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EFFECTS OF DRILLING DEGREES OF FREEDOM IN THE FINITE ELEMENT MODELING OF INFINITE DOMAINS

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This paper deals with a hybrid finite element modeling of wave scattering problems in infinite domains. Scattering of waves involving complex geometries in conjunction with infinite domains is modeled by introducing a mathematical boundary. On the mathematical boundary, the finite element representation is matched with analytical solution in the infinite domain in terms of fields and their derivatives. Drilling degrees of freedom at each nodes of the finite element model are introduced to take into account the transverse component of the elastodynamic field more precisely. To verify the roles of drilling degrees freedom and the slope constraint, normal incidences of P and SV waves are considered. For the P-wave incidence, the use of slope constraint suppresses artificial reflection at the mathematical boundary and for the SV-wave case, the use of drilling degrees freedom reduces numerical error at irregular frequencies.

1. INTRODUCTION

Numerical modeling of wave propagation and scattering in infinite domain has been of interest in many fields. There are several numerical approaches which have been used to treat wave problems in infinite domain involving complex geometries: boundary element method(BEM) [1,2], use of infinite element [3], matching technique with finite element method [4] and T-matrix method [5]. BEM approach has been widely used in radiation and scattering of waves. The introduction of an absorbing boundary condition (ABC) has been used to minimize the non-physical reflections from the mathematical boundary [6,7]. Recently, Givoli and Keller [8] and Harari and Hughes [9] derived exact non-local type non-reflecting boundary conditions, so called to Dirichlet-to-Neumann (DtN) boundary condition. However, when the DtN operator is truncated for implementation, uniqueness is lost at characteristic frequencies.

The finite element method has been widely used in investigating wave propagation and scattering problems because it can be used for inhomogeneous and anisotropic materials with geometric difficulties. Since the finite element technique is an approximation technique, it has some limitations. The dispersion and spurious oscillation phenomena in finite element solutions have been investigated. However, the effects of displacement-based conventional finite elements are rarely investigated. There are basically three kinds of fields; irrotational acoustic fields, solenoidal electromagnetic fields and elastodynamic fields. For acoustic fields, the use of conventional finite elements based upon nodal displacements is sufficient. For solenoidal fields, it has been found that node-based classical finite elements have drawbacks [10]. Representations of solenoidal fields as piece-wise linear, continuous, functions of space are not adequate. Such representations are too rigid across interfaces and result in loss of accuracy in computations.

Thus, recently, Kim et al. [11] proposed the use of drilling degrees of freedom (d.o.f.) in addition to nodal displacements in the finite element approximation to take into account the solenoidal component of the elastic field more precisely. For further transparency at the mathematical boundary, the continuity of all derivatives are introduced via a penalty method. It is found that by using slope constraint on the mathematical boundary and introducing drilling d.o.f. at the finite element nodes, the error due to artificial reflection can be reduced remarkably. However, the roles of the drilling d.o.f. and the slope constraint are not clearly proven because oblique incidence of P-wave is considered.

Therefore, this paper aims at proving the effects of slope constraint and drilling d.o.f. in the finite element modeling of wave scattering problems associated with infinite domain. Instead of oblique incidence, normal incidence is considered because normal incidence does not occur mode conversion in elastic media if the boundary is flat.

2. FINITE ELEMENT FORMULATION

The finite element formulation in this paper is not much different from the conventional formulation for elastodynamic problems except drilling degrees of freedom and slope constraints. Details of the whole derivation is in the reference[11] and a brief summary will be shown.

2.1. Drilling degrees of freedom

Rotational degrees of freedom at corner nodes of finite elements are considered to alleviate the stiff behavior of linear finite element in bending motion. Figure 1 shows a linear element with drilling d.o.f. in the two-dimensional case.

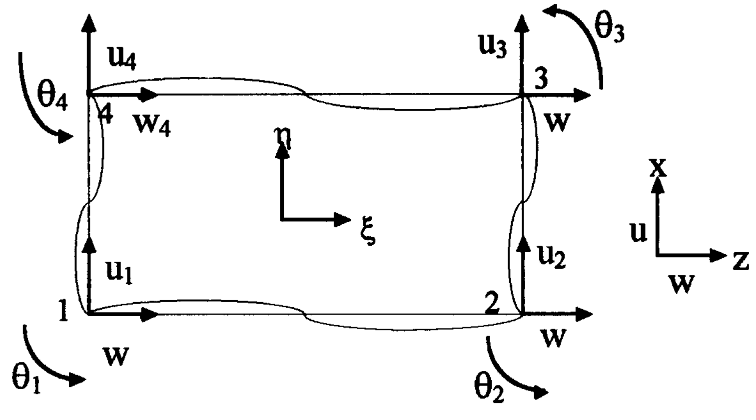

Figure 1. Linear element with drilling d.o.f.

From the nodal values, the displacement can be represented as follows by using proper interpolation functions:

$$\mathbf{u}=\begin{Bmatrix} w \\ u \end{Bmatrix}=\mathbf{N}_t\hat{\mathbf{u}}=\begin{bmatrix} N_1 & 0 & N_5 & N_2 & 0 & N_6 & N_3 & 0 & N_7 & N_4 & 0 & N_8 \\ 0 & N_1 & N_9 & 0 & N_2 & N_{10} & 0 & N_3 & N_{11} & 0 & N_4 & N_{12} \end{bmatrix}\begin{Bmatrix} w_1 \\ u_1 \\ \theta_1 \\ \vdots \end{Bmatrix} \quad (1)$$

where $N_1 - N_4$ are conventional interpolation functions for linear element and $N_9 - N_{12}$ are the functions that incorpolate displacement and drilling d.o.f. A detailed procedure to derive these relationships is given in the reference[11].

The drilling d.o.f. are simply the rotation about a given axis. In the two-dimensional case, the axis of rotation is the y-axis. The physical rotation in a continuum, Ω, is defined as

$$\Omega_y=\frac{1}{2}\left(-\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)=\mathbf{B}_\omega\hat{\mathbf{u}} \quad (2)$$

Since the rotation of drilling d.o.f. should be the physical rotation, a penalty factor γ is used to enforce the equality of θ and Ω, where θ is the rotation of drilling d.o.f.

2.2. Continuity of derivatives of the fields at the mathematical boundary

To eliminate the irregular frequencies, one can impose continuity of displacement fields and their derivatives on the mathematical boundary. This turns out the mathematical boundary to be non-reflecting. The slope constraints that has to be imposed on the mathematical boundary Γ, is:

$$\frac{\partial \mathbf{u}}{\partial \mathbf{x}}=\bar{\mathbf{u}}_\Gamma \qquad \text{on } \Gamma \quad (3)$$

where $\mathbf{x}=[x,\ z]$. The right side of above equation, $\bar{\mathbf{u}}_\Gamma$ is a given value at the boundary which is a derivative of the plane-wave solution, **u,** and the left side represents the derivatives of the displacements in the finite element model. By using the penalty method, the slope constraint is implemented in the finite element equation.

For the total finite element equations, if we assume that this is the steady state case, the equations turn out to be

$$\left\{-\omega^2\mathbf{M}+\mathbf{K}+\gamma\mathbf{K}_{\varpi}+\alpha\mathbf{K}_{\Gamma}\right\}\hat{\mathbf{U}}=\hat{\mathbf{F}}+\alpha\mathbf{F}_{\Gamma} \tag{4}$$

where **M**, **K** are mass and stiffness matrices of the elastic medium, and

$$\mathbf{K}_{\Gamma}=\int_{\Gamma}\mathbf{B}_n^T\mathbf{B}_n d\Gamma, \tag{5}$$

$$\mathbf{F}_{\Gamma}=\int_{\Gamma}\overline{\mathbf{u}}_{\Gamma}^T\mathbf{B}_n d\Gamma, \tag{6}$$

$$\mathbf{K}_{\varpi}=\int\left[\mathbf{B}_{\varpi}-\mathbf{N}_{\gamma}\right]^T\left[\mathbf{B}_{\varpi}-\mathbf{N}_{\gamma}\right]dV, \tag{7}$$

and ω is the excitation frequency.

2.3. Analytical solution of plane-wave problem

To obtain an analytical representation for the fields in the infinite domain, the problem of plane-wave scattering in the half space in the absence of any structure is considered. Accordingly, we consider a semi-infinite elastic domain with P- and SV-wave incidences, respectively.

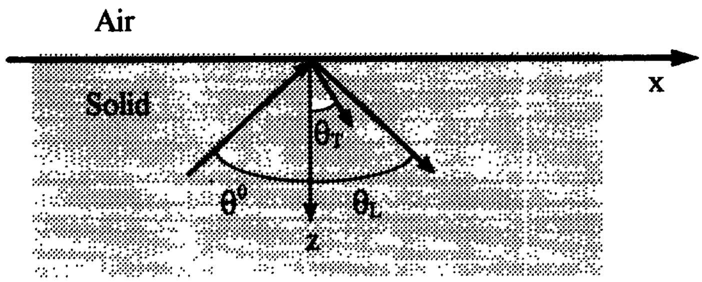

Figure 2. Semi-infinite elastic domain problem

Displacements in the solid region can be written as

$$\mathbf{u}=\mathbf{p}_i A_i\exp(j\mathbf{k}_i\cdot\mathbf{r})+\mathbf{p}_L A_L\exp(j\mathbf{k}_L\cdot\mathbf{r})+\mathbf{p}_T A_T\exp(j\mathbf{k}_T\cdot\mathbf{r}) \tag{8}$$

where **p** is polarization vector, **k** is the wave vector, A is amplitude of wave, θ_0 is the angle of incidence, and the time factor, $\exp(j\omega t)$ is contained in all fields, but omitted for convenience. Subscript i is for incidence, L is longitudinal and T is transverse waves respectively. Snell's law puts Eq.(8) in much simpler forms. The boundary conditions at the top surface are

$$\text{at } z=0,\quad \tau_{zz}=0,\qquad \tau_{zx}=0 \tag{9}$$

Thus, two unknown amplitudes, A_L, A_T can be found from the boundary conditions. Once the unknowns are found, after substituting these unknowns into (8), we can obtain the displacement at the mathematical boundary of the finite element model.

3. NUMERICAL EXAMPLES

An elastic half space problem is considered to prove the efficiency of enforcing slope constraints and endowing the finite elements with drilling d.o.f. to realize a transparent mathematical boundary. P and SV-wave incidences are considered and a finite element model is chosen such that the finite region will be solved with proper boundary conditions. Parameters used in the analysis are same as in the reference [11]. A four node linear element with drilling d.o.f. was used and 341 nodes are made. Constraint parameters for the slope constraint and drilling d.o.f. are searched by an optimization technique at each frequency.

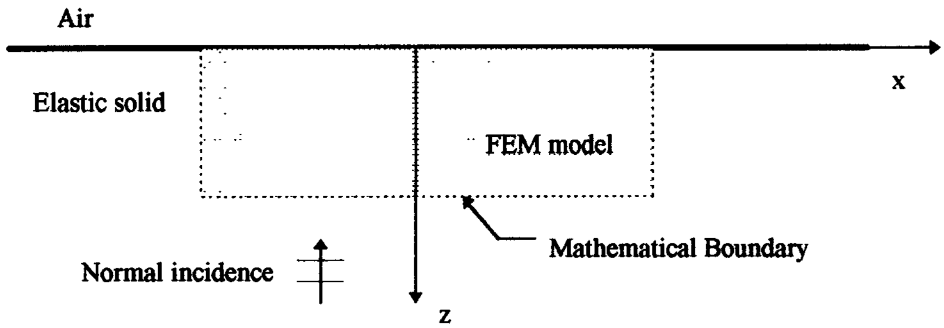

Figure 3. Elastic half space problem.

To judge the finite element analysis results, the average error has been defined in the previous study as

$$\text{Error}=\frac{1}{n}\sum_{i=1}^{n}\frac{\left|U_{FEM,i}-U_{Theory,i}\right|}{\left|U_{Theory,i}\right|} \tag{10}$$

for oblique incidence of P-wave. However, in normal incidence case, for example, P-wave normal incidence, the theoretical value of x displacement is zero. When SV-wave is normally impinging, in contrast, the theoretical y displacement is zero. This turns out the error defined in (10) to be infinite. Thus, the error criterion is slightly changed as follows,

$$\text{Error}=\frac{1}{n}\sum_{i=1}^{n}\frac{\left|w_{FEM,i}-w_{Theory,i}\right|+\left|u_{FEM,i}\right|}{A_i} \quad \text{for P-normal incidence} \tag{11}$$

$$\text{Error}=\frac{1}{n}\sum_{i=1}^{n}\frac{\left|u_{FEM,i}-u_{Theory,i}\right|+\left|w_{FEM,i}\right|}{A_i} \quad \text{for SV-normal incidence} \tag{12}$$

where $U_{FEM,i}=[w,u]_{FEM,i}$ is the displacement found from the finite element analysis and $U_{Theory,i}=[w,u]_{Theory,i}$ is the plane-wave solution at node I. Here, n is the number of nodes.

3.1. P-normal incidence

To distinguish the effects of drilling d.o.f. and slope constraints, P and SV-normal incidences are considered separately. P-wave generates particle movement in propagation direction. Normal P-wave incidence is some what simple case in the elastic half space problem because there is only one mode, P-wave.

When the displacements are specified at the mathematical boundary in the absence of slope constraint and drilling d.o.f., the error peaks are observed ('original' case in Figure 4). These peaks coincide with the natural frequencies of the rectangular slab associated with the mathematical domain. When the slope constraints are enforced at the mathematical boundary, these irregular frequencies are suppressed completely ('slope cnstr' case in Figure 4). This reduction is comparable with the oblique incidence case[11]. In the oblique incidence case, the peaks in the error curves were shifted down considerably but the errors are still high and irregular frequencies are still present. This result shows that the slope constraint helps to reduce the artificial reflections at the irregular frequencies of the model. The penalty factor, α is searched optimally and its range is 10^{10}-10^{14}.

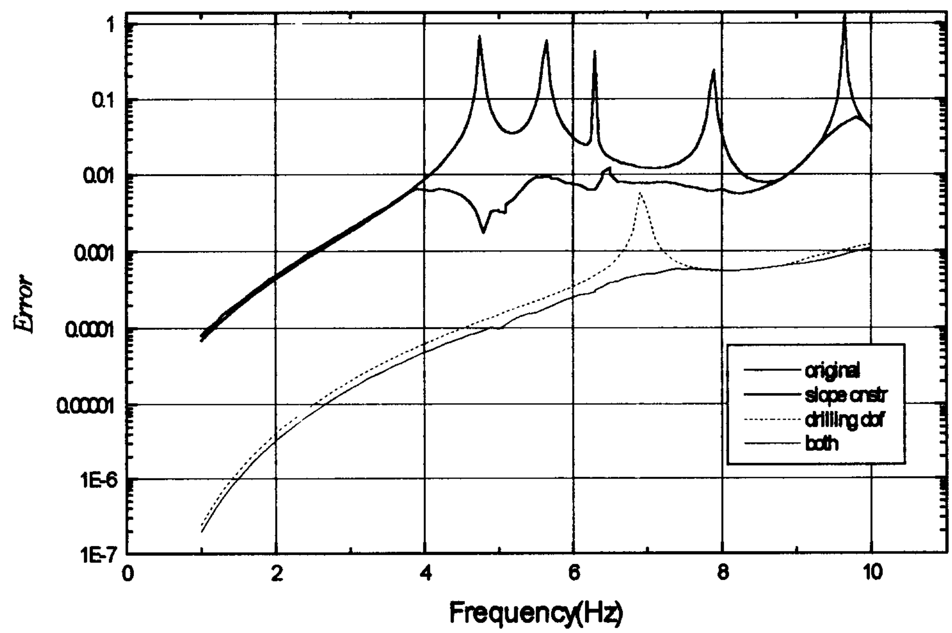

Figure 4. P-normal incidence case.

Next, the slope constraint and drilling d.o.f. are used simultaneously to eliminate artificial reflection. It is found that the error is reduced remarkably and at the same time, the peaks are completely eliminated ('both' case in Figure 4). This shows that slope constraint tends to eliminate irregular frequency while drilling d.o.f. reduce the error bound.

With the four-node linear finite element, drilling d.o.f. are used at each node without slope constraint. The rotational parameter, γ is in the range of 10^{15}-10^{18} . It is found that the error as well as the number of peaks are reduced ('drilling dof' case in Figure 4). The error reduction is due to the fact that the use of drilling d.o.f. avoids the stiff behavior of nodal-displacement-based finite elements.

3.2. SV-normal incidence

In SV-wave, the particle moves vertically with respect to the traveling direction. In contrast with normal incidence of P-wave, SV-wave travels similar to bending wave. In other words, while normal P-wave produces compression or tension in finite element, SV-wave incidence does bending motion. Thus, to take into account SV-wave, finite element needs to be flexible, which means the role of drilling d.o.f. are more important than P-wave incidence.

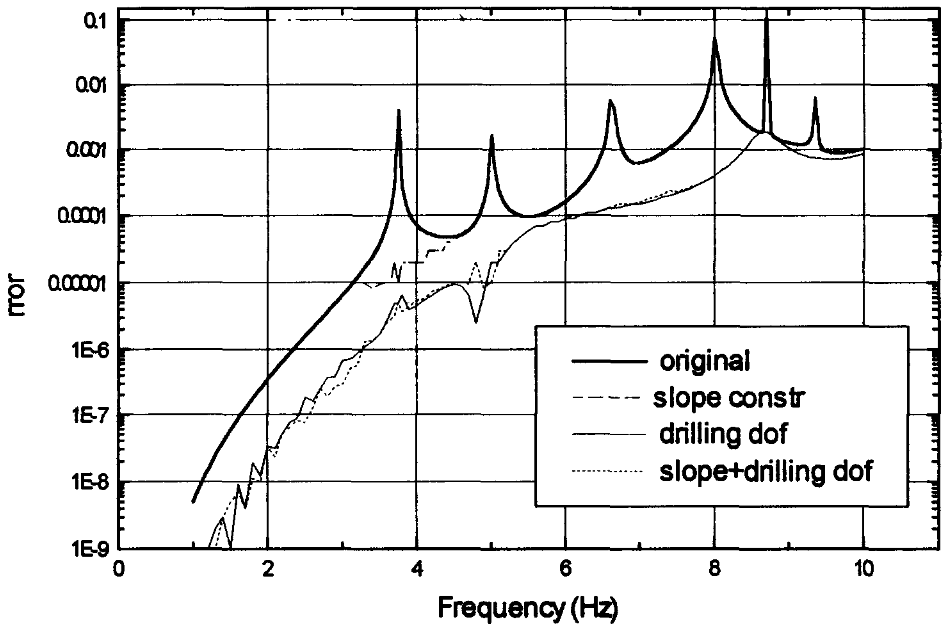

Figure 5. SV-normal incidence case.

In Figure 5, the 'original' means the error when displacement is specified at the mathematical boundary in the absence of slope constraint and drilling d.o.f. Several peaks are observed in the error curve. When slope constraint is imposed on the mathematical boundary, the first peak in the error curve is reduced, but other peaks are still remained ('slope constr' case in Figure 5). However, by using drilling d.o.f. at each node without slope constraint, the peaks are nearly eliminated and the level of error is shifted down ('drilling dof' case in Figure 5). When the slope constraint and drilling d.o.f. are used at the same time, no improvement than the use of drilling d.o.f. is observed. Thus, from these results, we can conclude that the use of drilling d.o.f. in modeling of SV-wave propagation is much important than modeling of P-wave propagation case. This fact proves that the use of drilling d.o.f. in addition to nodal displacements in the finite element is right to take into account the solenoidal component of the elastic field more precisely.

4. CONCLUSIONS

To model semi-infinite domain problems, the region is subdivided by introducing a mathematical boundary and the finite element method is used in the bounded region. To alleviate artificial reflection at the mathematical boundary, slope constraint are used on the mathematical boundary and drilling d.o.f are introduced at the finite element nodes. For normal incidence of P-wave, the use of slope constraint completely suppresses the peaks in the error curve. For normal incidence of SV-wave, the introduction of drilling d.o.f. reduces error peaks remarkably. This proves that the use of slope constraint is important in longitudinal wave propagation and the use of drilling d.o.f. takes into account transverse wave propagation more precisely.

ACKNOWLEDGEMENTS

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