FIFTH INTERNATIONAL CONGRESS ON SOUND AND VIBRATION
DECEMBER 15-18, 1997
ADELAIDE, SOUTH AUSTRALIA

# VIBRATION-ISOLATION PERFORMANCE OF FLOATING SLAB TRACK USED IN UNDERGROUND RAILWAYS

*James A. Forrest*
Department of Engineering
University of Cambridge, England.
Email: jaf23@eng.cam.ac.uk

## ABSTRACT

Floating slab track is a popular approach used to try to reduce vibration transmitted from underground railways into the surrounding soil and thence into nearby buildings. The rails are fixed to a concrete slab foundation which is supported on a resilient mounting, so isolating the track from the tunnel invert. However, the effectiveness of the vibration attenuation does not compare well with the simple mass-spring models typically used in the design of these tracks. This paper uses models of infinite length to investigate the performance of floating slab track in more detail. A track model constructed from an infinite series of repeating units is presented, each unit being two beams (for the rail and the slab) separated by elastic layers (for the resilient elements used in the track construction). An improved model consisting of an infinite slab beam mounted inside a tunnel of infinite length in soil of infinite extent is also presented.

## INTRODUCTION

Figure 1 shows the typical arrangement of an underground railway line using floating slab track (FST). The rails are fixed directly to a massive concrete slab by means of rubber rail pads which provide protection to the concrete as well as some reduction in noise produced. The slab is mounted on resilient elements (usually rubber bearings or steel springs) to isolate the track from the tunnel invert. The slab may be continuous, or it may be composed of discrete sections. A train running along this track provides input excitation to the system both through its moving static load and through irregularities such as rail-wheel roughness, wheel flats and rail joints.

There are many variants of FST and some of these are described by Esveld (1989) and the ORE D 151 Specialists Committee (1982). Grootenhuis (1976) presents measured results from the London Underground Barbican track system, which show that FST does provide significant attenuation of transmitted vibration.

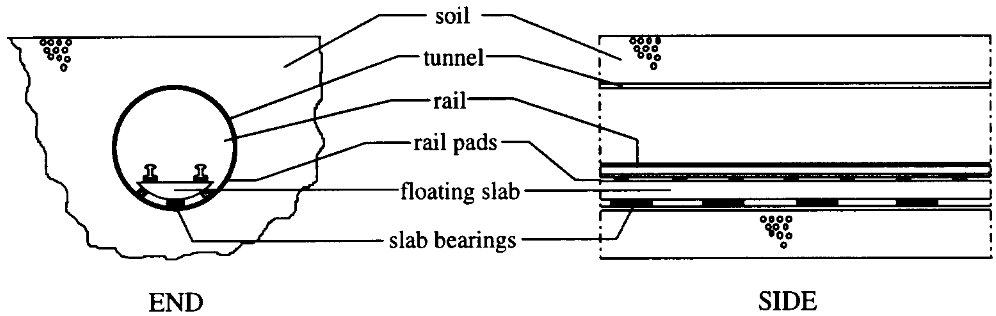


FIG. 1: The layout of an underground railway line and its various components

Several models for the transmission of vibration from underground railways into adjacent buildings have been proposed. Trochides (1991) and Kraemer (1984) use simple impedances to model everything between the excitation on the track and the response in the building. A more complicated model based on the finite-element method is given in Chua et al. (1992). This is a two-dimensional plane-strain model comprising a cross-section of the track, tunnel, ground and building. Balendra et al. (1991) derive an analytical model of a similar two-dimensional system by means of a substructure technique. Hunt (1995) uses a repeating-unit method to examine building models of infinite length in the context of vibration generated by underground railways.

Design methods for railway track are often based on simple lumped mass-spring models. Capponi (1995) summarises some of these approaches as used for FST. Wettschureck (1995) describes a similar method used to select elastomeric elements to reduce vibration and noise generation in conventional ballasted track with sleepers. Similarly, Jones (1994) examines the vibration attenuation performance of different configurations of ballasted track by means of a two-dimensional model of a mass and spring on an elastic halfspace as foundation.

Jones (1994) also presents a model consisting of a two-dimensional infinite beam with elastic foundation placed on a three-dimensional elastic halfspace. Beams on various foundations are often used to determine railway track responses. Grassie et al. (1982) give models for ballasted track based on infinite beams on simple elastic support, on continuous two-layer support incorporating sleeper mass, and on discrete support including sleeper masses. Dahlberg et al. (1993) use a finite-element approach to obtain a beam model of ballasted track which includes rail irregularities. Patil (1988) uses an infinite beam on elastic foundation to determine the response of railway track to a vibrating mass, and Duffy (1996) extends this to the case of a moving vibrating mass. Beams are combined in Esveld et al. (1996) to represent a light-rail track mounted in a concrete slab supported by layers of asphalt and soil. Courage and van Staalduinen (1996) construct a model of an elevated railway using nine sections, each section comprising a beam for the rail mounted on another beam for the viaduct.

An analytical solution for a tunnel of infinite length surrounded by soil of infinite extent is given in the companion paper by Hunt and May (1997). This model is based on the earlier work by Köpke (1993), who developed a model for the driving-point response of a pipe in infinite soil from a theory of elasticity formulation as part of an investigation into vibrating pipeline inspection devices (PIGs).

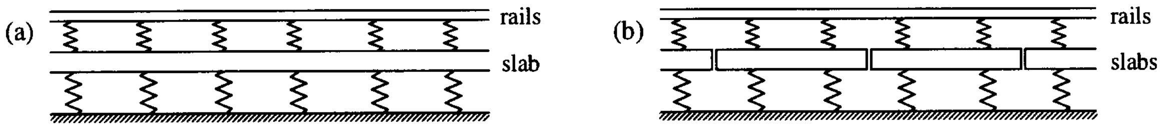


FIG. 2: Double beam models of FST using (a) a continuous slab and (b) discrete slabs

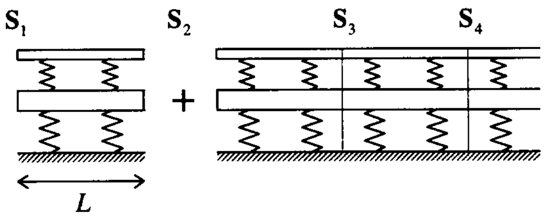


FIG. 3: Adding double-beam units to create a semi-infinite structure, showing the states **S** at the ends of the units.

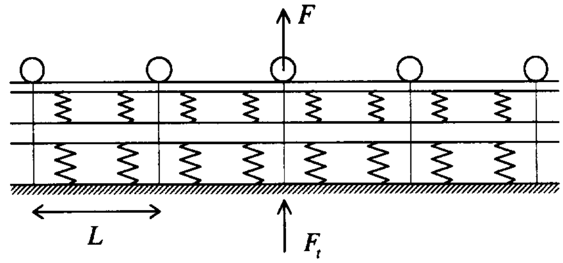


FIG. 4: A more realistic model of a railway includes masses on the rail to represent the axle-wheel assemblies of a passing train.

## MODELLING FLOATING SLAB TRACK

The three-dimensional underground railway track shown in Figure 1 can be thought of as a two-dimensional model consisting of an infinite beam, representing the two rails, mounted on another infinite beam, representing the slab. This is shown in Figure 2(a). The elastic layers between the two beams represent the rail pads and the slab bearings. In the first instance, the foundation is considered rigid. This is an extension of a simple beam on elastic foundation (Winkler beam). The lower beam may be divided into finite sections to model discrete slabs, as in Figure 2(b).

### The Repeating-Unit Method

Either of the two models given in Figure 2 can be constructed of an infinite number of finite-length double-beam units. For continuous slab track, both the top and bottom beams of the unit are joined end-to-end; for discrete slab track, only the top beams are joined. The details of this method are given in Forrest (1997), so only a brief description is given here.

Figure 3 illustrates the principle of the repeating unit method. The aim is to obtain the dynamic stiffness matrix (DSM) of the semi-infinite structure. Conceptually, a semi-infinite series of double-beam units are added together; it is clear that adding one more unit will not change the dynamics of the structure.

Euler beam equations and generalised boundary conditions can be used to find the DSM of the double-beam unit. This DSM can be used to get the transfer matrix relating the state **S** of forces and displacements at one end of the unit to that at the other. The crux of the method is that states are assumed to propagate along the structure unchanged except in magnitude and phase. This gives an eigenvalue problem and a solution for the semi-infinite DSM which condenses the dynamics of the whole structure to a relationship between the forces and displacements at the free end. An infinite track is composed of two semi-infinite halves, one extending to the right and one extending to the left.

To model the effect of railway vehicles on the track, masses representing axle-wheel assemblies can be added to each repeated unit to give a model like that in Figure 4. This shows a force input at only one axle. More realistic still is the use of roughness-displacement inputs between axles and rails, and multiple inputs; these are dealt with in Forrest (1997).

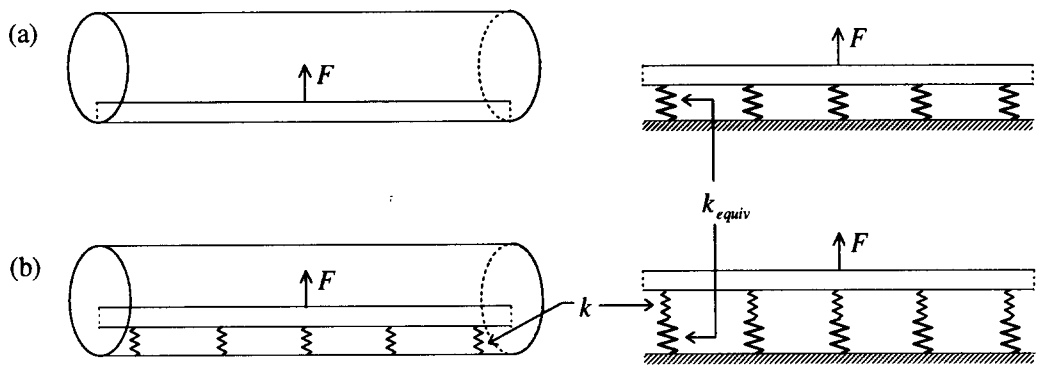


FIG. 5: A slab beam fixed (a) directly to the tunnel invert and (b) with a resilient layer between. Both cases are compared to an "equivalent" Winkler beam with the stiffness per unit length $k_{equiv}$ to simulate the effect of the tunnel.

The parameters used in the double-beam model are as follows. The rail beam has mass per unit length $m_1 = 100$ kg/m, bending stiffness $EI_1 = 10\times10^6$ Pa.m$^4$, support stiffness per unit length $k_1 = 40\times10^6$ N/m$^2$ and damping parameter $\zeta_1 = 0.5\times10^{-3}$ s. The slab beam has $m_2 = 3500$ kg/m, $EI_2 = 1430\times10^6$ Pa.m$^4$, $k_2 = 50\times10^6$ N/m$^2$ and $\zeta_1 = 1\times10^{-3}$ s. The damping is included in the stiffnesses as terms like $k(1+i\omega\zeta)$.

**The Track Mounted in a Tunnel**

The FST is in reality supported on a tunnel invert rather than a rigid base as has been assumed in the preceding section. In this paper, the tunnel is represented as an infinitely long tube surrounded by a medium of infinite extent to represent the soil. This is similar to the situation depicted in Figure 1, except that there is no free surface of the soil. The tunnel model is derived from a theory of elasticity solution for the problem, and details are given in the companion paper by Hunt and May (1997).

To examine the effect of the tunnel, a track must be joined to it. The current work considers only the track slab. Figure 5(a) illustrates a slab being directly fixed to the tunnel invert, with an "equivalent" Winkler beam for comparison. The "equivalent stiffness" to use in the Winkler model is found by equating the static driving point deflections of the slab in the two cases. Figure 5(b) shows the insertion of a resilient layer between the slab and tunnel, and the same addition in the "equivalent" Winkler beam.

Figure 6 gives the responses of interest in the slab and tunnel system. The slab and tunnel-floor displacements $y_1$ and $y_2$ are required in the joining of the two. The displacement $y_3$ in the far soil is important as it provides vibration input to building foundations situated there. All these depend on longitudinal position $z$ in the tunnel and on time. (The soil response is also a function of radial and axial position; radius and angle are fixed for the tunnel floor.)

The response calculations can be most conveniently done in the wavenumber ($\gamma$) domain, rather than the space ($z$) domain. This is because joining the infinite slab beam along its length to the infinite tunnel involves a convolution in the $z$-domain, and hence simple multiplication in the $\gamma$-domain. The calculations are done on discrete variables, so the total sample length and spacing between samples in space must be chosen carefully as in any other situation using discrete Fourier transforms. The variables are also already in the frequency domain by virtue of the harmonic analysis approach used.

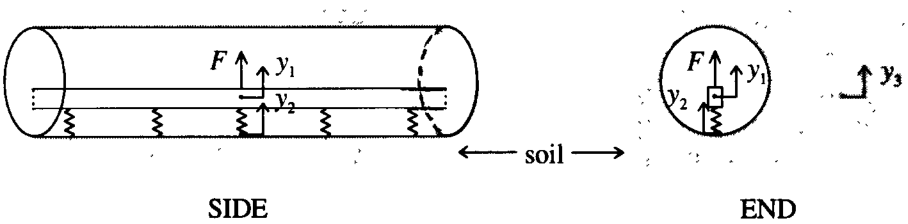


FIG. 6: Schematic of the tunnel plus slab showing the three required responses: along the slab beam, along the tunnel floor, and in the soil.

When the slab is excited by a single unit point force as shown in Figure 6, the equations governing the combined system in the $\gamma,\omega$-domain are

$$\left.\begin{aligned} & Y_1 = H_1 \cdot (G_1 + 1) \\ & Y_2 = H_2 \cdot G_2 \\ & G_2 = -G_1 \\ \text{and}\quad & Y_2 = Y_1 \quad \text{if no resilient layer present} \\ \text{or}\quad & G_1 = -2\pi k (Y_1 - Y_2) \quad \text{if there is one} \end{aligned}\right\} \tag{1}$$

where $Y_1$ is the response along the slab, $Y_2$ is the response along the tunnel floor, $G_1$ is the interaction force acting on the slab, $G_2$ is the interaction force acting on the tunnel floor, $H_1$ is the FRF along the slab to a point force acting on the slab, and $H_2$ is the FRF along the tunnel floor to a point force acting on the tunnel floor.

Solution of equations (7) for the case with no resilient layer give the result

$$Y_1 = Y_2 = H_1 \cdot H_2 / (H_1 + H_2) \tag{2}$$

which can be used to find the "equivalent stiffness". For the case of an added resilient layer of stiffness $k$ per unit length they give

$$\left.\begin{aligned} Y_1 &= H_1 \cdot (1 + 2\pi k \cdot H_2)/(1 + 2\pi k \cdot H_1 + 2\pi k \cdot H_2) \\ Y_2 &= 2\pi k \cdot H_1 \cdot H_2 / (1 + 2\pi k \cdot H_1 + 2\pi k \cdot H_2) \end{aligned}\right\} \tag{3}$$

where the factors of $2\pi$ arise from the definition of Fourier transform used. The response $Y_3$ along a line in the soil and parallel to the tunnel is determined for either case above by

$$Y_3 = (H_3 / H_2) \cdot Y_2 \tag{4}$$

where $H_3$ is the FRF along that line due to a point load acting on the tunnel floor.

The displacements of the combined system in the $z$-domain are obtained by taking the inverse Fourier transform of the results from equations (2), (3) and (4).

The parameters used in the slab-plus-tunnel model are as follows. The slab has mass per unit length $m = 3500$ kg/m and bending stiffness $EI = 1430 \times 10^6$ Pa.m$^4$. The tunnel has density $\rho_t = 2500$ kg/m$^3$, Young's modulus $E_t = 50 \times 10^9$ Pa, Poisson's ratio $\nu_t = 0.30$ and damping parameter $D_t = 0$ s. The soil has $\rho_s = 2000$ kg/m$^3$, $E_s = 470 \times 10^6$ Pa, $\nu_s = 0.44$ and $D_s = 1 \times 10^{-5}$ s. Damping is included in $E$ in terms like $E(1 + i\omega D)$, as well as in a loss factor $\eta = 0.1$ in the stiffness per unit length $k$ in the form $k(1 + i\eta)$.

## RESULTS

Several results for the double-beam models on rigid base are given in Forrest (1997). The following two graphs are taken from that source.

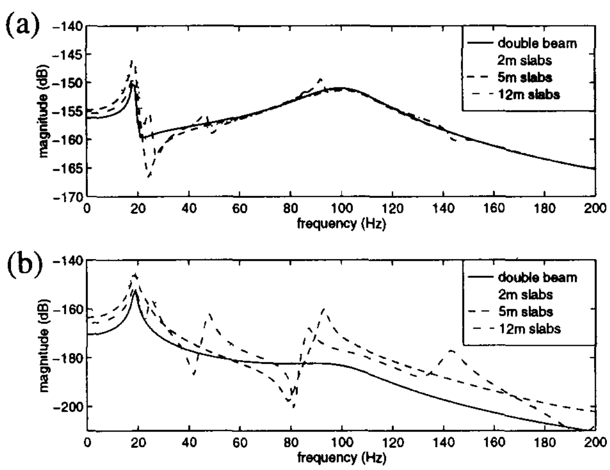


FIG. 7: Responses of (a) the top beam and (b) the bottom beam, directly under a unit harmonic point load applied to the rail beam (dB ref m/N).

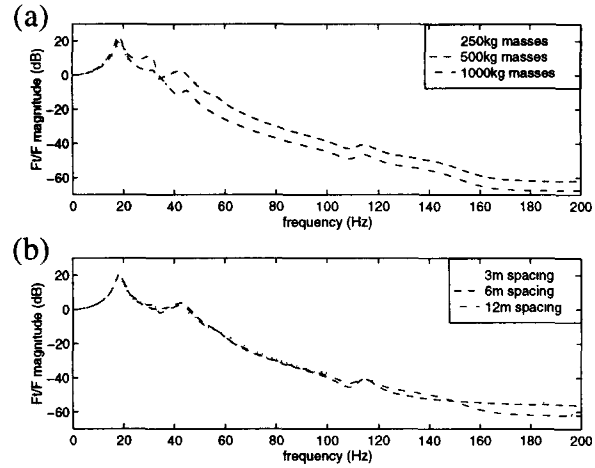


FIG. 8: Total transmitted force FRF magnitude for the continuous-slab model with (a) varying masses at 12m spacing and (b) varying spacing of 500kg masses.

Figure 7 shows the responses of the top and bottom beams directly underneath a point force applied to the top beam, for various slab lengths. The continuous slab results show resonances at about 20 Hz, corresponding to the resonance of the rail beam on its railpads, and 100 Hz, corresponding to the resonance of the slab on the slab bearings (and the onset of travelling waves in the respective beams in both cases). These can be predicted by a simple mass-spring system equivalent to one metre of track. However, the introduction of discrete slabs produces another set of resonances, particularly noticeable in the slab response, due to standing waves being set up in the slab above the frequency where travelling waves in the slab occur. Longer discrete slabs give more and lower-frequency resonances, since they can accommodate longer wavelength standing waves. Thus local displacement response is markedly affected by slab length. As slab response determines the force impressed on the ground, this is an important effect.

A better track model is that including axle masses, as depicted in Figure 4. Figure 8 illustrates the effects of axle spacing and axle mass on the total force (obtained by summing all the increments of force under the slab beam) transmitted to the foundation. The second resonance of the system is lowered due to the presence of the extra mass from the axles, but it is the axle mass and not axle spacing which changes this resonance's position. This suggests that the axles are interacting with a "characteristic length" of the rail beam rather than having a distributed effect. Spacing only begins to affect response above 100 Hz, the onset of travelling waves in the rail beam.

FST is mounted inside a tunnel rather than on a rigid base as assumed so far. Figure 9 gives the results of the slab-plus-tunnel model compared to an "equivalent" Winkler beam as discussed previously. Figure 9(a) shows the driving point response of the slab. When the slab is fixed directly to the tunnel, the response is very flat compared to the Winkler beam which shows a clear resonance. This is due to the large amount of radiation damping provided by the tunnel: energy is propagated away into the soil. As softer and softer resilient layers are added between slab and tunnel, the slab response gets closer to the Winkler approximation.

However, away from the load the picture is different. Figure 9(b) shows that when softer resilient layers are added, the slab response 20 m from the load increases, as well as being

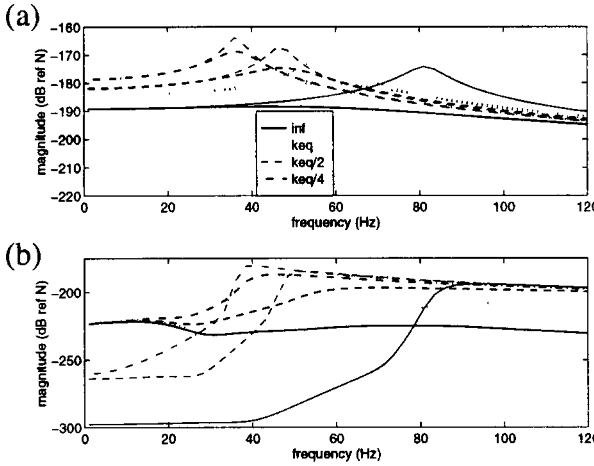


FIG. 9: The slab's response (a) at the driving point and (b) 20m along the slab, for varying added *k* ("inf" means slab direct to tunnel). Thick lines: slab-plus-tunnel model. Thin lines: "equivalent" Winkler beam.

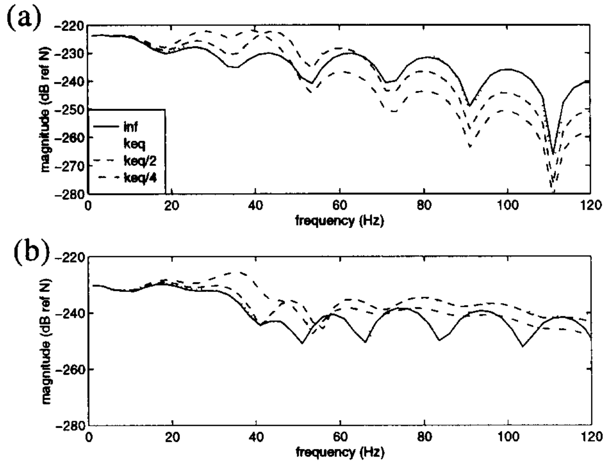


FIG. 10: Radial response of soil (a) at a 20 m radius from the driving point on the slab and (b) at a 20 m radius and 20 m longitudinal position from the driving point. Both at 90° circumferential rotation from tunnel invert.

different from the Winkler result. This higher response occurs because energy can travel down the slab rather than being radiated into the soil straight away.

The response in the soil 20 m from the tunnel, given in Figure 10, is greater for the lower frequency ranges for the cases with added resilient layers. Nevertheless, insertion of resilience between slab and tunnel provides reduction in vibration levels starting a little above the resonant frequency (clear in Figure 9(a)) of each particular case. This is comparable to what might be assumed from simple one-degree-of-freedom mass-spring systems.

## CONCLUSIONS

Initially it may seem that some features of FST can be explained by simple mass-spring models, but there are important ones which cannot. Investigation of double-beam track models reveals that local slab displacement is significantly affected by slab length, with extra resonances appearing, and addition of axle masses to the rail gives results which can be only partly explained by interaction with a "characteristic length" of rail. These effects can be ascribed to travelling waves in the beams. Mounting a track slab in a tunnel gives responses which are generally not well-predicted by an "equivalent" Winkler beam. Most importantly, soil displacements – the vibration inputs to buildings – can now be calculated. The qualitative aspects of when vibration attenuation begins are comparable to simple isolation theory, but the quantitative details of the soil response can only be gained from a model such as this.

## ACKNOWLEDGEMENTS

Thanks are due to Dr Hugh Hunt for all the guidance he has provided, and to Jonathan May, who developed the computer program for the tunnel used as part of the slab-plus-tunnel model presented here.

## REFERENCES

Balendra, T., Koh, C.G. and Ho, Y.C., 1991, "Dynamic response of buildings due to trains in underground tunnels", *Earthquake Engineering and Structural Dynamics*, Vol. 20, No. 3, pp. 275-291.

Capponi, G. 1995, "The Dynamic Behaviour of Underground Tracks, ISO document 108/2/3 N 167 draft" (Unpublished).

Chua, K.H., Balendra, T. and Lo, K.W., 1992, "Groundborne vibrations due to trains in tunnels", *Earthquake Engineering and Structural Dynamics*, Vol. 21, pp. 445-460.

Courage W.M.G. and van Staalduinen P.C., 1996, "Vibration analysis of an elevated railway track", *Structural Dynamics - EURODYN'96*, pp. 791-794. Balkema, Rotterdam, Netherlands.

Dahlberg, T., Åkesson, B. and Westberg, S., 1993, "Modelling the dynamic interaction between train and track", *Railway Gazette International*, June 1993, pp. 407-412.

Duffy, D.G., 1996, "The response of an infinite railroad track to a moving, vibrating mass", *Journal of Applied Mechanics, Transactions of the ASME*, Vol. 57, pp. 66-73.

Esveld, C., 1989, *Modern Railway Track*, Esveld Consulting Services, Zaltbommel, Netherlands.

Esveld, C., van't Zand, J., Scheepmaker, P.N. and Suiker, A.S.J., 1996, "Dynamic behaviour of railway track", *Rail Engineering International*, Edition 1996, No. 2, pp. 17-20.

Forrest, J.A., 1997, "Floating slab railway track for isolation of vibration: models of inifinite length", *Proceedings of DETC'97*, ASME Design Engineering Technical Conferences, September 14-17, Sacramento, California, USA.

Grassie, S.L., Gregory, R.W., Harrison, D. and Johnson, K.L., 1982, "The dynamic response of railway track to high frequency vertical excitation", *Journal of Mechanical Engineering Science*, Vol. 24, No. 2, pp. 77-90.

Grootenhuis, P., 1976, "Floating track slab isolation for railways", *Journal of Sound and Vibration*, Vol. 51, No. 3, pp. 443-448.

Hunt, H.E.M., 1995, "Prediction of vibration transmission from railways into buildings using models of infinite length", *Vehicle System Dynamics Supplement*, Vol. 24, pp. 234-247.

Hunt, H.E.M. and May, J.E., 1997, "Vibration generated by underground railway trains", *Proceedings of ICSV5*, Fifth International Conference on Sound and Vibration, December 15-18, Adelaide, Australia.

Jones, C.J.C., 1994, "Use of numerical models to determine the effectiveness of anti-vibration systems for railways", *Proceedings of the Institution of Civil Engineers, Transportation*, Vol. 105, pp. 43-51.

Köpke, U.G., 1993, "Transverse vibration of buried pipelines due to internal excitation at a point", *Proceedings of the IMechE, Part E: Journal of Process Mechanical Engineering*, Vol. 207, No. E1, pp. 41-58.

Kraemer, S., 1984, "Noise and Vibration in Buildings from Underground Railway Lines", PhD Thesis, University of London.

ORE D 151 Specialists Committee, 1982, "Question D 151: Vibrations transmitted through the ground. Report No. 2: An assessment of vibration counter-measures in current use.", Office for Research & Experiments of the International Union of Railways, Utrecht, Netherlands.

Patil, S.P., 1988, "Response of infinite railroad track to vibrating mass", *ASCE Journal of Engineering Mechanics*, Vol. 114, No. 4, pp. 688-703.

Trochides, A., 1991, "Ground-borne vibrations in buildings near subways", *Applied Acoustics*, Vol. 32, No. 4, pp. 289-296.

Wettschureck, R., 1995, "Vibration and structure-borne noise insulation by means of cellular polyurethane (PUR) elastomers in railway track applications", *Rail Engineering International*, Edition 1995, No. 2, pp. 7-14.