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ON STIFFENED PLATE VIBRATION

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ABSTRACT

On the theoretical analysis basis the principal trends and regularities in non-uniform plate vibration associated with the effects of bending and torque dynamic rigidity of stiffeners are studied. The relations between the parameters of plates and stiffeners at which a significant attenuation of acoustic radiation at low frequencies can be realized.

INTRODUCTION

Vibration of thin plates with stiffeners are of great interest for solution of a number of tasks associated, in particular, with acoustic fatigue of thin-walled elements of aircraft structures and their sound energy radiation into the cabin. The investigations of elasto-inertial features of stiffeners from the standpoint of their effect on thin-walled structures in application to the task of their acoustic radiation attenuation have become recently very urgent. This is determined by the necessity of essential reduction of low-frequency sound pressure levels inside cabins of airplanes with propeller power plants which is not provided with traditional soundinsulating structures. Varying the dynamic rigidity of stiffeners one can regulate the aircraft fuselage acoustic radiation achieving its significant attenuation. For example, varying the stiffener rigidity and their step, the space between the knot lines of the lowest forms of panel eigen-vibration can be reduced and thereby their acoustic radiation at low frequencies will be sharply attenuated. However, it is necessary for this aim to know at least the main trends and regularities associated with the elasto-inertial stiffener feature effect on thin-walled structures vibration. The above said trends and regularities, us one can judge from publications, are not yet studied properly and are the subject of the present investigation.

STATEMENT OF THE TASK AND SOLUTION METHOD

Similarly to [1, 2], we consider a thin freely-supported plate stiffened by a system of rigidity ribs in one direction which separate it into N spans of equal extension (l). Assume that the contact between stiffeners and the plate is realized along the straight lines, the cross forces arising on the contact line cause only a bending deformation of the stiffeners and the bending moments cause only a torque deformation. Refer the orthogonal system of coordinates $\vec{x} = \{x_1, x_2\}$ to the middle plate surface, bringing the coordinate origin into coincidence with one of its contour peaks. Coordinate x_2 in this case will be counted in the direction orthogonal to the contact lines. The plate length in x_1- direction is indicated as l_1 and in x_2- direction as $l_2 = Nl$.

The plate vibration equation is written down for normal displacements of w_n in the span with number n relative to the dimensionless coordinate $y = x_2/l - (n-1)$:

$$\frac{1}{l^4}\frac{\partial^4 w_n}{\partial y^4} + \frac{2}{l^2}\frac{\partial^4 w_n}{\partial y^2 \partial x_1^2} + \frac{\partial^4 w_n}{\partial x_1^4} + \frac{\rho h}{D}\frac{\partial^2 w}{\partial t^2} = 0 \tag{1}$$

Here $D = Eh^3/12(1-\nu^2)$ is the cylindrical rigidity, E is the elasticity module, ν is Poisson coefficient, h is the thickness and ρ is the density of plate material. Eq. (1) is supplemented by boundary conditions on the plate contour:

$$w_n\big|_{x_1=0} = \frac{\partial^2 w_n}{\partial x_1^2}\bigg|_{x_1=0} = w_n\big|_{x_1=l_1} = \frac{\partial^2 w_n}{\partial x_1^2}\bigg|_{x_1=l_1} = 0; \tag{2}$$

$$w_1\big|_{y=0} = \frac{\partial^2 w_1}{\partial y^2}\bigg|_{y=0} = w_N\big|_{y=l} = \frac{\partial^2 w_N}{\partial y^2}\bigg|_{y=l} = 0, \tag{3}$$

and by equations expressing the connection between flexures, turning angles, forces and moments on the contact line between two neighbouring spans:

$$w_n\big|_{y=0} = w_{n-1}\big|_{y=1}; \quad \frac{\partial w_n}{\partial y}\bigg|_{y=0} = \frac{\partial w_{n-1}}{\partial y}\bigg|_{y=l}; \tag{4}$$

$$\frac{D}{l}\frac{\partial}{\partial y}\left\{\frac{1}{l^2}\frac{\partial^2 w_n}{\partial y^2}+(2-\nu)\frac{\partial^2 w_n}{\partial x_1^2}\right\}\Bigg|_{y=0}-$$

$$-\frac{D}{l}\frac{\partial}{\partial y}\left\{\frac{1}{l^2}\frac{\partial^2 w_{n-1}}{\partial y^2}+(2-\nu)\frac{\partial^2 w_{n-1}}{\partial x_1^2}\right\}\Bigg|_{y=1}=\left\{E_1 J\frac{\partial^4 w_n}{\partial x_1^4}+\rho_1 F\frac{\partial^2 w_n}{\partial t^2}\right\}\Bigg|_{y=0}; \quad (5)$$

$$D\left\{\frac{1}{l^2}\frac{\partial^2 w_n}{\partial y^2}+\nu\frac{\partial^2 w_n}{\partial x_1^2}\right\}\Bigg|_{y=0}-D\left\{\frac{1}{l^2}\frac{\partial^2 w_{n-1}}{\partial y^2}+\nu\frac{\partial^2 w_{n-1}}{\partial x_1^2}\right\}\Bigg|_{y=1}=$$

$$=\frac{1}{l}\frac{\partial}{\partial y}\left\{GJ_k\frac{\partial^2 w_n}{\partial x_1^2}+\rho_1 J_p\frac{\partial^2 w_n}{\partial t^2}\right\}\Bigg|_{y=0}. \quad (6)$$

Here E_1J is the bending rigidity, GJ_k is the torque rigidity, J_p is the polar moment of inertia, F is the cross-section area of the stiffener, ρ_1 is the density of the stiffener.

For boundary conditions (2) the solution of Eq. (1) can be found as follows:

$$\begin{aligned} w_n &= W_n(y)\sin k_1x_1\exp(i\omega t); \\ k_1 &= \pi m/l_1, \quad m=1,\ 2,\ \dots . \end{aligned} \quad (7)$$

After substituting Eq.(7) into Eq.(1) we get a common differential equation the solution of which is the following:

$$\left.\begin{aligned} W_n(y) &= a_{1n}\sin k_2 ly + a_{2n}\cos k_2 ly + a_{3n}\,\mathrm{sh}\,kly + a_{4n}\,\mathrm{ch}\,kly, \\ k_2 &= \left(\kappa^2-k_1^2\right)^{1/2},\ k=\left(\kappa^2+k_1^2\right)^{1/2},\ \kappa=\left(\rho h\omega^2/D\right)^{1/2}, \end{aligned}\right\} \quad (8)$$

where a_{in} are the arbitrary constants $(i=1,\ 2,\ 3,\ 4)$.

From the conditions (4)-(6) we obtain four equations connecting the arbitrary constants for neighbouring spans which can be written in a matrix form:

$$a_n = Aa_{n-1},$$

$$a_n=\begin{vmatrix} a_{1n} \\ a_{2n} \\ a_{3n} \\ a_{4n} \end{vmatrix}.$$

Matrix A is expressed as follows:

$$A=\begin{Vmatrix} c-2k\varepsilon s & -(s+2k\varepsilon c) & -2k\varepsilon S & -2k\varepsilon C \\ s-2k_2\chi c & c+2k_2\chi s & -2k\chi C & -2k\chi S \\ 2k_2\varepsilon s & 2k_2\varepsilon c & C+2k_2\varepsilon S & s+2k_2\varepsilon C \\ 2k_2\chi c & -2k_2\chi s & S+2k\chi C & C+2k\chi S \end{Vmatrix}.$$

Here additional symbols are derived:

$$s=\sin k_2 l,\quad c=\cos k_2 l,\quad S=shkl,\quad C=chkl;$$

$$\varepsilon=\frac{E_1 J k_1^4-\rho_1\omega^2 F}{4k_2 k\kappa^2 D};\quad \chi=\frac{GJ_k k_1^2-\rho_1\omega^2 J_p}{4\kappa^2 D}.$$

For the task solution we'll use the method developed in [2]. According to this method matrix A is written in the basis of its own vectors or is transformed into Jordan form when it has multiples to eigen-numbers. As a result we get analytical expressions for determining the eigen-numbers and arbitrary constants at different values of ε and χ which can be directly used for the effect evaluation of the elasto-inertial stiffener features on stiffened plate vibrations.

INVESTIGATION RESULTS

At $\chi\to 0$ two equations for determining the eigen-numbers are obtained:

$$\sin k_2 l=0; \tag{9}$$

$$\frac{b-\left(b^2-4a\right)^{1/2}}{2}=\cos\frac{q\pi}{N}\quad at\quad s\neq 0; \tag{10}$$

$$a=cC+k_2\varepsilon cS-k\varepsilon sC,\ b=C+c+k_2\varepsilon S-k\varepsilon s;\ q=1,\ 2,\ \ldots,\ (N-1).$$

The respective expressions for a_{jn} at $s\neq 0$ are given as follows:

$$a_{1n}=2k\varepsilon S\left[\sin\frac{qn\pi}{N}-c\sin\frac{q(n-1)\pi}{N}\right];\quad a_{2n}=2k\varepsilon sS\sin\frac{q(n-1)\pi}{N};$$

$$a_{3n}=\left[C+2(c-k\varepsilon s)\right]\sin\frac{qn\pi}{N}-\left[2C(c-k\varepsilon s)+1\right]\sin\frac{q(n-1)\pi}{N}+$$
$$+C\sin\frac{q(n-2)\pi}{N}-\sin\frac{\pi q(n+1)}{N};$$

$$a_{4n}=S\left[2(c-k\varepsilon s)\sin\frac{q(n-1)\pi}{N}-\sin\frac{q(n-2)\pi}{N}-\sin\frac{qn\pi}{N}\right]; \tag{11}$$

and at $s=0$ they are:

$$a_{1n}=(-1)^n,\quad a_{2n}=a_{3n}=a_{4n}=0. \tag{12}$$

From Eq.(9) the eigen-numbers follow which correspond to the freely-supported plate span vibration and from (8) and (12) the sinusoidal vibration forms follow.

For arbitrary eigen-wave number k_1 in x_1- coordinate the system of equations (9), (10) gives an infinite number of eigen-number $\hat{k}_2$ groups in x_2 - coordinate within the limits of $\pi r/l\leq\hat{k}_2<\pi(r+1)/l$. Here $r=0,\ 1,\ 2\ldots$ are the numbers of groups which, by convention, will be referred to as zero, first, second, etc. The quantity of eigen-numbers in the group at $r>0$ is equal to the number of spans. The quantity of eigen-numbers in the zero group depends on the dynamic rigidity but never exceeds the value of $N-1$.

In a general case an arbitrary fixed value of ml/l_1 can be set in correlation with dependencies of dimensionless reduced dynamic rigidity $(k_2\varepsilon)$ on a dimensionless wave number $(k_2 l)$ at different values of q/N. Such dependencies for the case of $ml/l_1 = 0.3$ are presented in Fig.1. Abscissas of intersection points of the obtained curves with the curve

$$k_2\varepsilon = \left[(k_2 l)^2 + 2(\pi ml/l_1)^2\right]^{-1/2}\left\{\frac{3(1-\nu^2)(\pi ml/l_1)JE_1}{lh^3 E\left[(k_2 l)^2 + (\pi ml/l_1)^2\right]} - \frac{\rho_1 F\left[(k_2 l)^2 + (\pi ml/l_1)^2\right]}{4\rho lh}\right\} \quad (13)$$

correspond to dimensionless eigen-numbers $(\hat{k}_2 l)$ of the stiffened plate vibration.

The positive values of $k_2\varepsilon$ correspond to the determining part of elastic terms and negative ones correspond to that part of inertial terms. At high bending rigidity of stiffeners there is possible a case when the curve predicted according to Eq.(13) does not cross the region of eigen-number existence which correspond to the zero group, i.e. at $k_2 l < \pi$ there will be no eigen-numbers. Taking into account the relationship between eigen-numbers and frequencies

$$\omega_\alpha = k_\alpha^2 (D/\rho h)^{1/2}, \; k_\alpha^2 = k_1^2 + \hat{k}_2^2, \; \hat{k}_2 = \hat{k}_2(k_1, \varepsilon, N, r, q), \quad (14)$$

one can state that in this case the stiffened plate will not have eigen-frequencies lower than the first eigen-frequency of a freely-supported separate span.

Fig.2,a shows as an example some predicted eigen-functions (forms of eigen-vibration) of the plate with three stiffeners of the same material (aluminium alloy) of which the plate is made, at $h = 5\times10^{-4}$ m, $ml_1/l = 0.6$, $F = 6.3\times10^{-3}$ m^2, $J = 1.18\times10^{-5}$ m^4; Fig.2,b - at $h = 2\times10^{-3}$ m, $ml_1/l = 0.3$, $F = 4.5\times10^{-3}$ m^2, $J = 3.75\times10^{-6}$ m^4.

In the first case the stiffeners are characterized by a high bending rigidity, in comparison with the plate, and function $k_2\varepsilon$ does not cross the existence region of eigen-numbers of the zero group. In the second case the bending rigidity of stiffeners is small. Function $k_2\varepsilon$ crosses the existence region of eigen-numbers of the zero group. Therefore two eigen-numbers of the zero group appear the value of which is smaller than the first eigen-value of a freely-supported separate span. The first two vibration forms of the four ones shown in Fig.2b relate to the zero group and the other two refer to the first one. It is well seen in Fig.2b that at small values of $k_2\varepsilon$ the eigen-functions are close to those sinusoidal i.e. the stiffeners only slightly deform eigen-vibrations of the plate with general sizes $l_1 \times l_2$.

At $\varepsilon \to \infty$ and $\chi \neq 0$ the following expressions for determining the eigen-numbers and constants a_j in Eq.(8) for the forms of eigen-vibration are obtained:

$$\frac{k_2 cS - ksC + 2kk_2\chi cC}{k_2 S - ks} = \cos\frac{q\pi}{N}; \tag{15}$$

$$a_{1n} = (C - c)b_1,\ a_{2n} = -a_{4n} = b_1 s + b_2 S,\ a_{3n} = (C - c)b_2, \tag{16}$$

$$b_1 = Sc\sin\frac{q(n-1)\pi}{N} - [S + 2k\chi(C - c)]\sin\frac{qn\pi}{N},$$

$$b_2 = Cs\sin\frac{q(n-1)\pi}{N} + [S + 2k\chi(C - c)]\sin\frac{qn\pi}{N}.$$

The forced vibration and acoustic radiation of arbitrary elasto-inertial systems are sharply reduced at the frequencies lower than its lowest eigen-frequency. Therefore its lowest eigen-frequency increase leads to significant reduction of vibration and acoustic power radiated at low frequencies. The lowest eigen-frequency of the stiffened plate can be displaced into the higher frequency range by reducing the stiffener step. This stiffener step reduction is usually accompanied by their rigidity decrease and this can lead to occurrence of eigen-frequencies of the zero group the value of which can appear to be even much less than the lowest eigen-frequency of the plate with a large step of stiffeners.

The above obtained expressions can be used not only for the evaluation of plate regularities and trends associated with the effects of bending and torque rigidity of stiffeners but also for determining the relation between the plate and stiffener parameters at which the zero group frequencies will be deliberately absent. This relation can be written in the form of condition:

$$\left| b - \left(b^2 - 4a\right)^{1/2} \right| > 2 \ \ at\ \ k_2 l < \pi. \tag{17}$$

It is obtained without regard for dynamic torque rigidity of stiffeners. Taking into account its additional influence one can obtaine some increase of the lowest eigen-frequency, so that the lowest eigen-frequency of the stiffened plate at fulfilling the condition (17) will be deliberately higher than the first eigen-frequency of the freely-supported separate span. If we substitute the parameters of stiffened plates the vibration forms of which are shown in Fig.2 into (17) we can see that in the first case (see Fig.2a) it is fulfilled and in the second case (see Fig.2b) it is broken.

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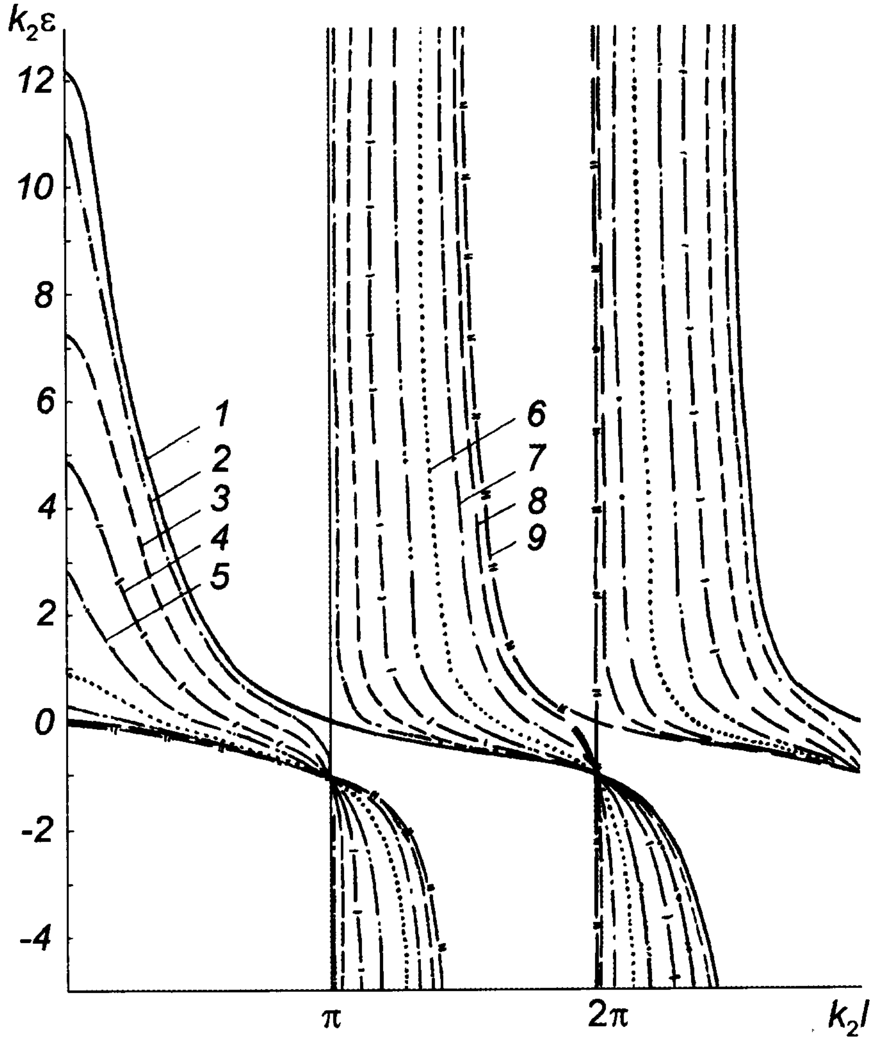

Fig.1. Effect of flexural rigidity of stiffeners on eigen-numbers of the stiffened plate: *1-q/N =1.0; 2-0.875; 3-0.75; 4-0.625; 5-0.5; 6-0.375; 7-0.25; 8-0.125; 9-0*

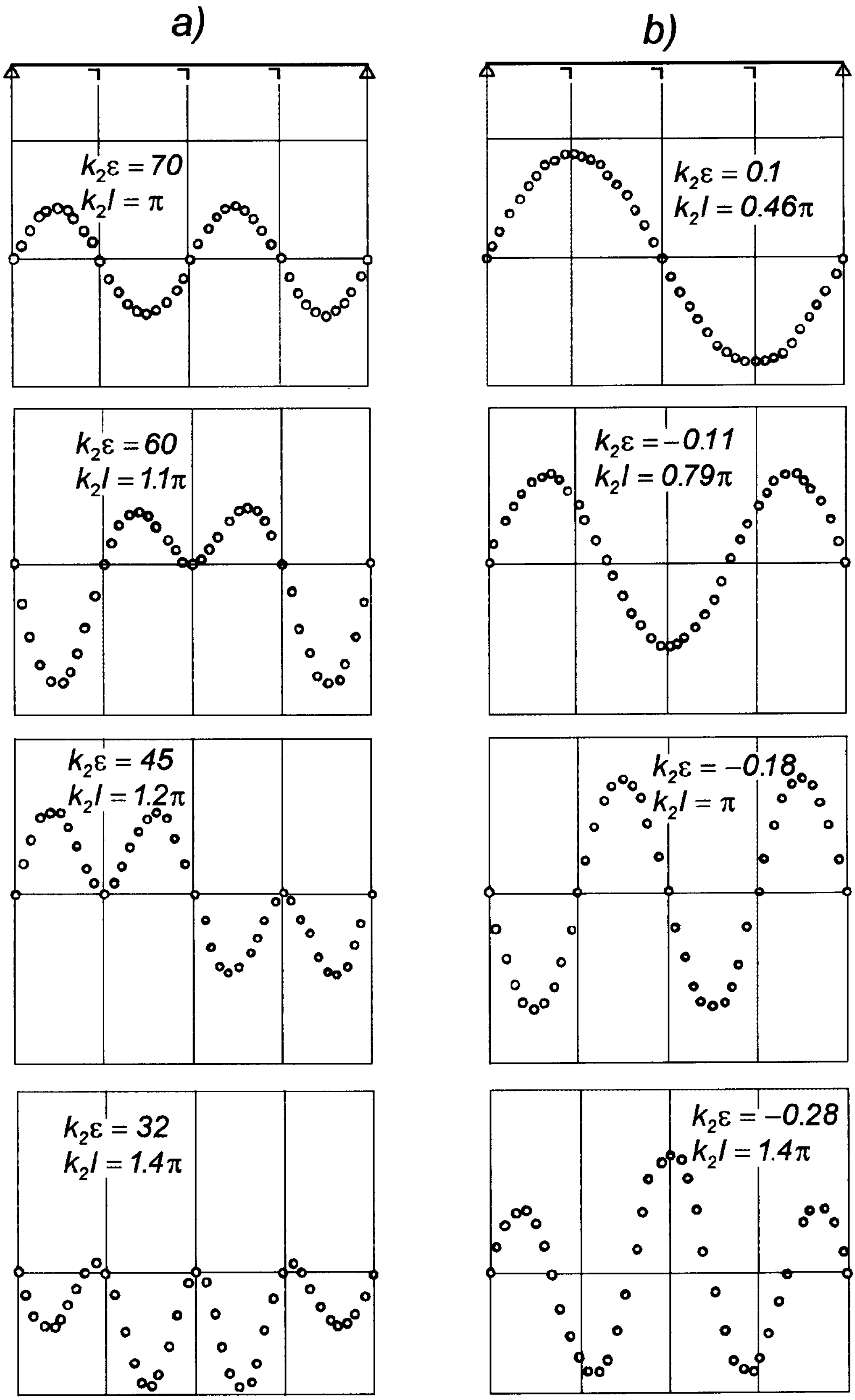

Fig.2. Forms of stiffened plate eigen-vibrations at high (a) and low (b) flexural rigidity of stiffeners (in comparison with rigidity of plates)