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Invited Paper

VIBRATION GENERATED BY UNDERGROUND RAILWAY TRAINS

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ABSTRACT

The control of vibration generated by railways in tunnels is complicated by the dynamic interaction between tunnel and track. Conventional methods (floating slab track, ballast mats, under-sleeper pads) do not perform as well as might be expected and it is the purpose of this paper to explore the sources of error. The major factors which must be taken into account pertain to the three-dimensional vibration modes of the tunnel and their interaction with the track and with the surrounding soil. These factors are usually omitted under existing design methodologies and supposed improvements in track design are often not effective. The model used to quantify tunnel effects is based on exact solutions to the 3-dimensional wave equation as applied to a tunnel of infinite length. For a typical tunnel there are many resonance frequencies to be found in the range from 5 to 100Hz, most significant for railway vibration calculations.

INTRODUCTION

In railway stations or city-centre shops the noise and vibration generated by passing trains does not generally cause great irritation but for those who work or live in buildings near (or even above) underground railway lines vibration and re-radiated noise are a source of significant discomfort. This is most especially true in European cities – though less of a problem in Adelaide – and in order to create quieter railways and better-isolated buildings a good understanding of the mechanisms of vibration generation and transmission is vital.

This paper is concerned with a theoretical representation of the vibration of an underground railway tunnel and the surrounding soil. The tunnel and soil are treated as infinitely long concentric visco-elastic cylinders. An analytical solution based on the theory of elasticity is obtained which allows the displacements of track, tunnel and surrounding soil to be evaluated for harmonic force input. Multiple moving axle loads can be computed by superposition in the normal way and random-process theory is applicable here when random track roughness is the source of excitation.

A great deal of research has been carried out before and since the publication of nine reports entitled "Question D 151 Vibrations transmitted through the ground" [1]. Then it was clear that the mechanisms of vibration generation and transmission were complicated and poorly understood. Over the past decade and a half new and often very complex computational models have been developed and many measurements have been made to gain a greater understanding of the problem. Yet ground-vibration control for underground-railway track is still based on unfounded assumptions and as a result vibration counter-measures do not perform anywhere near as well as "in theory" they should. Underground-railway operators recognise that there are two distinct ultimate objectives. The first is to develop models which can be used accurately to predict vibration levels in buildings near a railway track, taking into account such factors as soil type, track and tunnel construction, that the vibration generated by trains running on surface and underground railways are distinctly different, that slow freight trains are different from high-speed passenger trains. Such models may be complex, take a long time to run and require a large amount of data to be gathered. The second is to create an operational model as a tool for designers to enable them to develop the best track for different operating conditions. Such a model must be fast to run on a PC so that manufacturers, consultants, designers, builders and operators can all make rational decisions about track design for reduced vibration transmission. It is this second objective that is of great interest in Cambridge University and a schematic of a typical model under investigation is shown in Figure 1.

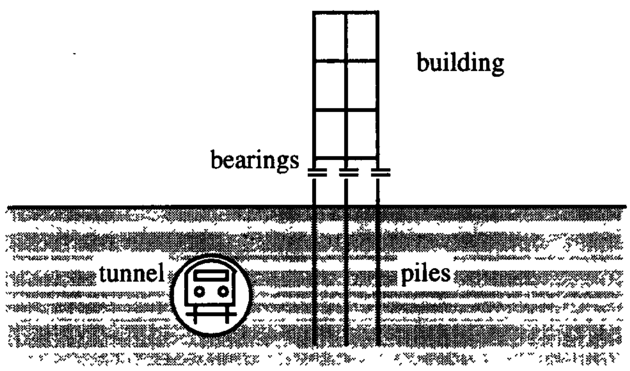


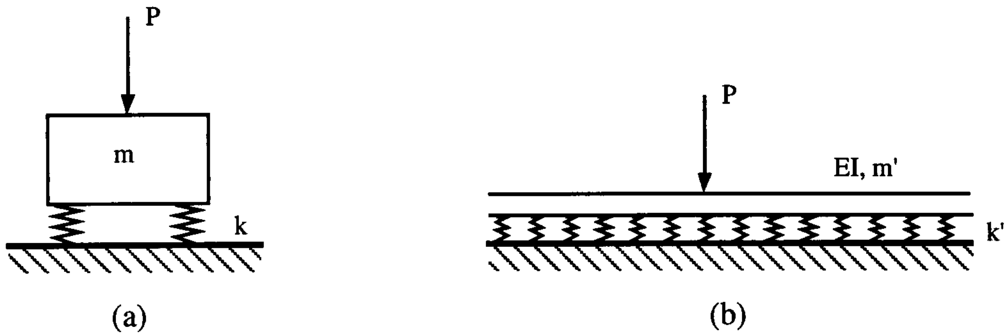
Figure 1. An end view (simplified) of an infinitely-long railway in a tunnel alongside an infinitely long building.

There have been many attempts to simplify the problem of vibration transmission from railways and in particular from railways in tunnels. For instance, Gutowski and Dym [2] use a geometric decay law to make vibration predictions at some distance from a railway but their methods give no hint as to the approach one might take to reduce vibrations at source. Trochides [3] makes an analysis of an underground railway tunnel of box construction and by using statistical energy analysis makes rather good predictions. But again it is hard to see how one might make recommendations for specific vibration counter measures in the track. Balendra et al. [4] have taken a 2D finite element model of a tunnel-building system in Singapore and have looked at its various characteristics, but it is the author's belief that important effects in the third dimension (along the track) have been ignored. The finite-element method is also rather cumbersome; useful in the particular, but not good for drawing generalisations.

With whatever model, many parameter studies have been carried out. Alabi [5] takes a relatively simple halfspace model with a moving load. Again, the details at the point of application of the load are absent and so vibration countermeasures cannot be designed. Krylov et al [6] go to great lengths to produce mathematically rigorous closed-form solutions to the problem of surface waves generated by moving loads, again of great interest but not leading to practical solutions for track construction. Hunt [7] makes use of random-process theory to reduce 3-dimensional problems into 2D. This is based on earlier work for computing ground vibration from road vehicles [8] and it forms the simplifying basis of the proposed models of infinite length described in this paper. Wheel loads are modelled as spatially-distributed random sources with a variable degree of spatial correlation. The essential assumption is that the spatial variation is statistically stationary and that, at some distance from the railway, no individual axles can be perceived. This stochastic framework simplifies the analysis of vibration from trains in tunnels and its propagation into buildings.

INSERTION-LOSS MODELS

When considering the response of isolated railway track it is usual first to consider the one-degree-of-freedom oscillator just as if it were a typical vibration-isolation system. Aware of the limitations of such a representation of the track it is considered an improvement in the calculation of insertion loss to consider a railway track supported on a continuous elastomeric support. For this purpose reference is usually made to the theory for beams supported on a Winkler foundation as shown in Figure 2.



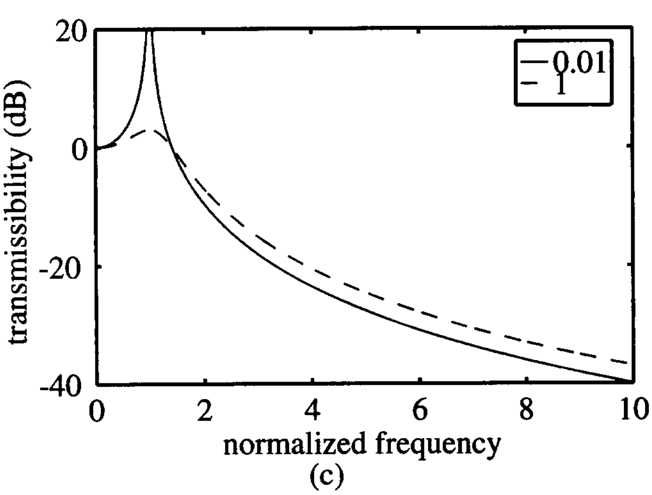


Figure 2. (a) a one-degree-of-freedom oscillator. (b) Euler beam on a Winkler foundation. (c) Force transmitted through to the base (transmissibility) which is identical for the two models. Both models predict that 'isolation' occurs above √2 time the natural frequency. Shown for loss factor η = 0.01 and η = 1 suggesting that low damping gives the best isolation..

As it turns out, the calculation of vibration transmission using the more elaborate Winkler model is *identical* to that which is obtained by using the simple oscillator model. This is a quirk of the Winkler theory and for more detail the reader is referred to Hunt [9]. More elaborate infinite-beam models have been evaluated by Forrest [10] with similar conclusions. It is not possible to evaluate the vibration-isolation effectiveness of a railway track system by means of a Winkler-type model. This is because the modelling of transmission away from the track and of interaction with the tunnel wall is not included and these effects are very significant. It is naïve to assume that the transmission occurs entirely in the vicinity of the applied force and it is also unreasonable to assume that the vibration propagating along the track and that is then transmitted into the ground can be accounted for by simply adding it to the total transmission,. In addition, it is fair to challenge the assumptions that the track bed and tunnel wall are rigid (for the purpose of calculating transmitted force) and that the load is applied along the centreline of the track.

A first step towards accounting for off-centre application of wheel loads is to consider travelling torsional waves in the beams representing the rail and floating slab. A typical arrangement for a floating slab track in a tunnel is shown in Figure 3. Note that the rails do not pass down the centreline of the track. This gives rise to both bending and torsional excitation of the floating slab. Moderately short wavelength rail surface irregularity is largely responsible for vibration generation and it follows that the excitation from each rail need not be correlated and therefore that torsional excitation should be considered. On the other hand, long-wavelength irregularities which derive from tunnel or track characteristics will be correlated and may generate only bending waves at low frequencies.

The response of a typical beam with a unit eccentric load has been computed and the results shown in Figure 4. The load is applied at the left edge of the beam and, as expected, note that the response on this side is greater than at the centre. It is interesting to note also that at higher frequencies the centre of the beam moves less than do the two edges. These are significant effects that must be taken into account when designing track isolation systems.

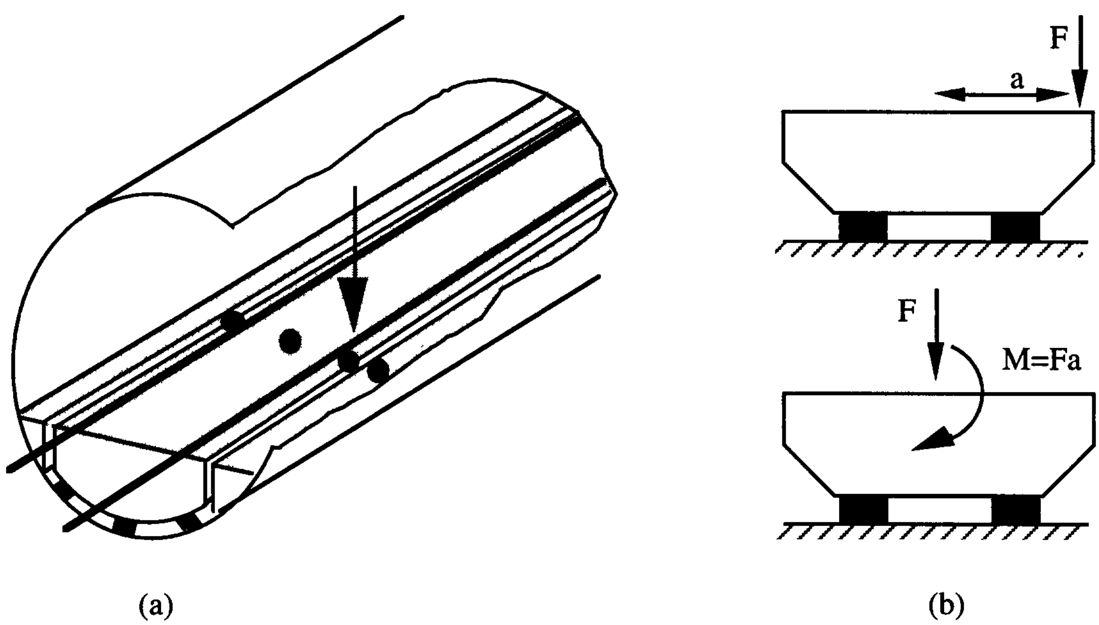


Figure 3. A schematic diagram of a floating-slab track in a tunnel. Note in (a) that wheel loads are not applied to the centreline of the track. A simple approach is to decouple torsional and bending excitation as in (b).

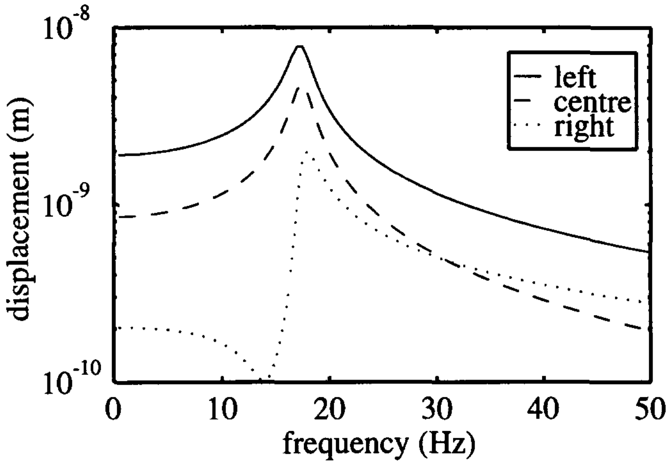


Figure 4. Variation with frequency of edge displacement of a beam on a Winkler foundation including the effect of torsional waves. Note that the centre, right and left responses of the slab are distinctly different.

Care must be taken when choosing values for the foundation modulus and flexural rigidity as it is not clear where the shear centre will be for a given section. This will depend very much on individual sections and on the distribution of elastomeric elements. The propagation of combined bending and torsional waves is discussed further in Hunt [9]

In January 1997 some measurements were made on a floating-slab track by means of an instrumented impulse hammer. The hammer was used to excite the floating slab at various points and the measured response can be compared with the predictions made above. A sample of the measurements is to be found in an earlier paper by Hunt [9] and further details are given by Forrest [11]. The measurements differ substantially from the predicted form of Figure 4 and there are two factors primarily responsible for these prominent differences. Firstly, the tunnel is interacting with the slab and it cannot be assumed that the slab is resting on a rigid base. Secondly, the propagation of waves along the slab is being governed by the interaction between the tunnel and the surrounding soil. An effect analogous to acoustic coincidence (see for example Fahy [12]) is important here when the speed of travelling waves in the floating slab is greater than the speed of waves in the supporting medium (tunnel/soil). In this case energy is radiated very efficiently from the track into the tunnel and surrounding soil. This has the effect of increasing damping and so stifling the resonant peak.

TUNNELS

Work that is in progress in Cambridge takes into account the deformations of the tunnel wall as shown in Figure 5. The model that is used is based on the work of Köpke [13] and it allows for the exact determination of the modes of vibration of a tunnel surrounded by an isotropic elastic continuum. The findings of this research are to be published shortly and they confirm the hypothesis that tunnel interactions are responsible for the poor predictions made by the simpler models presented in this paper. Some results are presented below and in a companion paper by Forrest [11].

Kopke's work was not related to railway tunnels; rather it's application is to the vibration of buried pipelines. The methodology is perfectly well applicable here and a brief summary is given below. For details see Kopke [13.] and Graff [14.].

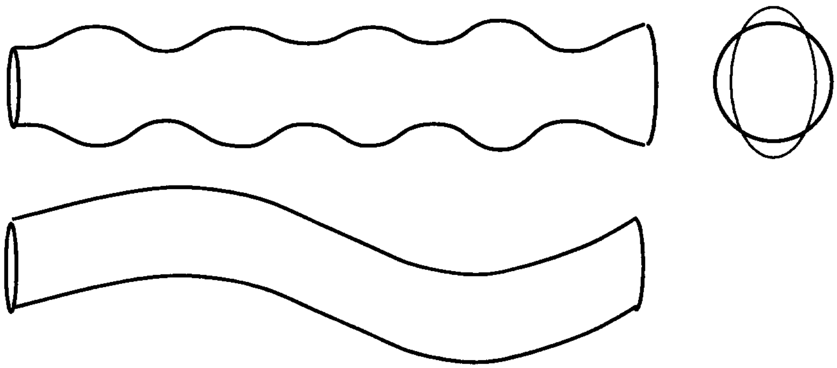


Figure 5. The two main types of tunnel motion that influence the propagation of vibration from railways.

INFINITE-CONCENTRIC-CYLINDER MODEL

A tunnel buried in a halfspace can be represented approximately as two concentric cylinders, one to represent the tunnel and another to represent the soil as shown in Figure 6. While clearly the model does not include a free surface it is useful for the evaluation of rail-tunnel interactions. Far-field details such as surface topography and the configuration of building foundations do not affect wheel-rail-tunnel interaction. It is not intended that 'exact' answers for vibration at ground level can be obtained but a comparison between track vibration isolation measures is possible with the model.

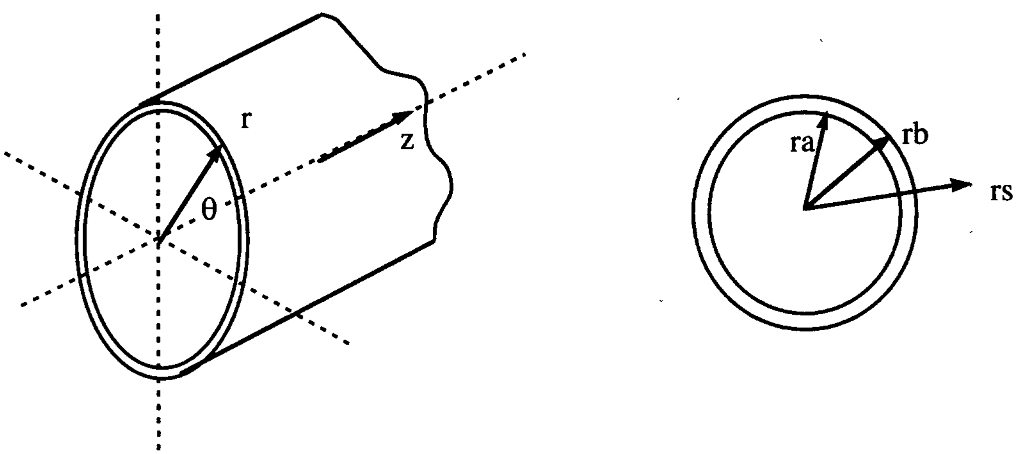


Figure 6. The concentric-cylinder model. Cylindrical co-ordinates z, r, θ are used. Two concentric cylinders are used to model the tunnel and the infinite soil ($r_s \rightarrow \infty$).

The governing equation for a homogeneous isotropic elastic solid are used in an axisymmetric form for a thick walled cylinder. Boundary conditions are used to obtain solutions based on a discrete Fourier series in θ and a continuous Fourier integral from z-space into γ-space. A matrix **S** contains terms such as sin(nθ) and cos(nθ) for each of the circumferential 'modes' and a summation over n gives the following solutions in matrix form. The inclusion of n is not merely a mathematical device but describes by summation the physical modes of the system. **U** and **T** are functions of the elastic properties and of r, γ, ω and n. A vector **c** contains constants determined by boundary conditions. The displacement and stress functions are written as follows:

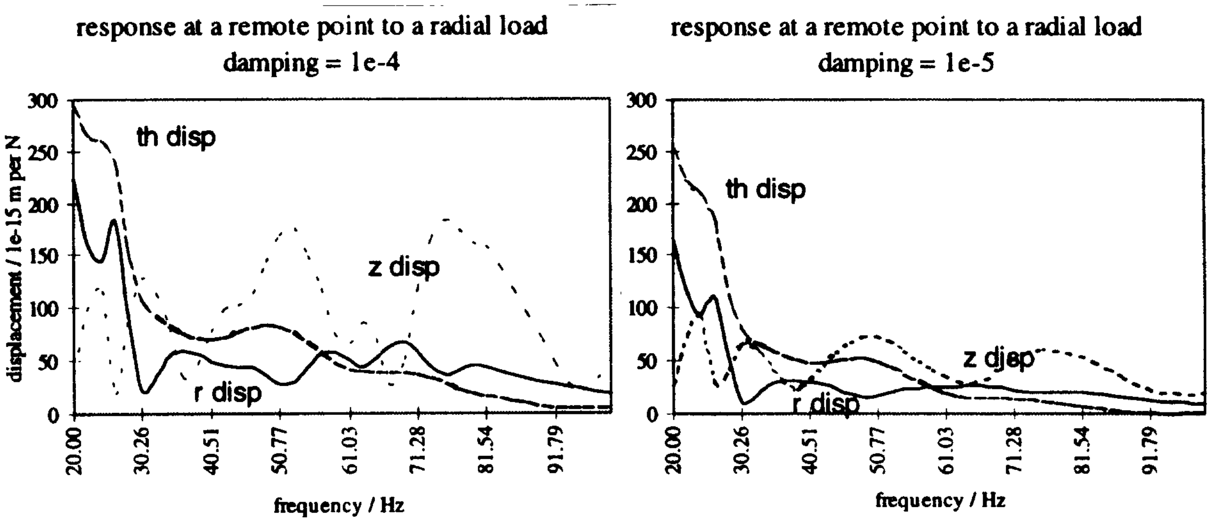
$$u = [u_r \; u_\theta \; u_z]^T = \sum_{n=0}^{\infty} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathbf{S}\,\mathbf{U}\,\mathbf{c}\; e^{i(\gamma z-\omega t)}\, d\gamma\, d\omega \qquad [1]$$

$$\tau = [\tau_{rr}\; \tau_{r\theta}\; \tau_{rz}\; \tau_{\theta\theta}\; \tau_{\theta z}\; \tau_{zz}]^T = \sum_{n=0}^{\infty} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \begin{bmatrix} \mathbf{S} & \mathbf{0} \\ \mathbf{0} & \mathbf{S} \end{bmatrix} \mathbf{T}\,\mathbf{c}\; e^{i(\gamma z-\omega t)}\, d\gamma\, d\omega \qquad [2]$$

A solution is obtained by first taking the fft of the applied load to find the loading in γ–space. For each value of n, set up **S** as a function of $n\theta$ and then find **U** and **T** for given r, γ, ω and n. Set up a matrix equation defining boundary conditions for each cylinder (the displacements and stresses at the outer surface of the inner cylinder and those at the inner surface of the outer cylinder are equal and the displacements and stresses in the outer cylinder are zero at infinity), invert the matrix, find constants **c** and finally find displacements and stresses using equations 1 and 2. Having done this for many values of γ take the inverse fft of displacements and stresses to find results in z space. A summation over n will allow any desired circumferential loading as a sum of modes.

For a railway tunnel the material damping in the tunnel walls will be negligible compared to that in the surrounding soil. Damping in the soil is introduced into the model by using a complex value of the elastic moduli, for example $G = G(1+i\eta\omega)$, where η is a damping factor. Its inclusion has confirmed that radiation damping is much more significant than the damping afforded by material loss.

The model has been validated against certain standard solutions: (1) a two dimensional ring with its flexural and extensional resonances; (2) a beam on a Winkler foundation; (3) propagation of shear and compressive waves in the soil; (4) an elastic cylinder with a band of external pressure. The results obtained confirm to well-established analytical solutions.



(a) (b)

Figure 7. Displacement at a remote point (30m from tunnel). Tunnel parameters: E=50GPa, ν=0.3, ρ=2500kgm^{-3}, η=0; soil parameters: E=470MPa, ν=0.44, ρ=2000kgm^{-3}, (a) η=10 , (b) η=100 μs. Tunnel diameter=6m, wall thickness=250mm.

In this paper, only four terms are included in the modal summation (n=0, 1, 2, 3). In this way a 'point load' is spread over a range of about 80°. For a tunnel of internal diameter of 6 m this a length of approximately 8.5 m. This compares with an axial spacing of 5.5 m. Figure 7 shows all components of displacement at a remote point 30 m distant from the tunnel, and 60 m down the tunnel from the loading position. Two damping conditions are shown. These are the same two which were shown in the driving point response seen above . The higher value of damping makes a significant difference, moreso at higher frequencies as expected. The various peaks and troughs correspond to the various modes of the tunnel which cannot easily be predicted without a complete tunnel-soil model

CONCLUSIONS

The principle conclusions to be drawn from this paper are:

1. Measurements of the response of floating slab track in railway tunnels indicates that there are mechanisms of vibration attenuation that cannot be explained by the usual models;
2. A Winkler-beam model is no more useful for making predictions of insertion loss than is the single-degree-of-freedom model;
3. It is important to include the off-centre excitation effects due to the rails not being central to the slab.
4. A theory-of-elasticity solution for two concentric infinitely-long cylinders can be used to represent an underground railway and the surrounding soil.

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