig.7 Correction factor: $\nu = 0.2$

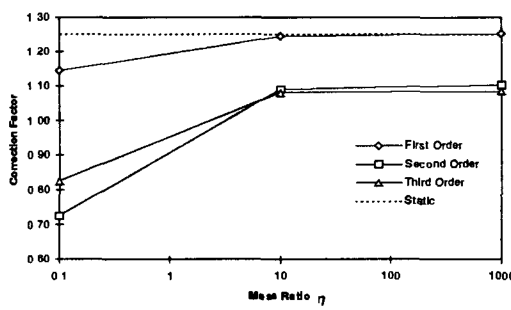

Fig.8 Correction factor: $\nu = 0.49$

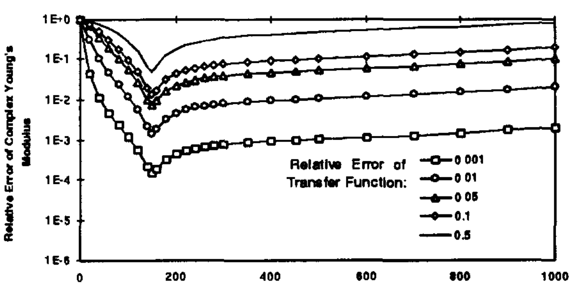

Fig.9 Error curve for the rod theory

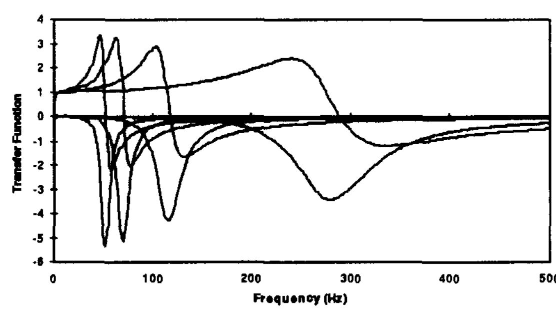

Fig.10 Typical measured transfer functions

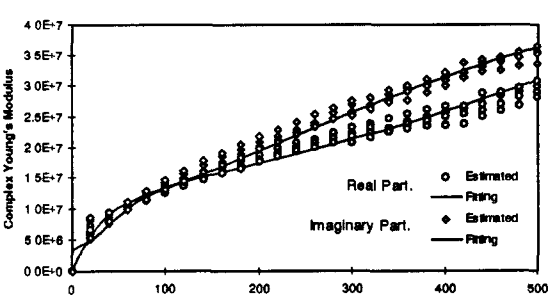

Fig.11 Complex Young's modulus of EAR

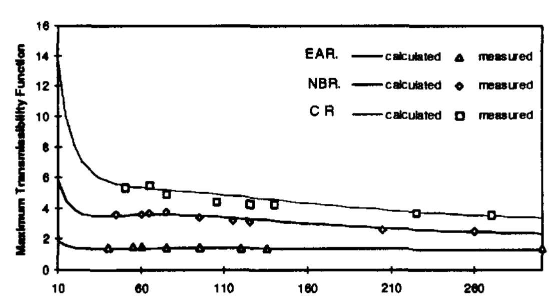

Fig.12 Comparison of maximum transmissibility