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Non-Linear Response of Acoustically Excited Panels

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The design of reinforced panel structures for enhanced sonic fatigue durability requires consideration of their non-linear response due primarily to the combined action of bending and membrane effects for large amplitude response. The paper presents the numerical and analytical methods used to predict vibration levels and provides a comparison with measured responses. It is shown that non-linear effects can greatly reduce the maximum response and to ignore them results in large overestimates on inservice strain levels together with an associated underestimate of panel fatigue life.

Introduction

The need to improve the reliability of commercial and military aircraft is becoming increasingly important in an atmosphere of civil airliner deregulation and reduced defence budgets worldwide. Pressure for ever increased maintenance and inspection intervals, widespread use of Carbon Fibre Reinforced Composite (CFRP) components, and greater reliance upon Computer Aided Design and Manufacturing (CADAM) with less emphasis on prototype testing has heightened the need for reliable design and analysis processes to safely predict the fatigue life of airframe structures.

The problem of acoustically induced fatigue failure has also been accentuated with minimum weight designs leading to higher component strain levels and increased engine performance yielding high intensity Sound Pressure Levels (SPL's) in excess of 150dB. It has been shown [5] that CFRP panels exhibit a higher degree of non-linearity than traditional aluminium panels, further accentuating the need for reliable, useable prediction tools to design efficient structures.

For random excitation, traditional analytical methods using small deflection theory and assuming linear behaviour have been shown to over estimate the response of panels as noise levels exceed 120dB [1]. Panel resonance frequencies tend to increase with increasing excitation level, their bandwidth also increase and the strain response becomes non-Gaussian [5].

For tonal excitation, which emanates from the power plant, for example, as propeller tones, and for a high modal density panel, a series of resonances will potentially coincide with the exciting tones. In practice however and as has been observed in laboratory studies [6], increasing panel displacements increase the panel stiffness and panel resonances will move away from the excitation frequencies.

Numerical methods have been used to improve predictions of panel response subject to high intensity tonal acoustic excitation. Comparisons of predictions with experimental results are discussed, leading to recommendations for further investigation.

Response of a Simple Clamped Panel

Analysis methods using elementary linear plate bending theory for determining the static or dynamic response of typical panels used in aircraft (where the thickness is small compared to the other dimensions of the panel) grossly overestimate both the deflections and the plate strains unless the deflections are small compared to the plate thickness. Elementary linear plate bending theory assumes that there are no mid-surface (ie membrane) stresses. In practice this is true only if the plate boundaries are free to move in a direction parallel to the plate surface so that membrane stresses cannot develop and linear plate theory is appropriate. However if the plate boundaries are constrained from movement in the plane of the plate then some membrane stress can develop. For acoustically excited panels the applied pressure is partially resisted by plate bending and partially by plate stretching. The bending and membrane effects are coupled theoretically, however a rigorous mathematical analysis is not warranted as results are known to be sensitive to boundary support conditions which in practice can only be approximated.

Some appreciation of the combined membrane and bending behaviour is provided by the following elementary discussion of a strip with clamped and immoveable ends (so as to restrict rotation and displacement parallel to the strip respectively). The structure is assumed to consist of two elements occupying the same space, one element being a beam with flexural rigidity (EI) but no axial stiffness, and the other a stretched cable with axial rigidity (EA) but no flexural stiffness. Then the required lateral pressure p_b, to bend the beam with maximum deflection ζ, is:

$$p_b = \frac{Et^3}{l^4} \cdot \zeta^2$$

where the span and thickness are given by ℓ and t respectively and E is Young's Modulus. To determine the membrane effect calculate the increase in length of the beam which is approximately given by:

$$\delta = \frac{\pi^2 \zeta^2}{8\ell}$$

Hence the membrane strain ε is:

$$\varepsilon = \frac{\delta}{\ell} = \frac{\pi^2 \zeta^2}{16\,\ell^2}$$

and the membrane tension force T is:

$$T = \varepsilon E t = \frac{\pi^2 \zeta^2 E t}{16\,\ell^2}$$

So the pressure required to stretch the beam for this membrane strain p_e is approximately given by: where the factor k depends on curvature (which is varying). Appropriate values for k range from 4 from small deflections to 2 for large deflections. For deflections about equal to strip thickness a value of about k of 3 is appropriate. Then the total pressure to bend and stretch the beam is:

$$p = p_b + p_e = (2\frac{Et^3}{\ell^4})\cdot\zeta + (k\pi^2\frac{Et}{16\,\ell^4})\cdot\zeta^3$$

Note that the effective 'stiffness' ($\partial p/\partial\zeta$) in the limit of larger deflections ie ($\zeta > t$) varies with the square of displacement. Thus the response to fluctuating harmonic loading is not harmonic and elementary linear dynamic analysis (ignoring membrane effects) can grossly overestimate both deflections and strains - provided the supports allow development of membrane stresses.

Smith, Malme, and Gogos [4] tested an aluminium strip under high intensity progressive acoustic wave fields. The panel tested was 2" wide, rigidly clamped at each end via 1" thick aluminium walls giving a free span of 12" (the long edges of the strip were left free). The strip was mounted in the side of a 1ft square test duct with a fundamental flexural resonance observed at 97.5 Hz (Q ~ 80). The strip was driven by a progressive sound wave from two test sirens: the duct sound levels were maintained at constant level and the excitation frequency increased slowly from ~80Hz (well below the lowest strip resonance) to ~300Hz. The measured strain levels, plotted against frequency for a sound pressure level of 150 dB are shown in Figure 1. The measured response shows typically non-linear behaviour with a distinct jump at about 140 Hz.

The strip strain response was analysed numerically using Finite Element (FE) techniques. A linear analysis using standard modal superposition methods and the measured Q, was used as well as a non-linear transient dynamic analysis using full numerical integration of the non-linear equations of motion.

Figure 1 also shows the results from the numerical analyses compared with the measured response. Note that the predicted natural frequency of the strip using linear analysis is about 95Hz, so the measured results show both a shift upwards of the resonance frequency and a much reduced response from the linear prediction.

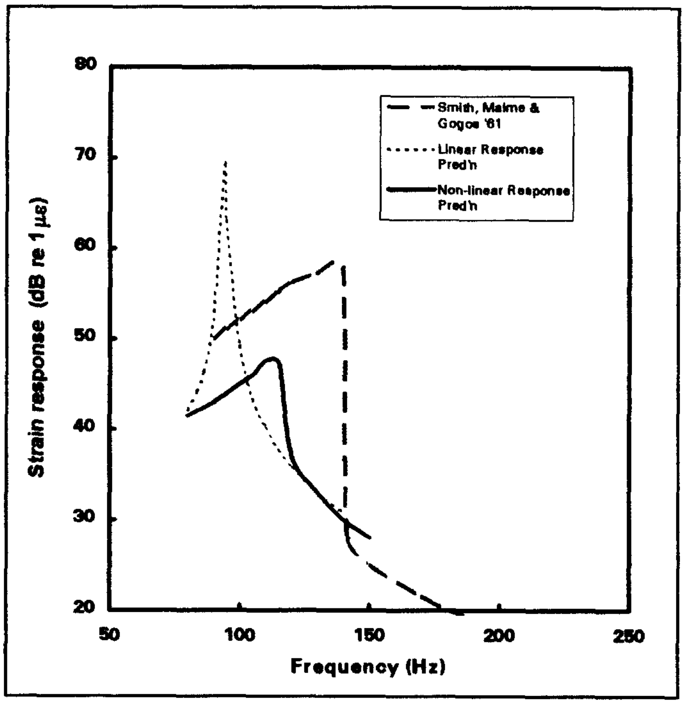

Figure 1: Strain Response Spectra for Tonal Excitation (140dB)

The linear predicted response level is 20dB above the measured level. The non-linear transient analysis yields results qualitatively consistent with experiment in that it does predict the jump in the response - albeit at a frequency of about 115 Hz. The numerical non-linear response is also significantly below the measured data. The reason is unclear although at this level of excitation the membrane strains would be about 50 microstrain corresponding to an elongation of 0.015mm. Hence such a movement of the supports would be sufficient to relieve the membrane stiffening effects completely.

Verification on a Ribbed Panel Structure

The numerical methods developed for the strip were then applied to a CFRP panel structure more typical of the type found on aircraft. A CFRP box structure was manufactured consisting of a three-bay span with each bay bounded by two ribs and a front and rear spar. Acoustic testing was performed upon the structure in conjunction with a numerical analysis using Finite Element (FE) methods.

To simulate the appropriate boundary conditions formed by the ribs (in a spanwise direction) and the spars (in the fore/aft direction) the model had all edges fully clamped re rotation, translation perpendicular to the plate surface and re translation in plane in a direction parallel to the plate edges. All edges were free re translation in plane in a direction perpendicular to the plate edges.

The panel loading was applied in two stages. First a static pressure was applied with non-linear geometry effects and stress stiffening effects included but with transient dynamic effects excluded, ie. the analysis was static non-linear. Second, the acoustic pressures were applied in a transient dynamic analysis which included non-linear geometry and stress stiffening effects

The methodology employed in the non-linear analysis numerically integrated the structural equations of motion in the time domain (transient analysis) for a period of time sufficient for any starting transients to be damped out. The generated time histories were then analysed to determine 'steady state' maximum strain levels. A constant loss factor value of 0.08 was used for all analyses. This non-linear analysis was performed separately for every frequency, every static pressure value, and every sonic pressure combination considered.

Figure 2 shows typical results from the non-linear analysis at a static pressure of 0.33 psi for the four levels of tonal excitation. By way of comparison, linear analysis at 150dB acoustic excitation level yields a peak strain of approximately 1900 με at the natural frequency of the panel (approximately 82Hz). This compares with approximately 350 με from the non-linear analysis.

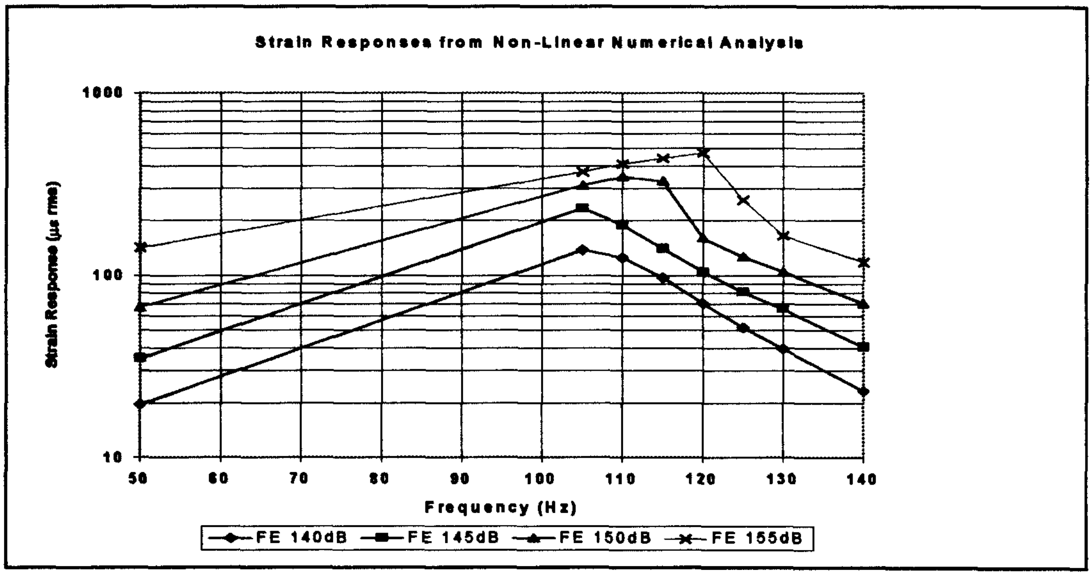

Figure 2: Non linear Strain Response to Acoustic Excitation with 2kPa Static Load

In addition to the numerical analysis a test panel with similar dimensions, properties, and boundary conditions was manufactured and tested in an acoustic chamber so as to provide a comparison with the numerical analysis. Testing consisted of loading the panel with static differential pressures similar to those analysed and exciting the panel with random and tonal excitations up to levels of 153dB. Results for an excitation frequency of 100 Hz and static pressure of 2kPa are shown in Figure 3.

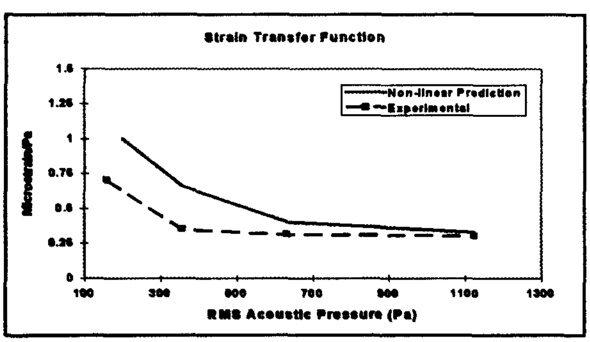

Figure 3: Comparison of Measured and Non-linear Predicted Strain Levels in Rib-Stiffened CFRP Panel

There is reasonable agreement between the predicted non-linear strain response and the measured data. The non-linear response is an order of magnitude smaller than the predicted linear strains.

Conclusions

Failure to recognise the non-linearities present in the response of thin aerospace panel structures results in gross over-estimates of the strain levels and a consequent structural over-design. The numerical approach has provided a significant improvement over the traditional linear methods of analysis in predicting the strain response for high intensity tonal acoustic excitation where the panel response is large compared to the thickness. Numerical methods can be used appropriately to analyse structures where the boundary conditions are too complex to describe analytically. Further work, however, should be performed to show if these methods are effective in predicting the broadband strain response of panel structures to high intensity random acoustic excitation [5].

References

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