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ANALYZING TIME-VARYING AND TRANSIENT VIBRATION PROPRTIES IN TECHNOLOGICAL SYSTEMS

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Abstract: Nowadays the joint time-frequency analysis and wavelet techniques have stepped out from the academic research sites and spread out to several industrial applications. The static nature of the conventional spectral techniques used in the analysis of vibrating structures means considerable limitation in certain type of systems and problems arisen on them. Fast changes in the vibrating structures and the presence of transient effects need advanced methods for a compromise-free analysis. The startup or shutdown processes of large rotational machines are typical examples. Continuous monitoring and failure detection of the startup processes is a significant means to avoid serious damages or accidents, hence to save costs of operation and maintenance. The application of wavelets in analyzing the transient processes by preserving the harmonic behavior of vibrations results in the joint time-frequency (JTFA) methods. A new, novel approach with wavelets stems from the approximation properties of these type constructs. The approximations of signals belonging to L^2, L^∞, H^2, H^∞ spaces form nowadays a rapidly evolving field with applications in the identification and detection. This paper is devoted to the investigation how can a priori information (derived from the physical and technological knowledge) be used in wavelet approximations with the purpose to achieve efficient detection of specific phenomena in the vibration signals.

1. INTRODUCTION

Exploring dynamics of systems have got great significance in numerous fields of the industry and power production, e.g. in failure detection, inspection, maintenance, etc. Conventional (FFT-based) and model-based (AR and ARMA) spectral methods proved to be efficient tools in describing the dynamic of systems [1]. Spectral methods are also powerful in system identification, modeling and control design. However, these methods are mainly useful in investigation of slowly varying processes, when the signals can be supposed to be stationary during the period of analysis. Stepwise changes are allowed only at the boundaries of the stationary peaces of the time domain; this model is used in the detection of abrupt changes in the system tested. The conventional mode of analyzing spectral changes is as follow: applying spectral estimation methods to periodically measured short signal records, and drawing and investigating trend and/or waterfall diagrams of the resulting spectra. As it will be shown later, this style of the analysis is not suitable for the investigation of quickly changing processes, e.g. transients on the system behavior, or startup and shutdown processes. In vehicles the system behavior can often be characterized as instationary, and a rather significant task is just the examination of changes. In many cases changes are not satisfactory to be modeled as steps (abrupt changes), the 'shape' of them is not less important. Joint time-frequency description of dynamical systems - re-invented and elaborated recently - offer a good basis for investigations on the field of time-varying systems. Joint time-frequency analysis (JTFA) methods are originated from some extensions of the windowed Fourier-transform methods.

The joint time-frequency descriptions have now been embedded in an excellent framework to investigate them by introducing the concept of wavelets [5]. Within this framework the intuitively widely used and in physical sense correct time-varying spectra have gained an exact mathematical foundation, and several methods have been developed to investigate specific physical systems [6]. In addition wavelet bases offer new possibilities for system modeling and detection of changes or several other phenomena, which were not common in the conventional spectral methods. A new, novel approach with wavelets stems from the approximation properties of these type constructs. The approximations of functions (i.e. signals) belonging to L^2, L^∞, H^2, H^∞, as well as the unit disc form nowadays a rapidly evolving field with applications in the identification and detection [7].

Nowadays the joint time-frequency analysis and wavelet techniques have stepped out from the academic research sites and spread out to several industrial applications. The static nature of the conventional spectral techniques used in the analysis of vibrating structures means considerable limitation in certain type of systems and problems arisen on them. Fast changes in the vibrating structures and the presence of transient effects need advanced methods for a compromise-free analysis. The startup or shutdown processes of large rotational machines are typical examples. Continuous monitoring and failure detection of the startup process of large rotating machines (e.g. turbine - turbo generator machine group in conventional or nuclear power plants) is a significant means to avoid serious damages or accidents, hence to save costs of operation and maintenance.

2. JOINT TIME-FREQUENCY ANALYSIS

Frequency domain description of the behavior of systems is a very common in practice, since periodical motion, vibration, wave motion is very common in mechanical, electrical, etc. structures. Conventional spectral analysis is based upon the Fourier-transform. The Fourier transform is very informative in analyzing systems which the inherent energy is distributed around some frequencies in, i.e. the spectra contain formations localized in some frequency regions. Frequency localization capability of the Fourier spectrum is the property, which makes it useful for practical purposes. The more characteristic formations are peaks in the spectrum, which usually form an excellent basis to analyze the dynamic behavior of many systems, including most of mechanical, electric and electronic ones. Change detection upon spectral peaks can be the starting point of early failure detection in dynamic systems, i.e. can indicate evolving failures before they cause serious problems, e.g. breakdowns, accidents, disasters. However this statement contradicts with the time-independence of the Fourier spectra, which can directly be seen from its definition.

Time-dependency of spectral analysis is a very common assumption in practice, also in the field of several physical (industrial) systems. Some characteristics of a dynamic system can change within a shorter or longer time period. Most of the changes in the system characteristics influence the spectral behavior of the system, i.e. influences the distribution of energy in several frequency bands, e.g. a weakening spring changes the resonance characteristics of a vibrating structure. In practice the short-time Fourier-transform has been the means of realizing time-localization in spectra. Short-time Fourier-transform means that relatively short records of the analyzed signal are transformed subsequently, hence time-localization is assured through the series of the subsequent spectra. Slow changes in system parameters can excellently be analyzed by this technique. Applying distance measures or trend analysis of individual points or intervals of the series of spectra, as well as a waterfall-type graphical representation of them are adequate tools for analysis, change detection and decision upon failures.

However there are constraints in applying short-time Fourier-transform: fast changes in the system characteristics require shorter records to involve in Fourier-transform. Shorter records ensure a better time-localization, however results in the deterioration of frequency-localization characteristics. This fact is the consequence of the so-called uncertainty principle. The more common expression of it is as follow:

$$T_t B_\omega \geq C$$

where T_t is the duration of the signal on the time-scale, B_ω is its bandwidth on circular-frequency base, C is a constant. The uncertainty principle stated here is similar to that expressed by Heisenberg in the field of quantum mechanics in 1927. See [3, 4] for details. The uncertainty principle states, that time duration cannot be decreased without the bandwidth being increased, therefore while decreasing the time duration the frequency resolution will also be decreased, i.e. time-localization and frequency-localization on the basis of Fourier-transform are contradictory requirements.

Constructing a method that describes the resolution of the total energy of a system in time and frequency domain seems to be a rather natural demand. Energetical considerations can easily be translated to mathematical conditions by selecting signals to belong to $L^2(\mathbb{R})$ space of f(t) functions. The desired time-frequency distribution is a function of the form

$$W(t, \omega) : (\mathbb{R}, \mathbb{R}) \rightarrow (\mathbb{R})$$

The demand for the existence of an inverse transform and a Parseval-type theorem, i.e. the total energy must be the integral of the function, involves $W(t,\omega)$ to belong to $L^2(\mathbb{R}, \mathbb{R})$.

A possible generalization is the use of short-time Fourier-transform with a window function running along the time-scale. Let w(t) be a so-called window function, a function belonging to $L^2(\mathbb{R})$ space, symmetric to t, and finite duration at least in the sense of standard deviation (defined above in connection with the uncertainty principle). The joint time-frequency distribution is based upon the Fourier-transform of the windowed signal, i.e. $s_t(\tau) = s(\tau)w(t-\tau)$

$$\mathrm{S}_t(\omega) := \int_{-\infty}^{\infty} s(\tau) w(t-\tau) e^{-\iota\omega\tau} d\tau \qquad W_{Sg}(t,\omega) = \left|\mathrm{S}_t(\omega)\right|^2$$

The $W_{Sg}(t,\omega)$ function is called spectrogram. In the practice of course the infinite bounds of integral are replaced by finite ones, which can be done errorless, if the window function is finitely supported, or with small error if the functions have fast decay. The central point of the method is the optimal selection of the window function. Optimality criterion is not unique, because there can exist several user demands for the level of time and frequency localization, which cannot be selected independently as it is stated in the uncertainty principle. A good time localization results in poorer frequency resolution, and contrary, to obtain a less biased spectra, a wider window function is required.

D. Gabor in his famous paper written in 1946 [2], suggested as optimal window the Gauss function (with $\alpha > 0$):

$$w_\alpha(t) = \frac{1}{2\sqrt{\pi\alpha}} e^{-\frac{t^2}{4\alpha}}$$

This function is not finitely supported, however its duration is exactly 2α in the sense of the standard deviation. A remarkable property of this function is that its Fourier-transform is a similar function, i.e. $W_\alpha(\omega) = e^{-\alpha\omega^2}$ with bandwidth proportional to α^{-1}.

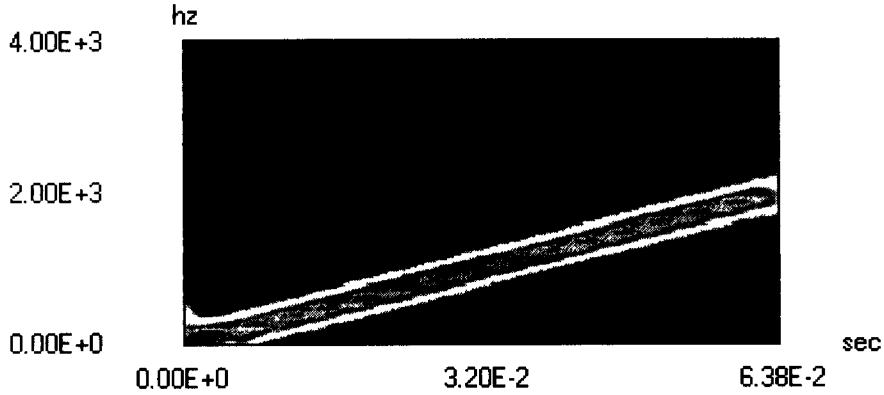

Fig. 1.: Gabor transform of a chirp signal

As a generalization of the windowed Fourier-transform applying Gauss-window, an orthogonal basis can be constructed in $L^2(\mathbb{R})$, which offers a framework of approximating functions with superior time-frequency resolution characteristics [3]. This transform is called Gabor-transform, and the basis resulted in Gabor-wavelet (see later for the concept of wavelets). The Gabor-spectrogram of a chirp function (i.e. a sine-signal with linearly increasing frequency) on an `intensity-plot` can be seen in figure 1. as illustration.

Another way of treating the problem of time-varying spectra has been suggested by Ville. A natural requirement for joint time-frequency distributions is to satisfy marginals, i.e.

$$\int_{-\infty}^{\infty} W(t,\omega)dt = |S(\omega)|^2 \qquad \frac{1}{2\pi}\int_{-\infty}^{\infty} W(t,\omega)d\omega = |s(t)|^2$$

Ville suggested the Wigner distribution as a suitable function. This joint time-frequency distribution - which plays a central role in time-frequency analysis - is called Wigner-Ville distribution. It is defined as follow:

$$W(t,\omega) = \int_{-\infty}^{\infty} s\left(t+\frac{\tau}{2}\right)\bar{s}\left(t+\frac{\tau}{2}\right)e^{-\iota\omega\tau}d\tau$$

The Wigner-Ville distribution is always real, shows symmetry - for real s(t) - both in t and ω, and satisfies the marginals introduced above. Many variations of this distribution have been developed to achieve better characteristics, e.g. the Choi-Willliams distribution that can be considered as a Gauss windowed variant of the Wigner-Ville distribution; see [4] for details.

3. WAVELETS

A more general approach to similar problems in physics has been emerged in the past decade, the wavelet decomposition of signals. In general wavelet (i.e. small wave) is a signal of finite (or almost finite) duration which is dilated and translated on the axis of the independent variable. If some conditions (discussed later) are hold, any signal representing finite energy can be decomposed into the dilated and translated form of this function the so-called mother-wavelet denoted here $\psi(t)$. The wavelet decomposition of the f(t) $f \in L^2(\mathbb{R})$ (more exactly the continuous form of the transform) is done by the following formula:

$$W_\psi(a,b) = |a|^{-1/2} \int_{-\infty}^{\infty} f(t)\psi\left(\frac{t-b}{a}\right)dt$$

where $[(t-b)/a]$ represents the dilated form (at scale **a**) and the translated form (by translation **b**) of the mother wavelet. $W_\psi(a,b)$ is called the continuous wavelet transform of f(t) respective to mother wavelet ψ. Variable **a** is said to be the scale variable, b represents the time displacement, therefore the general wavelet transform of a signal (function of time) is said to be a time-scale decomposition.

Scale variable shows strict correspondence with frequency, however there is no equivalence between them except for specific mother wavelets. Time-frequency descriptions outlined above can also be considered as wavelet decomposition's based upon mother vavelets which keep the physical interpretation of frequency, i.e. keep connection to harmonic vibrations. For example the Gabor wavelet (inherent in the Gabor transform), roughly speaking, has the form of a trigonometric system multiplied by a Gaussian window function, which ensures the time localization.

Time-scale descriptions that do not correspond to time-frequency descriptions are not able to describe energy distribution of the signal trough the time-frequency plane, however they can have other remarkable advantages. For example they can serve as a basis for minimal size approximation of signals or pattern recognition with unknown length (scaling) of the pattern to look for. Using specific type of (admissible) functions as mother wavelet provide a highly sensitive tool for detecting similar patterns in the signal. Mathematically the admissibility condition for a mother wavelet is as follow - $\hat{\psi}(\omega)$ is the Fourier-transform of the $\psi(t)$ $\psi \in L^1(\mathbb{R}) \cap L^2(\mathbb{R})$ function:

$$C_\psi = \int_{-\infty}^{\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega > \infty$$

i.e. the mother wavelet must be a bounded function with fast decay, something similar to that shown on the small figure on right. Finite support is advantageous in many applications (wavelets with finite support has been constructed by Daubechies - see for details in [5]), however it is not necessary for a good description and makes practical computations too complicated. The transient phenomena occurring in signals in most cases can be expressed as functions satisfying the admissibility conditions. Selecting a mother wavelet satisfying further conditions originated from a priori assumptions of the system results in detection methods very sensitive to the desired characteristics.

For the practice the discrete representation of wavelet decomposition is advantageous. After Daubechies [5] the following form is conventionally used:

$$\psi_{jk}(t) = 2^{j/2}\,\psi(2^j t - k) \qquad j, k \in \mathbf{Z}$$

i.e. dyadic dilations and integer translations are used. By applying this form the wavelet transform of any admissible function is an infinite two-dimensional sequence of numbers w_{jk}. It can be proved that wavelet elements form an orthonormal basis in $L^2(\mathbb{R})$, i.e. the f(t) function can be expressed with the wavelet elements and the w_{jk} coefficients as follow:

$$f(t)=\sum_{j,k} w_{jk}\psi_{jk}(t)$$

The coefficients representing the wavelet transform can be computed by the scalar product of the f(t) function with the corresponding wavelet element, i.e. the dilated and translated form of the mother wavelet. For realizing the wavelet transform numerous algorithms, including fast transforms (analogous to fast Fourier-transform) have been developed.

4. APPROXIMATIONS IN NONSTANDARD BASES

Exploring the frequency domain behavior of a signal or system can be viewed as approximation of the concerned time-functions in a basis consisting of trigonometric functions. The set of the approximation coefficients can be interpreted as the distribution of energy-like characteristics of the system upon frequency domain. The trigonometric basis (built up of sinusoids of several frequencies) has become standard in the signal and system theory, however it is also the source of some problems which has arisen with the requirements of the robustness in system control and other areas – including the detection-type problems. The trigonometric basis well approximates the signals with finite energy – or to be expressed mathematically: the functions belonging to L^2 or H^2 spaces, however arises problems in signals belonging to L^∞ or H^∞ spaces, emerging in the robust control system design. The control society has concluded to use so called nonstandard basis functions in describing signals and systems to eliminate problems in L^∞ and H^∞ system description (see e.g. [7] for a survey on this field). The use of nonstandard bases also proved to be efficient in other fields than to solve robust control problems: e.g. the description of signals and systems in nonstandard bases seems to be advantageous also in the detection problems arisen in failure monitoring and detection of industrial systems, as it will be investigated below.

As a general framework we introduce the generalized Kautz basis for discrete-time systems as follows. The generalized Kautz system Φ_m (m = 0, ±1, ±2, ...) is a system of rational functions defined on the complex plain $\mathbb{C}$. This system can be obtained from the functions

$$r_{k,l}(z)=\frac{z^l}{(1-\bar{a}_k z)^{l+1}} \qquad k=0,1,\ldots,N-1 \qquad l=0,1,2,\ldots \qquad z\in\mathbb{C}$$

by the Gramm-Schmidt orthogonalization procedure, where $a=(a_0,a_1,\ldots,a_{N-1})$ is a set of poles ordered into conjugated complex pairs belonging all to the open unit disk This system is orthonormal with respect of the scalar product

$$\langle F,G\rangle := \frac{1}{2\pi}\int_{-\pi}^{\pi} F(e^{jt})G(e^{-jt})dt$$

Φ_m can be expressed by the Blaschke functions

$$B_b(z)=e^{jb}\frac{z-b}{1-\bar{b}z} \qquad z,b\in\mathbb{C}$$

i.e. with $m=lN+k$

$$\Phi_m(z)=\Phi_k(z)B_a^l(z) \qquad \text{and more direclty} \qquad \Phi_k(z)=\frac{\sqrt{1-|a_k|^2}}{1-\bar{a}_k z}\prod_{j=0}^{k-1}B_{a_j}(z).$$

In the case of $N=1$ and real $a=a_0$, the so-called Laguerre system is obtained. In the case of $N=2$, and $a_0=\bar{a}_1$ we get the Kautz system.

The description of signals and systems in Laguerre, Kautz, and generalized Kautz bases is advantageous if a priori information is available of the system behavior, e.g. there exist – at least qualitative - information of system poles. The system can be described by Laguerre, Kautz, or generalized Kautz bases if the system possesses dominantly one real pole, one conjugate pair of poles, or a finite series of conjugate complex pairs of poles respectively. Approximation of signals in these cases results in few dominant parameters with quite fast convergence, while in other cases there are no dominant parameters, and the approximations converge slowly. Considering the time-domain behaviour of the system: if the analyzed signal consist of aperiodic exponentionals, it can well be approximated by the Laguerre system; similarly if the signal consist of exponentially damped periodic components, it can well be approximated by the Kautz system; a mixed more complex case can be covered by the generalized Kautz basis.

The approximation of signals and systems on the basis the generalized Kautz system can be performed by applying a nonlinear argument transformation, as it has been shown in [8]. It has also been verified, that this procedure can be realized in practice by applying Discrete Fourier Transform, which can be performed effectively by using FFT algorithms. The use of the generalized Kautz system results also in other advantages, e.g. robust detection algorithms can be derived upon it in the sense that L^∞ or H^∞ criteria can be applied to suppress unwanted disturbances. This fact is originated from the proper approximating properties of the generalized Kautz system in the L^∞ and H^∞ spaces (which is not valid for the standard trigonometric base), as it has been shown e.g. in the survey [7].

The nonstandard basis to be used for describing the signals or system can also be a wavelet basis, i.e. it can be built up of the dilated and translated forms of a mother wavelet. The mother wavelet can be selected to include properties based upon a priori knowledge belonging to the system to be examined. A priori properties to be considered can be e.g. the shape of an effect on the signal, at least qualitatively, or it can be based upon the assumed behavior of the system in the frequency domain, the assumed position of the poles, etc. The detection procedures are based upon the dominant coefficients of the wavelet approximation in any displacement (location of time in most cases), i.e. upon peaks on the time-scale domain. These types of approximations - mainly based upon the wavelet forms of the generalized Kautz system - are examined now, and will be published soon.

5. ADVANCED DETECTION METHODS

In the present paper we consider the goal of a signal and system analysis to be a means of identifying specific characteristics or the changes on them. The detection method to be applied should emphasize the effect to be identified with the purpose to achieve high sensitivity in the detection, while the unwanted disturbances were suppressed. A characteristic classical example to this type of methods is the Fourier-transform based spectral ones: they are highly sensitive in the detection of weakly damped harmonic motion, and resonance phenomena; these effects can be identified as spectral peaks on distinguished frequencies. The Fourier-transform based methods use the standard trigonometric basis for interpreting the signals. The Fourier-transform based methods suffer from some obvious disadvantages, which stem from the static nature of the Fourier-transform, and the uncertainty principle, as it has been described in the previous sections. The advanced methods introduced above mean development in two directions.

Some methods while try to preserve the harmonic nature of the description, i.e. they remain loyal to the classical frequency based spectral view, by keeping the trigonometric system as basis, they offer time dependence, and joint localization in time and frequency domain. These approaches constitute the class of Joint Time-Frequency Analysis methods. They can be used instead of the classical spectral methods in systems, which can be characterized as harmonically vibrating ones, however rapid and continuous changes in their operation arise problems. The transient behavior between state changes, the startup and shutdown processes in most of industrial systems belong to this class. A typical example, which possesses great significance in the practice, is the startup process of a turbo-generator machine group. The real-time monitoring of the changing spectral behavior and fast identification of harmful effects occurring in the spectra can prevent from serious damages and accidents.

An alternative direction of the development realizes detection methods which can be sensitive to specific a priori known (or assumed) effects on the systems, even by giving up the frequency based spectral interpretation of the phenomena. The use of specific nonstandard basis functions in approximating signals or applying wavelet decompositions by using specific mother wavelets both fit to some a priori known characteristics of the phenomena to be detected forms the basis of the methods. For example loose part detection applied in most nuclear power plants can be realized by approximating the recorded vibration signals in a Kautz type basis, because the effect of a collision of a loose part with the walls of the structure results approximately in an exponentially damped sinusoid. The detection procedure can be based upon the fact, that the normal signal (which can be considered as noise) cannot be well approximated in the selected basis, i.e. there are no dominant coefficients. When collision effect is present, some parameters become dominant. A statistical decision method (e.g. GLR) can be built upon the transform to realize efficient detection.

6. CONCLUSIONS

Besides the joint time-frequency descriptions, which can supercede the classical Fourier-transform based spectral methods, the use of nonstandard basis functions and wavelets constructed upon the a priori knowledge available of the phenomena to be identified form a novel approach of the description of systems and signals, with the purpose of sensitive detection of specific effects, e.g. state changes, failures, etc. The elaboration of efficient methods on specific systems and phenomena, for example detection of harmful effects on turbine startup processes and loose part detection in nuclear power plants, as well as testing them on real records are performed now.

7. REFERENCES

[1] Bokor, J., S. Veres and I. Nagy (1988). Reactor noise analysis applying multivariable AR and ARMA methods in NPP Paks. In: Singh M.G., K.S.Hindi et al. eds., Fault Detection and Reliability: Knowledge Based and Other Approaches, Pergamon Press, 207-214.

[2] Gabor, D.: Theory of communication, J. IEE (London) Vol. 93, 1946. (p. 429-457)

[3] Chui, Ch. K.: An Introduction to Wavelets, Academic Press, Inc., Boston etc., 1992.

[4] Cohen, L.: Time-Frequency Analysis. Prentice-Hall PTR, Englewood Cliffs, 1995.

[5] Daubechies, I.: Ten Lectures of Wavelets. SIAM, Philadelphia, 1992.

[6] Meyer, Y.: Wavelets: algorithms and applications. SIAM, Philadelphia, 1993.

[7] Bokor, J.: Approximate Identification for Robust Control, 2nd IFAC Symposium on Robust Control Design, Budapest, Hungary, June 25-27, 1997.

[8] Soumelidis, A., J. Bokor, F. Schipp: Representation and Approximation of Signals and Systems Using Generalized Kautz Functions, 36th IEEE Conference on Decision an Control, San Diego, CA, Dec 10-12, 1997, (to appear).