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Study of Vibrating Systems Resulting from Collision.
- Rebound Characteristics of the Equivalent Stiff Surface -

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This paper deals with the analysis of the chattering phenomenon exhibited by the one-degree-of-freedom system resulting from collision.

The objective of the authors is to investigate the state of rebound and contact time when the mass element of the vibratory system comes into contact with the collision surface. The collision surface is represented on paper, by the stiff surface that comprises a spring and a damper.

The chattering phenomenon is characterized by some properties of the system model, and moreover the considerations, for the analysis of collision systems by the application of nonlinear equivalent equations for the stiff surface, as indicated by the results of our experiments.

1. Introduction

The chattering phenomenon of a point of contact exhibits extremely complicated vibrations resulting from collision. In fact, the point of contact itself behaves like a nonlinear spring system, therefore a vibrating system resulting from collision can be analyzed more accurately by simulating the point of contact in terms of a multi-degree-of-freedom vibrating system considering the coefficient of loss/damping and that of rebound as a function of velocity. In this study, the point of contact is modeled as a one-degree-of-freedom vibrating system composed of factors such as stiffness and damping coefficient. The "equivalent stiff surface" whereby the rebound mechanism exhibits energy loss as a result of collision was employed. Under these conditions, we report the results of a test for determining the completion condition of chattering through the behavior of two systems that interfere under arbitrary conditions.

2. Method for the computation of the dynamic behavior of a vibrating system resulting from collision

In the case of a real spring system, Hertz's stiffness exists at the point of collision and the equation of motion is described as a non-linear function with respect to displacement. Thus it should be treated as a non-linear system. In addition, to obtain the dynamic behavior of the vibrating system resulting from collision, during a stepwise computation with a constant time step $\Delta\tau$, the minute time step $\Delta\tau$ should be corrected and the rebound velocity of mass points should be considered because mass points of the spring system collide with the stiff surface. According to the principle of topological analysis, information regarding the displacement and velocity of mass points can be obtained on the orbit of a topological surface, which simplifies the study. Accordingly, we derived the following stepwise equations required for computing the dynamic behavior of the vibrating system resulting from collision.

$$\left.\begin{aligned} \dot{x}_{k+1} &= \lambda_2 \dot{x}_k + \lambda_1 \ddot{x}_k \\ x_{k+1} &= x_k + \lambda_1 \dot{x}_k + \lambda_3 \ddot{x}_k \end{aligned}\right\} \qquad (1)$$

$$\left.\begin{aligned} \lambda_1 &= \sin\omega_n \Delta t \\ \lambda_2 &= \cos\omega_n \Delta t \\ \lambda_3 &= 1 - \lambda_2 \end{aligned}\right\} \qquad (2)$$

Using the above stepwise equations, the next state of P_{k+1} can be calculated if the trajectory at any initial state of P_k is given.

3. One-degree-of-freedom vibration system collides with a stiff solid

In this section, we describe a case in which the mass points of a one-degree-of-freedom vibrating system collide with a stiff surface, the rebound coefficient of which remains constant regardless of the collision velocity.

The vibrating system resulting from collision is shown in Fig. 1. The equation of motion for the system is given by

$$\ddot{x}(\tau)+2\zeta\dot{x}(\tau)+x(\tau)=F \qquad (3)$$

where s is the stiffness of the spring system, r is the damping coefficient, m is the mass point mass, f is the external force exerted on the mass point, the damping ratio is $\zeta=r/(2\sqrt{ms})$, the undamped natural frequency is $\omega_n=\sqrt{s/m}$, the constant displacement is F = f/s, and the non-dimensional time is $\tau = \omega_n t$. In Fig.1, k_0F is the pushing distance.

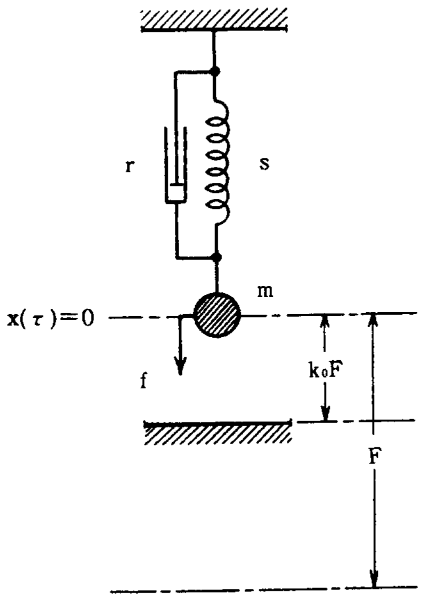

Fig.1 Generalized model of one-degree-of-freedom vibrating system resulting from collision.

Figure 2 shows the dynamic behavior and the topological orbit of the system obtained by a computation, where the rebound coefficient of the stiff surface is set as 0.6, $\omega_n = 0.25$, $F=1$, $k_0F=0.75$, and the time step for stepwise computation $\Delta\tau = 0.01$.

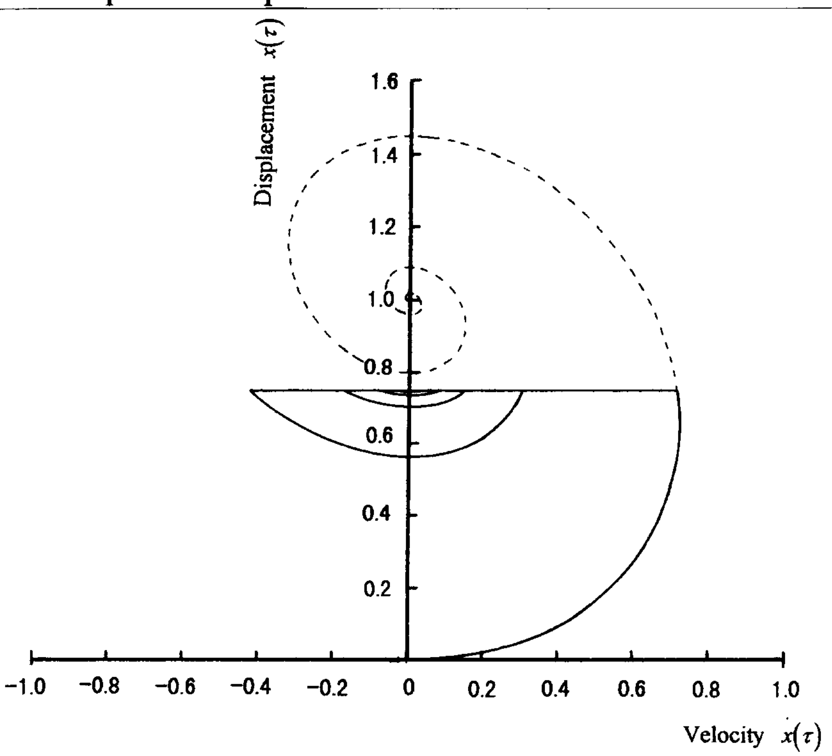

Fig.2 (a) Phase-plane trajectory ; one-degree-of-freedom vibrating system resulting from collision.

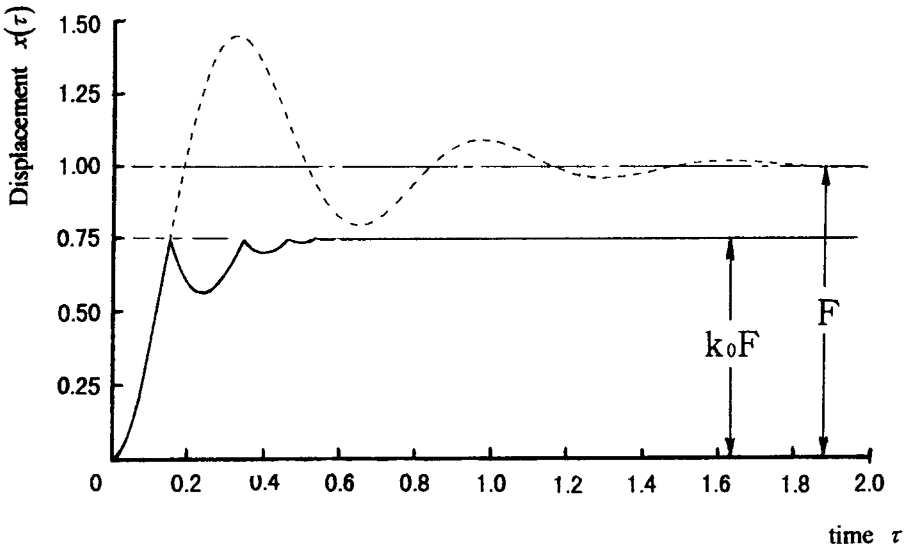

Fig.2 (b) Time response curve ; one-degree-of-freedom vibrating system resulting from collision.

3-1. One-degree-of-freedom vibrating system collides with an equivalent stiff surface

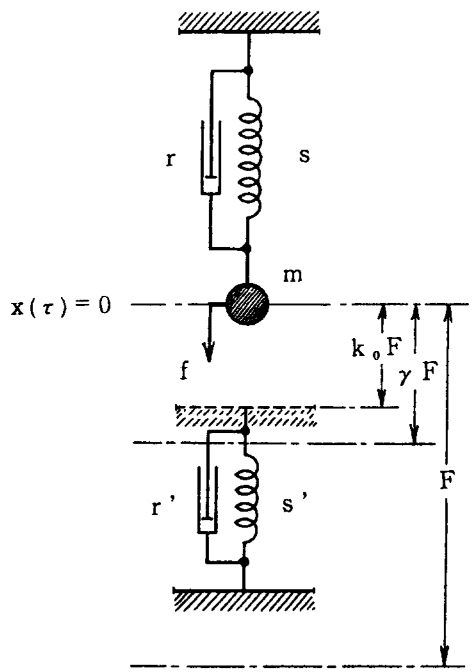

Fig.3 Generalized model of one-degree-of-freedom vibrating system and equivalent stiff surface.

As shown in Fig.3, we consider a fixed plan model having stiffness s' and damping coefficient r' in which the stiff surface determines the rebound. We call this stiff surface an "equivalent stiff surface". If we assume that γ represents the constant displacement at the time of coupling of the spring system and the equivalent stiff surface, the equation of motion for a coupled vibrating system is represented as

$$\ddot{x}(\tau)+2\alpha\zeta\dot{x}(\tau)+\beta x(\tau)=\gamma F \quad (4)$$

where $\alpha=1+(r'/r)$,

$\beta=1+(s'/s)$,

$\gamma=\{1+k_0\,(s'/s)\}/\beta$.

3-2. Chattering completion condition on linear equivalent stiff surface

A mass point that collides with an equivalent stiff surface at an arbitrary velocity remains in the region where $\{x(t)-k_0F\}$ until it is pushed back by the equivalent stiff surface. The mass point of the vibrating system sinks into the collision surface as a result of a minute displacement. This means that the surface is elastic in terms of rebound mechanism and yet rigid enough not to cause plastic deformation.

We can locate a boundary point at which rebound of the mass point ceases and transition to coupled vibration begins, through an investigation of the contact time and rebound coefficient on the equivalent stiff surface. The presence of the boundary point is the condition for the completion of chattering for a contact point. We call this collision velocity v_c "critical velocity" and it is given as a function of the damping ratio ζ. The computation curve in Figure 4 shows the critical velocity when $k_0=0$. The critical velocity with any k_0 is given by enlarging the curve for k_0 by $(1-k_0)$ fold.

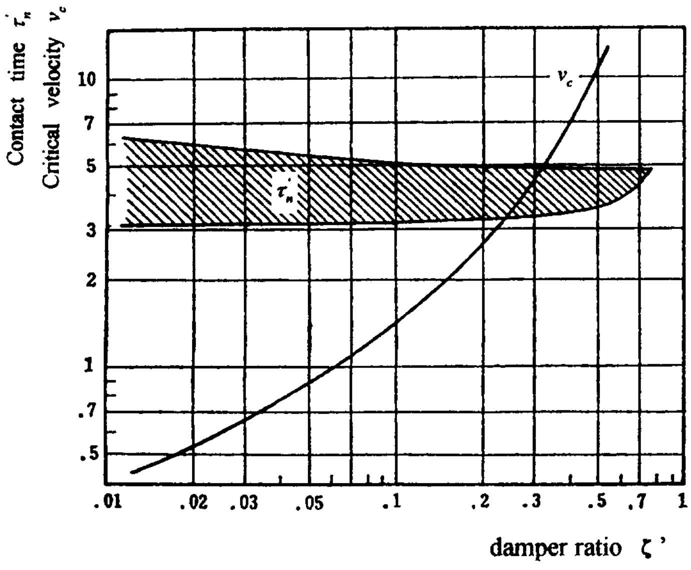

Fig.4 Boundary diagram of contact time and critical velocity for damping ratio.

3-3. Collision velocity, rebound coefficient and contact time

The contact time τ'_n becomes infinity at the end of chattering. This is shown in Fig.4 as the upper limit curve of τ'_n. When the collision surface is placed at the free equilibrium point of $k_0=1$, the rebound coefficient e_0 and contact time are given by

$$e_0 = \exp\left(-\frac{\pi\zeta'}{\sqrt{1-\zeta'^2}}\right) \tag{5}$$

$$\text{where} \quad \zeta' = \frac{\alpha}{\sqrt{\beta}}$$

$$\tau' = \frac{\pi}{\sqrt{1-\zeta'^2}} \bullet \frac{1}{\sqrt{\beta}} \tag{6}$$

which indicates that the collision velocity has no effect on them. As a result, τ'_n can exist in the hatched area in Fig.4.

As an example, Fig.5(a) shows the relationship between the collision velocity and rebound coefficient, Fig.5(b) shows the relationship between the collision velocity and contact time when damping ratio is fixed at $\zeta = 0.05$ and parameter k_0 lies between 0.5 and 1.0, Fig.5(c) shows the relationship between rebound coefficient and contact time when parameter ζ' lies between 0.02 and 0.2.

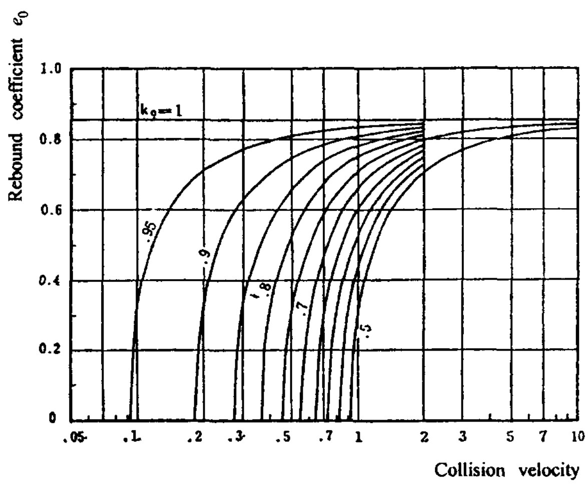

Fig.5 (a) The rebound coefficient versus collision velocity in case of linear equivalent stiff surface.

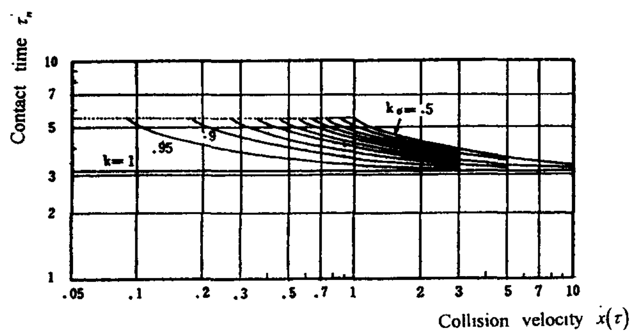

Fig.5 (b) Contact time versus collision velocity in case of linear equivalent stiff surface.

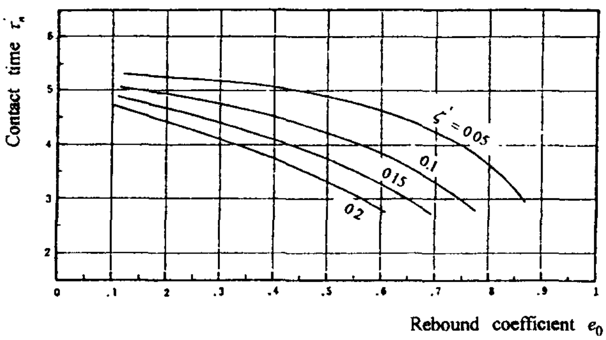

Fig.5 (c) Contact time versus rebound coefficient.

4. Chattering completion condition on a nonlinear equivalent stiff surface

The measured rebound coefficients of metals such as copper and brass generally average 0.85 and tend to decrease as the collision velocity increases. The contact time tends to decrease as the collision velocity increases and gradually levels off at a certain value. This tendency is generally considered to be "a result of the higher order of vibration which occurs simultaneously at the time of collision".

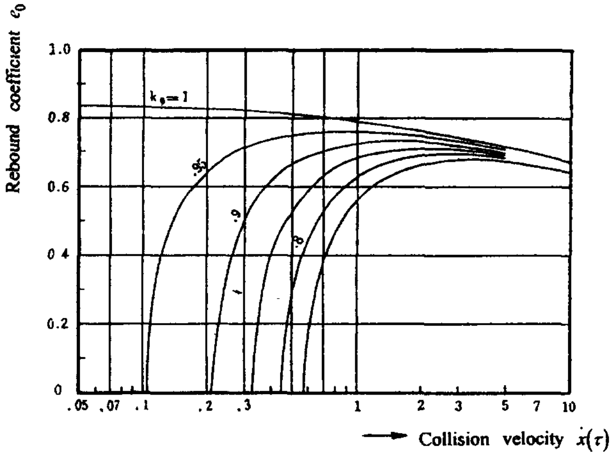

Fig.6 (a) The rebound coefficient versus collision velocity in case of nonlinear equivalent stiff surface.

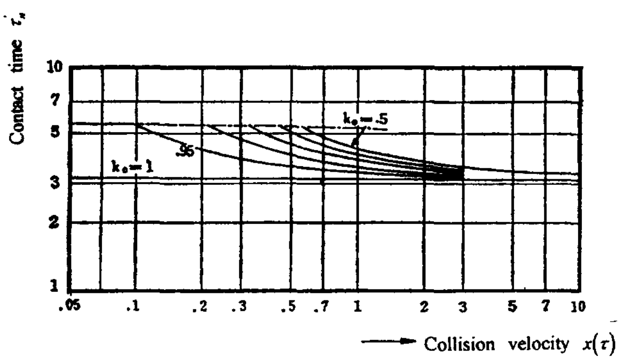

Fig.6 (b) Contact time versus collision velocity in case of nonlinear equivalent stiff surface.

Figure 6 shows two characteristics curves for a case in which the damping ratio ζ' on the equivalent stiff surface is a nonlinear function of the velocity, the damping ratio ζ' of the spring system is fixed at ζ'=0.05, and parameter k_0 is given by $0.5 \geqq k_0 \geqq 1.0$.

5. Conclusion

In this report, we simulated the chattering phenomenon which is induced by the collision of mass points with a stiff surface by introducing the equivalent stiff surface. The mass points are components of the one-degree-of-freedom vibration system that were subjected to a stepwise external force. The simulation revealed the characteristics of the rebound mechanism; and the condition for the completion of chattering was obtained.

We conclude the following results of this study.

(1) The rebound coefficient can be determined by the pushing distance, damping ratio of equivalent stiff surface, and damping coefficient.

(2) The contact time can be determined by the pushing distance, damping ratio of equivalent stiff surface, damping coefficient and collision velocity.

(3) A non-linear vibrating system resulting from collision must be used to construct an appropriate computational model with respect to the actual vibrating system resulting from collision.

Further studies should be carried out to analyze multi-degree-of-freedom vibrating systems resulting from collision that induce a higher order of vibratory behavior using a new numerical model.