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METHOD FOR PROBLEM OF KHLADNI FIGURES SOLUTION

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ABSTRACT

Method for problem of Khladni figures on rectangular plates solution based on the wave notion of energy transmission is presented. The run of phases in propagation of free flexural waves along the closed path in the plate with the smallest energy expenditure gives information about the density of natural vibration spectrum and about the process of figure formation for each natural freguency It was found out that for a free plate the figures in the shape of a circle or a cross are considered to be complex and simple as it has been thought untill now.

INTRODUCTION

The existing classic method of accurate Solution of problems of naturel vibrations of the plate is based on L.Euler solution for free vibration of the beam resting on rigid supports. Vibration parameters are determined by means of direct Solution of wave equations of the 4 $^{\text{th}}$ order for a beam. However, the problem is solved only, for the simpliest conditions. Approximate methods for complex boundary conditions provide contradictory conclusions. That is why the problem still arouse interest.

Theory and Experimental Results

The most complex boundary conditions are those for rectangular plates with all edges free. It may be supposed that wave eigenfilds are here more varied that in the plate with other classical boundary conditions.

Suggestion 1. For a thin free rectangular plate naturel vibrations are being formed by three kinds of free flexural waves They are well-known homogeneous flexural waves ζ_1, some non-homogeneous flexural waves, ζ_2, and the waves discovered and introduced into the given paper possessing hte signs both of homogeneous and non-homogeneous waves ζ_3. Let free flexural waves fall onto the edge of the free plate Y=0. Axes of coordinates are combined from the plate sides a, b.

$$\zeta_1 = A_1\, e^{i(\omega t + kx \sin\alpha + ky \cos\alpha)} \tag{1}$$

where $k=\omega/c$ - wave number,

c - velocity of wave propagation,

ω - circular frequency,

$A_1 = A\, e^{i(\varphi + \psi)}$,

φ, ψ - components of an initial phase angle,

α - angle of incidence of a flexural wave.

$$\zeta_2 = A_{12} e^{i(\omega t + kx\,\mathrm{ch}\,\alpha' + ky\, i\,\mathrm{sh}\,\alpha')} \tag{2}$$

where α' - material part of the non-homogeneous wave angle of incidence if it is presented in a complex form

$$\zeta_3 = A_{13} e^{i(\omega t + kx\sqrt{1 - sh^2\alpha''} - ky\, sh\,\alpha'')} \tag{3}$$

From the expressions (1), (2), (3) it follows that velocities of progation of the considered waves are different.

Suggestion 2. Natural vibrations of plates are those when free wave motions in them become closed with the minimum energy expend; ture on the formation of new propagating waves of the same kind.

Considering the propagation and the reflection (reverberation) of the waves from the plate edges and taking into account Suggestion 2, we receive a picture of disposition of main lines forming Khladni figures. At every reverberation there appear new waves of the neighbouring field which bring in originality into the distribution of displacements [1].

In case of forming natural vibrations by means of uniform flexural waves, we get natural wave field

$$\zeta_I = \pm AXY\cos\omega t, \tag{4}$$

where

$$X = \frac{2g_2}{d_2}(1 + \beta_a e^{-ka\sqrt{1+\cos^2\alpha}})\sin(kax_1 \sin\alpha) - \frac{2}{d_2}(1 - g_2^2\beta_a e^{-ka\sqrt{1+\cos^2\alpha}}) \times$$

$$\times \cos(kax_1 \sin\alpha) - \frac{\chi_2}{d_2}(2 - q_2\beta_a e^{-ka\sqrt{1+\cos^2\alpha}})e^{-kax_1\sqrt{1+\cos^2\alpha}} + \chi_2\beta_a e^{ka(x-1)\sqrt{1+\cos^2\alpha}};$$

$$Y = \frac{2g_1}{d_1}(1 + \beta_b e^{-kb\sqrt{1+\sin^2\alpha}})\sin(kby_1 \cos\alpha) - \frac{2}{d_1}(1 - g_1^2\beta_b e^{-kb\sqrt{1+\sin^2\alpha}}) \times$$

$$\times \cos(kby_1 \cos\alpha) - \frac{\chi_1}{d_1}(2 - q_1\beta_b e^{-kb\sqrt{1+\sin^2\alpha}})e^{-kby_1\sqrt{1+\sin^2\alpha}} + \chi_1\beta_b e^{kb(y_1-1)\sqrt{1+\sin^2\alpha}}; x_1 = x/a; y_1 = y/b$$

$$\beta_a = \frac{2[g_2 \sin(ka\sin\alpha) - \cos(ka\sin\alpha) + e^{-ka\sqrt{1+\cos^2\alpha}}]}{d_2 - 2g_2[\sin(ka\sin\alpha) + g_2\cos(ka\sin\alpha)]e^{-ka\sqrt{1+\cos^2\alpha}} + q_2 e^{-2ka\sqrt{1+\cos^2\alpha}}};$$

$$\beta_b = \frac{2[g_1 \sin(kb\cos\alpha) - \cos(kb\cos\alpha) + e^{-kb\sqrt{1+\sin^2\alpha}}]}{d_1 - 2g_1[\sin(kb\cos\alpha) + g_1\cos(kb\cos\alpha)]e^{-kb\sqrt{1+\sin^2\alpha}} + q_1 e^{-2kb\sqrt{1+\sin^2\alpha}}}$$

$$g_2 = g_a \chi_2^2 \; ; \; g_a = \frac{\sqrt{1+\cos^2\alpha}}{\sin\alpha} \; ; \; \chi_2 = \frac{1-(1-\sigma)\cos^2\alpha}{1+(1-\sigma)\cos^2\alpha} \; ; \; d_2 = g_2^2+1 \; ; \; q_2 = g_2^2 - 1$$

$$g_1 = g_b \chi_1^2 \; ; \; g_b = \frac{\sqrt{1+\sin^2\alpha}}{\cos\alpha} \; ; \; \chi_1 = \frac{1-(1-\sigma)\sin^2\alpha}{1+(1-\sigma)\sin^2\alpha} \; ; \; d_1 = g_1^2+1 \; ; \; q_1 = g_1^2 - 1$$

$$x_1 = x/a \; ; \; y_1 = y/b$$

The field of displacements (4) is charactererised by rectilinear main (knot) lines. The formation and propagation of non-uniform wave (2) finally develop the wave field

$$\zeta_{II} = \pm A X_2 Y_2 \cos\omega t \tag{5}$$

where

$$X_2 = (1+\beta_a' e^{kacha'})\sin(kax_1 ch\alpha') - (1-\beta_a e^{-kacha'}) \times \cos(kax_1 ch\alpha') - e^{-kax_1 ch\alpha'} + \beta_a' e^{ka(x_1-1)ch\alpha'}$$

$$Y_2 = e^{kby_1 sh\alpha'} + \frac{1}{1+g'}(1-g'+2g'\beta_b' e^{-kb\sqrt{1+ch^2\alpha'}})e^{kby_1 sh\alpha'} +$$
$$+\frac{\chi'}{1+g'}\left[2-(1-g')\beta_b' e^{-kb\sqrt{1+ch^2\alpha'}}\right]e^{-kby_1\sqrt{1+ch^2\alpha'}} + \chi'\beta_b' e^{kb(y_1-1)\sqrt{1+ch^2\alpha'}}$$

$$\beta'_a = \frac{\sin(kach\alpha') - \cos(kach\alpha') + e^{-kach\alpha'}}{1-\left[\sin(kach\alpha')+\cos(kach\alpha')\right]e^{-kach\alpha'}};$$

$$\beta'_b = \frac{(1-g')e^{kbsh\alpha'} + (1+g')e^{-kbsh\alpha'} - 2e^{-kb\sqrt{1+ch^2\alpha'}}}{1+g'-2g'e^{-kb\sqrt{1+ch^2\alpha'}}e^{kbsh\alpha'} - (1-g')e^{-2kb\sqrt{1+ch^2\alpha'}}}$$

$$g' = 3\chi'^2 \; ; \; \zeta = \frac{\sqrt{1+ch^2\alpha'}}{sh\alpha'} \; ; \chi = \frac{1-(1-\sigma)ch^2\alpha'}{1+(1-\sigma)ch^2\alpha'}$$

the first (from the plate edge) main lines of wich are curvilinear.

If we follow the propagation of the wave (3) fulfilling the conditions of the Suggestion 2 we shall get the wave field which is discribed by the function

$$\zeta_{III} = X_3 \, Y_3 \cos\omega t \tag{6}$$

where

$$X_3 = (1+\beta_a'' e^{-ka\sqrt{1-sh^2\alpha''}})\sin(kax_1\sqrt{1-sh^2\alpha''}) - (1-\beta_a'' e^{-ka\sqrt{1-sh^2\alpha''}})$$
$$\times\cos(kax_1\sqrt{1-sh^2\alpha''}) - e^{-kax_1\sqrt{1-sh^2\alpha''}} + \beta_a'' e^{ka(x-1)\sqrt{1-sh^2\alpha''}}$$

$$Y_3 = -\frac{2g''}{(1+g'')}(1+\beta_b'' e^{kb\,sh\alpha''})\sin(kby_1 sh\,\alpha'') - \frac{2}{1+g''}(1-g''^2\beta_b'' e^{kb\,sh\,\alpha''})\times$$
$$\times\cos(kby_1 sh\,\alpha'') - \frac{\chi_3''}{1+g''}\left[2-(g''^2-1)\beta_b'' e^{kb\,sh\alpha''}\right]\times e^{kby_1\,sh\alpha''} + \chi''\beta_b'' e^{-kb(y_1-1)sh\alpha''}$$

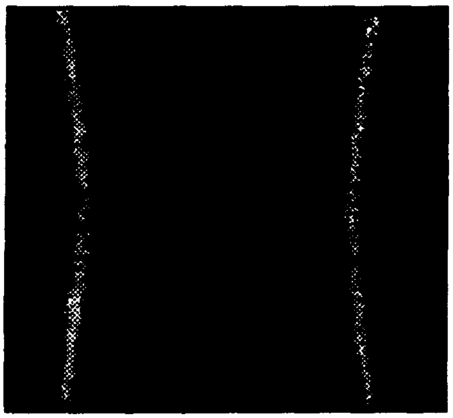
$$\beta_a'' = \frac{\sin(ka\sqrt{1-sh^2\alpha''}) - \cos(ka\sqrt{1-sh^2\alpha''}) + e^{-ka\sqrt{1-sh^2\alpha''}}}{1-\left[\sin(ka\sqrt{1-sh^2\alpha''}) = \cos(ka\sqrt{1-sh^2\alpha''})\right]e^{-ka\sqrt{1-sh^2\alpha''}}}$$

$$\beta_b'' = \frac{2\left[-g''\sin(kb\,sh\alpha'') - \cos(kb\,sh\alpha'') + e^{kb\,sh\alpha''}\right]}{1+g''-2g''\left[g''\cos(kb\,sh\alpha'') - sh(kb\,sh\alpha'')\right]e^{kb\,sh\alpha''} + (g''^2-1)e^{2kb\,sh\alpha''}}$$

$$g'' = g_3\chi_3^2 \quad ; g_3 = \frac{sh^2\alpha'' - (2-\sigma)(1-sh^2\alpha'')}{sh^2\alpha'' + (2-\sigma)(1-sh^2\alpha'')} \quad , \quad \chi_3 = \frac{sh^2\alpha'' + \sigma(1-sh^2\alpha'')}{sh^2\alpha'' - \sigma(1-sh^2\alpha'')}$$

It can be seen that here the main (knot) lines of the wave field have, in simple forms, rectilinear outlines along one coordinate and curvilinear shape on the other one.

Simple form (5) with two curvilinear knots (Pic.1.) is marked out on the plate from roofing steel with the dimensions in plan 0,20 x 0,20 m^2 .



Pic.1 - Form of natural vibrations ζ_{20} (experiment)

REFERENCES

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