FIFTH INTERNATIONAL CONGRESS ON SOUND AND VIBRATION
DECEMBER 15-18, 1997
ADELAIDE, SOUTH AUSTRALIA

NEW APPROACH TO THE THEORY OF AERODYNAMIC SOUND

A. T. Fedorchenko

*Department of Fluid Mechanics, Moscow Phys.-Tech. Institute,
141700 Dolgoprudnyi, Moscow Region, Russia.*

New nonlinear theory of sound in unsteady subsonic flow has been proposed. This two-medium theory is based on the nonlocal invariant procedure of separating out the acoustic components in high-unsteady flow, and so it departs radically from all the traditional approaches. A nonlocal mathematical model of *globally-compressible fluid flow* has been also designed for the simulation of unsteady subsonic flows with acoustics excluded. This model, which represents a fundamental extension of the classical model of incompressible fluid flow, forms a necessary basis of the above theory while being applied for the approximation of unsteady mean flow and in turn for the estimation of sound sources.

1. INTRODUCTION

To begin a brief review, let us recall "the most general linear equations of flow acoustics" proposed in [1]. There was assumed that $\tau >> \omega^{-1}$ (τ is the characteristic time of changes in the mean flow structure, ω is the sound frequency). Any procedure of time-averaging depends on the interval (t_1, t_2), and, as usual, it is taken so large that all the mean flow variables *are independent of time*. Thus, the approximate linear system has been derived with the aim to describe the evolution of small *unsteady* fluctuations $\mathbf{Z}_\varepsilon(\mathbf{r}, t)$ on the background of *steady mean flow* with variables $\mathbf{Z}_0(\mathbf{r})$ which represent the exact stationary solution of the basic nonlinear equations of fluid mechanics without any sources and forces (here $\mathbf{Z} = \{\mathbf{u}, p, \rho, s\}$). Surely, the mean flow is regarded as steady only in the unique reference frame, and any Galilean transformation will destroy the model (this defect is peculiar to any linearization). Such fluctuations describe simultaneously all kinds of waves: the sound as well as the disturbances of both entropy and vorticity, but this cannot be recognized as an advantage. So, any time-averaging procedure, formally applied to the basic evolutionary equations, is not able to give a way how to separate out the acoustic waves, except the case of uniform flow.

The absence of distributed mass and heat sources as well as of any body forces is a serious limitation of the above model. Otherwise, one ought to give an exact procedure, and this is far from trivial, how to define both the main part of any source and the fluctuating additive. This problem was considered later by Goldstein, but the approach he suggested in sec.1.2 of [2] (all the source terms, being assumed *small*, are to be attributed to the sound field, and $\mathbf{Z}_0(\mathbf{r})$ represents the stationary solution of basic nonlinear system without any sources and forces) cannot be accepted. This implies that the source terms by no means act on the mean flow.

Keeping such an approach (see eq.(1.18) in [2]) one could come to the wrong conclusion that no sound waves are generated by force $\mathbf{f}$ if $\nabla\mathbf{f} = 0$, although the contrary example was given in [3]. The similar problems also arise if we study the action of unsteady boundary conditions, for instance those assigned on permeable or moving walls.

Lighthill's famous paper [4] represented the first and the most influencing attempt to give a theoretical model for the mechanism of sound generation by flow. Let us write the second-order equation derived there for isentropic inviscid gas flow without any external sources

$$\partial^2 \rho_\varepsilon/\partial t^2 \;-\; a_0^2\,\Delta\rho_\varepsilon = \eta\,, \qquad \rho_\varepsilon = \rho - \rho_0\,, \qquad \rho_0\,,\, a_0 = \text{const}\,, \tag{1}$$

$$\eta = \partial^2 T_{ij}/\partial x_i \partial x_j\,, \qquad T_{ij} = \rho u_i u_j + \delta_{ij}[(p - p_0) - a_0^2(\rho - \rho_0)]\,,$$

where T_{ij} is *Lighthill's stress tensor.* This equation is exact since no terms were omitted in transforming the original equations of fluid mechanics. Unfortunately, the nonlinear term η , called as "quadrupole sound source", includes *unknown total variables* $\mathbf{u}$, p , ρ. Hence, equation (1) is not closed, and we cannot determine the value of η before solving all the problem. To soften this sharp question, the source region was assumed to be compact, and the only aim declared was to estimate the far sound field in ambient stationary medium.

A new idea has been set up by Powell [5] to simplify the Lighthill's approach. For inviscid isentropic flow at low Mach number he assumed that

$$p - p_0 \approx a_0^2(\rho - \rho_0)\,, \qquad T_{ij} \approx \rho_0 u_i u_j\,,$$

and the approximate expression for the "dipole-like sound source" η was suggested as

$$\eta \;\approx\; \eta_\beta = \rho_0 \nabla[(\nabla\times\mathbf{u}_\beta)\times\mathbf{u}_\beta]\,, \qquad \nabla\mathbf{u}_\beta = 0\,. \tag{2}$$

Thus, the *incompressible flow* variables $\mathbf{Z}_\beta = \{\, \mathbf{u}_\beta\,, p_\beta\,, \rho_\beta \,\}$ are used to estimate η as well as the far sound field. This means that we determine the nonlinear term η from the precedingly found solution of the evolutionary problem posed within the quite different model of incompressible fluid flow. Besides, like in [4], the difference $p - p_0$ was unfoundedly regarded as the "acoustic pressure". Anyway, this form of η_β had evoked a lot of further efforts to connect the sound sources solely with the region of nonzero vorticity, at least when $\nabla s = 0$.

Ribner [6] performed a modification of Lighthill's model by introducing the new variable $p_\omega = p - p_\beta$. As a result, for subsonic isentropic flow he obtained the equation

$$a_0^{-2}\,[\partial^2 p_\omega/\partial t^2] - \Delta p_\omega \;=\; \eta_\omega \;=\; -\,a_0^{-2}\,[\partial^2 p_\beta/\partial t^2\,] \;+\; \varepsilon\,, \tag{3}$$

and the term η_ω was called as the "monopole-type sound source". There was assumed that $|\,\varepsilon/\eta_\omega\,| << 1$, and so ε could be omitted. However, if we intend to use (3) for the analysis of sound processes, $|\,\varepsilon\,|$ may be not small in comparison with some terms containing the acoustic variables. Rather, all the acoustic terms could be neglected in comparison with η_ω .

It should be emphasized, that all the above mentioned versions of Lighthill's model do not contain any procedure for separating out the acoustic disturbances, and so this model by no means can give a convincing definition of sound sources in unsteady flows, much less in the internal ones. Nevertheless, irresistible attractiveness of this approach is explained by the appearance of equation (1) which resembles the routine linear acoustic equation with distributed *external* sources, so that even nowadays this model gathers innumerable followers.

Doing so, one could separate out a linear part from any equation if all *uncomfortable* nonlinear terms were transferred into the right-hand side and called as "sources".

Howe [7] suggested to take the stagnation enthalpy as the main variable for which the exact second-order nonlinear equation has been derived. Actually, the total variables $\mathbf{u}$, p , s were used again, and no way was given how to separate out the acoustic components. Therefore the habitual next step is done: the terms, which are regarded (without any convincing evidence) as the sound sources, are to be approximated with the use of $\mathbf{Z}_\beta(\mathbf{r}, t)$.

To comment the great number of experimental data which are usually drawn to confirm the validity of the above approaches, it is relevant to give a few introductory phrases from [8]: "Measurements in the far field, no matter how detailed and sophisticated, cannot lead to a unique picture concerning the nature of the acoustic sources. One is forced therefore to make measurements at the source location as well. This, however, proves to be a most elusive task. Since the noise production is associated with a volume integral, point measurements (or even two- or three-point correlation measurements) are insufficient to lead one to the desired picture of the sources". Thus, it is clear why diverse theoretical models of sound generation, even if those are evidently invalid, exist till now. One should also remember that some of those theories are resulted from *exact* transformations of the basic nonlinear equations of fluid mechanics, and so they are able to reflect *integrally* all the phenomena in fluids. But we have to be very careful in analysing any "agreement" of an aeroacoustic theory with experiment.

A new approach was offered by the author in [3] to the theory of sound generation and propagation in unsteady subsonic flow of inviscid gas with nonuniform entropy field, but without any external sources and forces. The *local* splitting procedure was there applied, and the expression given for the sound source was not invariant within Galilean transformation, more complex conditions were beyond consideration, etc. Nevertheless, that approach was *the first exact two-medium model* which did work in some particular cases and served as a starting point for the radically new *nonlocal* theory, proposed in [9].

Bearing in mind all the previous experience, now we pose two key questions which represent the fundamental problems in both fluid mechanics and aeroacoustics:

I. *What is the most general definition of the flow which does not radiate sound?* Instead of the classical notion of steady flow, when we demand $\partial \mathbf{Z}/\partial t = 0$ everywhere and this can be satisfied only in the unique reference frame, we should find an invariant definition of *the steady-structure flow* which does not change its spatial structure and hence does not emit sound.

II. The model of incompressible fluid flow is usually regarded as a limiting case (when the characteristic Mach number tends to zero) of the much more general model of compressible fluid flow. It seems illogical to apply these two radically different models *simultaneously* to a certain flow as it was done in the aeroacoustic theories mentioned above. So one could ask: *is it possible that a sub-model of globally-compressible fluid flow (something like unsteady mean flow with infinite sound velocity) can be separated out in a rigorous manner within the general model of compressible fluid flow at finite Mach numbers?*

2. BASIC EQUATIONS

Let us take the general system of nonlinear equations governing unsteady flow of inviscid gaseous medium in the spatial domain G (with the boundary Γ which can move) considered in a certain inertial coordinate system K_o within a time interval $J_t = (0\,,\, t_k)$

$$\partial \rho \mathbf{u}/\partial t + \nabla(\rho \mathbf{u}\,;\,\mathbf{u}) + \nabla p = \mathbf{f} + \mathbf{k} \,, \tag{4}$$

$$\partial \rho/\partial t + \nabla(\rho \mathbf{u}) = \xi \rho \,, \tag{5}$$

$$\partial s/\partial t + \mathbf{u}\nabla s = q\,, \tag{6}$$

$$F(s, p, \rho) = 0\,, \tag{7}$$

where $\nabla(\rho\mathbf{u}\,;\,\mathbf{u}) = (\rho\mathbf{u}\,,\nabla)\,\mathbf{u} + \mathbf{u}\nabla(\rho\mathbf{u})$, $\mathbf{f}$ is the assigned force, ξ is the mass source strength per unit mass, $\mathbf{k}$ is the rate of momentum change because of mass source, q is the entropy source per unit mass due to both volume heat release and a nonzero mass source. All source terms $\{\mathbf{f}\,, \mathbf{k}\,, \xi\,, q\}$ are assumed to be the functions of flow variables. The medium will, to fix the ideas, be regarded as a perfect gas, i. e. we take (7) as $s = c_v \ln(p/\rho^\gamma)$, $\gamma =$ const.

We can pose an initial-boundary-value problem for $\mathbf{Z}(\mathbf{r}, t) = \{\mathbf{u}, p, \rho, s\}$ in $G \times J_t$ by specifying $\mathbf{Z}(\mathbf{r}, 0) = \zeta(\mathbf{r})$, $\mathbf{r} \in G$ as well as the boundary conditions which we take as

$$\Phi(u_n, p, \rho, \mathbf{r}, t) = 0 \quad \text{at any sign of } u_f \tag{8}$$

$$s = \theta(\mathbf{r}, t) \quad \text{only if } u_f < 0\,, \tag{9}$$

where Φ and θ are the assigned functions, $\mathbf{r} \in \Gamma$, $t \in J_t$, $u_n = \mathbf{un}$, $\mathbf{n}$ is the outward normal to the smooth surface Γ, $u_f = u_n - u_b$, u_b is the assigned velocity of Γ along $\mathbf{n}$.

3. PROCEDURE OF DECOMPOSITION

We decompose all the flow variables as a sum $\mathbf{Z} = \mathbf{Z}_\nu + \mathbf{Z}_\alpha$ where the subscripts α and ν are used to label the acoustic components and the variables denoting the background unsteady flow. Following the basic concept [3] first we solve the separate initial-boundary-value problem for $\mathbf{Z}_\nu$ in a medium which is characteristically similar to the model of incompressible fluid flow. Then we pose the other problem for $\mathbf{Z}_\alpha$ taking $\mathbf{Z}_\nu$ as known function in $G \times J_t$.

A number of necessary requirements was imposed upon the splitting procedure to make sure that our model is unique. Among those were: all resulting equations should retain invariant form within Galilean transformation; sound waves are to be precluded within the system written for $\mathbf{Z}_\nu$ (i.e. the sound velocity is infinite there); all the source terms must be integrable square over the infinite flow domain; all sound sources must be equal to zero in *steady-structure flow* which does not change its spatial structure in time even if it is rotating; all sound sources should be minimized to eliminate the spurious quasisound effects; all these sources are not to change the local specific entropy, etc. Besides, the model has to reduce naturally to a series of limiting cases.

So we write the following general system for $\mathbf{Z}_\nu(\mathbf{r}, t)$

$$\partial\rho_\nu\mathbf{u}_\nu/\partial t + \nabla(\rho_\nu\mathbf{u}_\nu\,;\mathbf{u}_\nu) + \nabla p_\nu = (1 - A_1)(\mathbf{f}_\nu + \mathbf{k}_\nu) + m_\nu\mathbf{W}_\nu\,, \tag{10}$$

$$\partial\rho_\nu/\partial t + \nabla(\rho_\nu\mathbf{u}_\nu) = (1 - A_2)\,\xi_\nu\rho_\nu + m_\nu\,, \tag{11}$$

$$\partial s_\nu/\partial t + \mathbf{u}_\nu\nabla s = (1 - A_3)\,q_\nu\,, \tag{12}$$

$$F(s_\nu, p_\nu, \rho_\nu) = 0 \tag{13}$$

where $\{A_1, A_2, A_3\}$ are the weight functions. The main problem is to find the proper expressions for the sound sources m_ν , $m_\nu\mathbf{W}_\nu$.

To generalize the operator $\partial/\partial t$ (here applied to a certain function $p(\mathbf{r}, t)$ in $G \times J_t$), that will enable us to give a *nonlocal* definition of m_v and $\mathbf{W}_v$, we introduce the functional

$$\Psi(\mathbf{r}, t) = \Psi\{p\} = \partial p/\partial t + (\mathbf{V} + \boldsymbol{\Omega} \times \mathbf{r})\nabla p - H, \qquad H = H(t),$$

with unknown vector function $\mathbf{N}(t) = \{N_j\}$, $j = 1, \ldots 7$ if we denote

$$N_1 = V_1, \quad N_2 = V_2, \quad N_3 = V_3, \quad N_4 = \Omega_1, \quad N_5 = \Omega_2, \quad N_6 = \Omega_3, \quad N_7 = H.$$

Let us also introduce the other functional

$$Y(t) = \int_G [\,\partial p/\partial t + (\mathbf{V} + \boldsymbol{\Omega} \times \mathbf{r})\nabla p - H\,]^2 \, dx_1 dx_2\, dx_3 \tag{14}$$

to define the norm of Ψ as $\|\Psi(\mathbf{r}, t)\|_G^2 = Y(t)$. Then we have to find the set $\mathbf{N} = \mathbf{N}^*(t)$ which minimizes Y. It would be illogical to minimize the integral of $\Psi(\mathbf{r}, t)$ over G since it could be equal to zero although the local values of Ψ may be considerable. The values of unknown functions $N_j^*(t)$ at any time can be obtained from seven linear algebraic equations

$$\partial Y/\partial N_j = 0, \qquad j = 1, \ldots 7 \tag{15}$$

which correspond to the necessary conditions for the functional $Y(t)$ to reach its minimum on the set of all admissible values of $N_j(t)$. At the same time we demand all integrals to be convergent even if our spatial domain is infinite.

One can readily prove that conditions sufficient for Y (if $\nabla p \neq \mathbf{L}(t)$ in G) to reach its minimum are hereby satisfied as well. Further we will imply the functional $\Psi(\mathbf{r}, t)$ to be taken at $\mathbf{N}(t) = \mathbf{N}^*(t)$ that ensures the minimal value of $Y(t)$ at the moment. This means that we have found the unique reference frame K^* (it is translating at a velocity $\mathbf{U}(t)$ and rotating at angular velocity $\Omega(t)$ relative to K_o) in which we have

$$\Psi(\mathbf{r}^*, t) = \Psi\{p\} = \partial p/\partial t - H.$$

Clearly, the values of $\Psi(\mathbf{r}, t)$ are the same in any reference frame we have taken.

Let us consider some particular cases in the solution of system (15). For instance, if we analyse axisymmetric vortex structure convected by uniform background flow, we should take $\Omega = 0$, although any value of Ω satisfies (15).

When we have the case $\nabla p = \mathbf{L}(t)$, the unique solution of system

$$\partial Y/\partial V_i = 0, \quad \partial Y/\partial \Omega_i = 0, \quad i = 1, 2, 3$$

will be obtained if we define $H = dP_e/dt$ where $P_e(t)$ is the average value of p in G. This value is attained in the central point of plane $p(\mathbf{r}, t) = const$ (like in the centre of mass) within our domain G. Generally, function H has been introduced to exclude the contribution of $dP_e/dt \neq 0$ into the value of Ψ. This can be interpreted as if we observe the waved sea surface irrespective of the variable sea depth.

Let us also consider the limiting case when we analyse function $p(\mathbf{r}, t)$ in a domain G with its volume tending to zero. Then domain G can be regarded as a small vicinity of a fixed point $\mathbf{r}_c$. Suppose we know $\partial p/\partial t$ as well as $\nabla p \neq 0$ in that point at any time. If we take

$$\mathbf{W} = \beta \nabla p, \quad H = 0, \quad \text{sign}(\beta) = -\text{sign}(\partial p/\partial t), \quad |\beta| = |\partial p/\partial t|(\nabla p)^{-2},$$

then $\Psi\{p\} = \partial p/\partial t + \mathbf{W}\nabla p \equiv 0$. This means that we should take domain G rather extended to analyse function $p(\mathbf{r}, t)$ there. On the other hand, if we consider a vast spatial domain which contains $p(\mathbf{r}, t)$ with numerous nonuniformities as well as with many randomly distributed local extrema, the set $\mathbf{N}^*(t)$ may be overaveraged.

In the particular case of infinite spatial domain, if the value $|p - p_\infty|$ goes rapidly to zero while $\mathbf{r} \to \infty$, at least as $|\mathbf{r}|^{-2}$, we demand $H \equiv 0$ (then $P_e = p_\infty = \text{const}$).

As a result, we offer the radically new definition of the sound sources

$$m_v = a_v^{-2}\Psi\{p_v\}, \qquad a_v^2(\mathbf{r}, t) = \partial p_v/\partial \rho_v, \quad \mathbf{W}_v = \mathbf{V} + \Omega \times \mathbf{r}, \quad \nabla \mathbf{W}_v = 0.$$

Here the source m_v as well as $\mathbf{N}^*(t) = \{\mathbf{V}, \Omega, H\}$ are obtained implicitly from $\mathbf{Z}_v(\mathbf{r}, t)$, and consequently this model is essentially nonlocal, although in some particular cases (for instance, when the flow is spatially symmetrical) functions $\mathbf{V}$, Ω, H can be found immediately and then system (10)-(13) is much more simple.

When $\mathbf{f} = 0$, $\xi = 0$, we can write the following equation

$$\rho_v[\partial \mathbf{u}_v/\partial t + (\mathbf{u}_v, \nabla)\mathbf{u}_v] + \nabla p_v = m_v(\mathbf{W}_v - \mathbf{u}_v),$$

which can be readily derived from (10)-(11).

In the particular case $s = s_v = \text{const}$, $\xi = 0$ equation (11) takes the form

$$\nabla[\rho_v(\mathbf{u}_v - \mathbf{W}_v)] + H/a_v^2 = 0.$$

If one analyses the local characteristic properties of system (10)-(13), the fact will be proved (when $|\mathbf{u}_v - \mathbf{W}_v| < a_v$) that the sound waves cannot take place there. Doing this we can take $H = 0$, $\mathbf{f}_v = 0$, $\mathbf{k}_v = 0$, $q_v = 0$, $\xi_v = 0$ since all these represent zero-order forcing terms, and function $\mathbf{W}_v$ is assumed to be known during the *local* analysis. As a result, our model of unsteady subsonic mean flow does display the characteristic features similar to those of the model of *globally-compressible-fluid flow* [10].

Decomposition of boundary conditions is a separate and very delicate problem. For instance, one can formally split the condition $u_n = \phi(\mathbf{r}, t)$, $\mathbf{r} \in \Gamma_1 \subset \Gamma$ by introducing an arbitrary function $A_{\beta 1}(t)$ (even with $|A_{\beta 1}| > 1$) so that at each point on Γ_1 we have the conditions for $\mathbf{Z}_v$ and $\mathbf{Z}_\alpha$ respectively $u_{vn} = (1 - A_{\beta 1})\phi$, $u_{\alpha n} = A_{\beta 1}\phi$. Thus we have the total set of indefinite weight functions $\mathbf{A}(t) = \{A_1, A_2, A_3, A_{\beta 1}, A_{\beta 2}, \ldots\}$. We demand function $\mathbf{A}$ to be of minimal norm, and usually $\mathbf{A} \equiv 0$ is taken (that will be implied below). However, in some particular cases we cannot assign $\mathbf{A} \equiv 0$.

4. ACOUSTIC EQUATIONS

Taking $\mathbf{Z}_v(\mathbf{r}, t)$ as known function in $G \times J_t$, we can write the nonlinear system of equations for $\mathbf{Z}_\alpha(\mathbf{r}, t)$ that complements (10)-(13) to the initial system (4)-(7)

$$\partial \mathbf{w}_\alpha/\partial t + \Lambda_\alpha + \nabla p_\alpha = A_1(\mathbf{f}_v + \mathbf{k}_v) + \mathbf{f}_\alpha + \mathbf{k}_\alpha - m_v \mathbf{W}_v, \tag{16}$$

$$\partial \rho_\alpha/\partial t + \nabla \mathbf{w}_\alpha = A_2 \xi_v \rho_v + \xi_\alpha \rho_v + \xi_v \rho_\alpha + \xi_\alpha \rho_\alpha - m_v, \tag{17}$$

$$\partial s_\alpha/\partial t + \mathbf{u}_v \nabla s_\alpha + \mathbf{u}_\alpha \nabla s_v + \mathbf{u}_\alpha \nabla s_\alpha = A_3 q_v + q_\alpha, \tag{18}$$

$$F(s_\nu + s_\alpha \ , p_\nu + p_\alpha \ , \rho_\nu + \rho_\alpha \ ) \ = \ 0 \ , \tag{19}$$

where $\Lambda_\alpha = \nabla [\, (\mathbf{w}_\nu \, ; \mathbf{u}_\alpha) \ + \ (\mathbf{w}_\alpha \, ; \mathbf{u}_\nu) \ + \ (\mathbf{w}_\alpha \, ; \mathbf{u}_\alpha) \,]$,

$$\mathbf{w}_\alpha \ = \ \rho_\nu \mathbf{u}_\alpha \ + \ \rho_\alpha \mathbf{u}_\nu \ + \ \rho_\alpha \mathbf{u}_\alpha \ , \qquad \mathbf{w}_\nu \ = \rho_\nu \mathbf{u}_\nu \ .$$

Here we split $q_\alpha = q - q_\nu$, $q = q(\mathbf{Z})$, $q_\nu = q(\mathbf{Z}_\nu)$ and similarly for $\mathbf{f}, \mathbf{k}, \xi$.
If $\| \mathbf{Z}_\alpha / \mathbf{Z}_\nu \| << 1$ in $G \times J_t$, then we can write the linearized version of the above equations. Furthermore, one could simplify the model by assuming $s \equiv s_\nu$, $s_\alpha \equiv 0$.

5. APPROXIMATE VERSION OF THE MODEL

One can encounter a lot of difficulties in applying the most general version of our nonlocal theory to the solution of practical problems. Therefore it would be much desirable if we find an approximate procedure how to calculate the sources m_ν, $m_\nu \mathbf{W}_\nu$ *explicitly* from the ready solution of an initial-boundary-value problem where no sound sources were considered. At first sight, this may resemble the idea expressed in [5] that the analysis of an *incompressible fluid flow* could give an information about the sound sources in a *compressible fluid flow*. However, the main difference between our approach and all others must be emphasized: we are trying to simplify the model by approximating $\mathbf{Z}_\nu(\mathbf{r}, t)$ *after* obtaining the *exact expressions* for sound sources.

Let us apply the nonlocal model of *globally-compressible fluid flow* [10] to the solution of our problem. Within that model we write the following system of equations in $G \times J_t$

$$\partial \rho_\tau \mathbf{u}_\tau / \partial t \ + \ \nabla(\rho_\tau \mathbf{u}_\tau \, ; \mathbf{u}_\tau) \ + \ \nabla p_\tau \ = \ \mathbf{f}_\tau \ + \ \mathbf{k}_\tau \ , \tag{20}$$

$$\partial \rho_\tau / \partial t \ + \ \nabla(\rho_\tau \mathbf{u}_\tau) \ = \ \xi_\tau \rho_\tau \ , \tag{21}$$

$$\partial s_\tau / \partial t \ + \ \mathbf{u}_\tau \nabla s_\tau \ = \ q_\tau \ , \tag{22}$$

$$F(s_\tau \ , P_\tau \ , \rho_\tau) = 0 \ , \qquad P_\tau = P_\tau(t) >> |\, p_\tau | \ , \tag{23}$$

supplemented with boundary conditions like (8)-(9) where

$$\Phi(u_{\tau n}, \ P_\tau \ , \ \rho_\tau \ , \ \mathbf{r}, t) \ = \ 0, \ \ \mathbf{r} \in G, \ t \in J_t \ , \tag{24}$$

and with the initial field $\mathbf{Z}_\tau(\mathbf{r}, 0) = \zeta_\tau(\mathbf{r})$. The subscript τ remindes that we are now using the different model which is intended to approximate all variables $\mathbf{Z}_\nu(\mathbf{r}, t)$ of unsteady mean flow at $M^2 << 1$. From equations (21)-(23) we can also derive the following one

$$\nabla \mathbf{u}_\tau \ = \ \xi_\tau + \eta_\tau - (\, dP_\tau/dt \,)/(\gamma P_\tau) \ , \qquad \eta_\tau = q_\tau / c_p \ ,$$

which reflects the nonlocality of this model (see [10]). In consequence, we can obtain the solution $\mathbf{Z}_\tau(\mathbf{r}, t) = \{\mathbf{u}_\tau \, , p_\tau \, , \rho_\tau \, , s_\tau \, , P_\tau\}$ in $G \times J_t$. Then we define the relation between $\mathbf{Z}_\tau$ and $\mathbf{Z}_\nu$ as series with terms $O(\varepsilon^n)$, $n = 1, 2, 3, \ldots, \varepsilon = \| p_\tau \| / P_\tau \le O(M^2) << 1$

$$\begin{aligned} \mathbf{u}_\nu \ &= \ \mathbf{u}_\tau + \mathbf{u}_1 + \mathbf{u}_2 \ + \ldots, \qquad & \rho_\nu \ &= \ \rho_\tau \ + \rho_1 \ + \rho_2 \ + \ldots, \\ s_\nu \ &= \ s_\tau + s_1 \ + s_2 \ + \ldots, \qquad & p_\nu \ &= \ P_\tau \ + p_\tau \ + p_1 \ + p_2 \ \ldots, \end{aligned} \tag{25}$$

where $F(s_\tau, P_\tau + p_\tau, \rho_\tau + \rho_1) = 0$, i.e. for a perfect gas we have $\rho_1 \approx p_\tau / a_\tau^2$, $a_\tau^2 = \gamma P_\tau / \rho_\tau$. Then we can take $\mathbf{Z}_{v1} = \{ \mathbf{u}_\tau, s_\tau, P_\tau + p_\tau, \rho_\tau + \rho_1 \}$ as the first approximation so that with the same accuracy we can determine the sound source

$$m_v = a_\tau^{-2} \Psi\{ P_\tau + p_\tau \} = a_\tau^{-2} \Psi\{ p_\tau \}$$

Further we can find $\mathbf{Z}_{v2} = \{ \mathbf{u}_\tau + \mathbf{u}_1, s_\tau + s_1, P_\tau + p_\tau + p_1, \rho_\tau + \rho_1 + \rho_2 \}$ and so on, but the first approximation $\mathbf{Z}_{v1}$ may be sufficient to solve many aeroacoustic problems.

In this way we can study the great number of known solutions obtained within the classical model of incompressible fluid flow. As an example, let us take the two-dimensional solution given by Kelvin for the rotating elliptic vortex (the same for the elliptic cylinder rotating at constant angular velocity in potential flow). That solution was used in [7] to approximate the sound sources by following the approach [4-7]. However, if we take that solution to find $\mathbf{Z}_{v1}$, then we will come to the undeniable conclusion that such a mean flow does not radiate sound at all. Even if we estimate the possible contribution of further terms of expansion (25) into the far sound field, its intensity will be much less than that given in [7]. Actually, the spurious quadrupole sound source has appeared in [7] only due to the wrong aeroacoustic model applied there.

Besides, the important general conclusion is resulted from our model: changes in the structure of external potential shell of a vortex may give the substantial contribution into the total strength of the sound sources.

The above model was successfully applied to the analysis of the phenomena of sound generation in diverse unsteady subsonic flows, both internal and free, also taking into account the nonlinear interactions between coherent vortex structures and small-scale turbulence. Effective means of boundary control have been developed with the aim to create the high-stable nonuniform flows with minimum sound emission, that will promote a lot of practical applications.

REFERENCES

1. D. I. Blokhintsev 1946 *Acoustics of a nonhomogeneous moving medium.* (In Russian. Translated into English as NACA T. M. 1339, 1956).
2. M. E. Goldstein 1976 *Aeroacoustics*. McGrow-Hill.
3. A. T. Fedorchenko 1988 *Sov. Phys. Acoust.* **34**(6), 635-638. On the theory of generation and propagation of sound in an unsteady flow.
4. M. J. Lighthill 1952 *Proc. Roy. Soc. Lond.*, Ser. A, **211**(1106), 564-587. On sound generated aerodynamically. Part I. General theory.
5. A. Powell 1964 *J. Acoust. Soc. Amer.* **36**(1), 177-195. Theory of vortex sound.
6. H. S. Ribner 1959 *J. Acoust. Soc. Amer.* **31**(2), 245-246. New theory of jet noise generation, directionality, and spectra.
7. M. S. Howe 1975 *J. Fluid Mech.* **71**(4), 625-673. Contribution to the theory of aerodynamic sound, with application to excess jet noise and the theory of the flute.
8. J. Laufer & T.- C. Yen 1983 *J. Fluid Mech.* **134**, 1-31. Noise generation by a low-Mach-number jet.
9. A. T. Fedorchenko 1995 *Physics-Doklady* **40**(9), 487-490. On the nonlinear theory of aerodynamic sound sources.
10. A. T. Fedorchenko 1997 *J. Fluid Mech.* **334**, 135-155. A model of unsteady subsonic flow with acoustics excluded.