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Uncertainty of vibroacoustic behaviour of industrially identical structures. A new challenge for structural acoustic people

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Abstract :

Some experiments on populations of industrially identical structures have shown a large variation of their vibroacoustic behaviour. They indicates that small structural defects can lead to strong differences on vibration and acoustic radiation properties. This problem is presently important for automotive industry, because it leads to a non negligible percentage of cars that have acoustic problems.

The phenomenon was also theorically shown on poulations of coupled plates having small defects in their junction angle; and was called hypersensitivity. The explanation of such behaviour is a very strong change in the coupling of flexural and in plane motions, when the angle of connection vary. However, the phenomenon only exist for quasi flat junctions.

An experiment made on real body parts of car is presented to demonstrate that small geometric imperfections due to the process of assembling substructures, produce the hypersensitivity phenomenon.

The experiment was based on mobility measurements. We started with simple parts then coupled together in order to build a population of complex structures. It is then possible to see, at what level of the process of building cars the dispersion appears. The end of the paper is devoted to a theoretical approach of non perfectly described structures. We present how the uncertainty on the equation of motion, characterized through a residual energy, is related to the uncertainty on vibration response.

INTRODUCTION

When manufacturing cars, planes, machinery, etc..., it generally happens that a part of the production has poor vibroacoustic properties compared to the standard production. This is presently a big problem in industry and it seems necessary first to understand the physical reasons of the dispersion of acoustic behaviour, then to predict it. This phenomenon was shown by KOMPELLA and BERNHARD in the case of transfer function in cars.

Industrial structures are constructed from the assemblage of substructures, and one possible explanation of the dispersion is the accumulation of small defects in the substructures and in the junctions that produce finally a big uncertainty.

However this explanation seems not sufficient because, the vibrations and acoustic radiation of simple structures can be also, very difficult to predict. The hypersensitivity phenomenon was theoretically demonstrated on coupled plate by REBILLARD and GUYADER. A brief description of the phenomenon will be presented in the paper, it is related to the coupling of transverse and in plane motions, and appears with small defects of the angle of junction, when the angle of junction is around 5°. This suggest that the basic source of dispersion of vibroacoustic properties can be due to the hypersensitivity of one structure in the total assembly.

In order to demonstrate the reality of this explanation of observed dispersion on cars acoustic properties, an experiment was carried out on uncoupled parts of a car, that was then assembled. These experiments are presented in the paper.

The final part, and the more difficult, is the calculation of bounds of variation for vibrations of such structures having defects. An introduction to a possible approach based on the vibrations of imperfectly described structures will be presented.

THE HYPERSENSITIVITY PHENOMENON

For a detailed presentation one can read the original papers of REBILLARD GUYADER. Here just a brief description will be given.

The structure under study is constructed from several thin isotropic plates of the same thickness and width but of different lengths, connected along the widths at any angles. All the plates are simply supported on the lateral sides. The structure is excited by a pure tone force. A four plates example is shown in figure (1).

The basic result is presented in figure (2), where the transfer mobility of a population of two coupled plates are plotted. As one can see, even if the structures are very close; big differences appear in their mobilities, mainly because peaks and anti peaks are shifted. Let us also notice that the differences of behaviour can be observed also on the phase of the mobilities see figure (3).

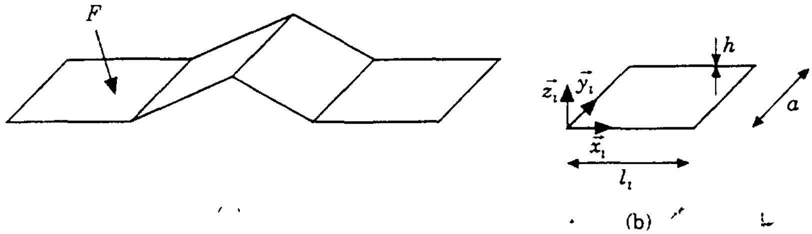


Figure 1. (a) A whole structure with four identical plates : (b) a particular plate of structure. $a = 0.4\ m$, $l = 0.5\ m$, $h = 2\times10^{-3}$ m, $E = 2.1\times10^{11}$ Nm^{-2}, $\rho = 7.85\times10^{3}$ kg m^{-3}, $\nu = 0.28, \eta = 10^{-2}$.

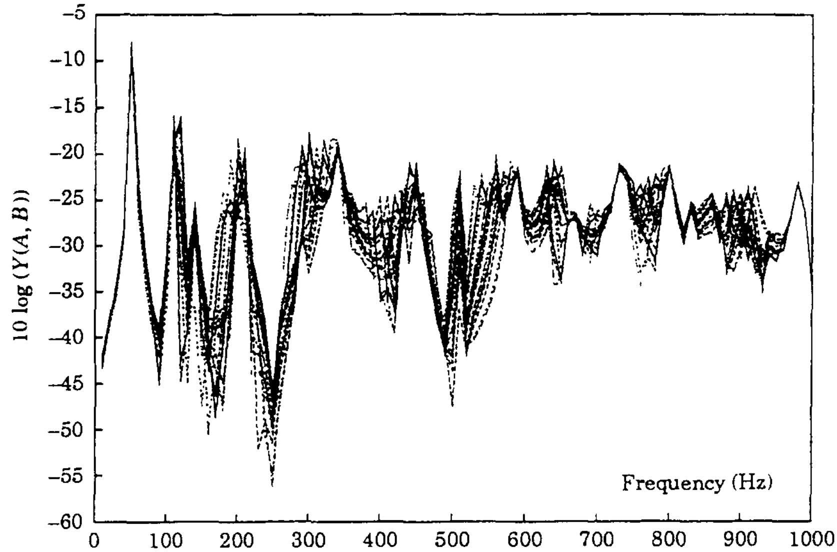


Figure 2 Modulus of transfer mobility; plate geometry as Figure 1. Connection angle $4 + \varepsilon^{\circ}$ where ε follows a Gaussian distribution (mean value 0°, standard deviation 1°).Driving force on the first plate at $x = 0.3,\ y = 0.17$ Response on the second plate at $x = 0.3,\ y = 0.17$.

The phenomenon of hypersensitivity appears for connection angles between plate around 4°, and does not exist when the angle of connection is bigger. The explanation is related to the coupling of transverse and is plane motions that vary rapidly with the connection angle when around 4°. This significate that quasi flat structure can behave very differently, when small geometrical defects appear.

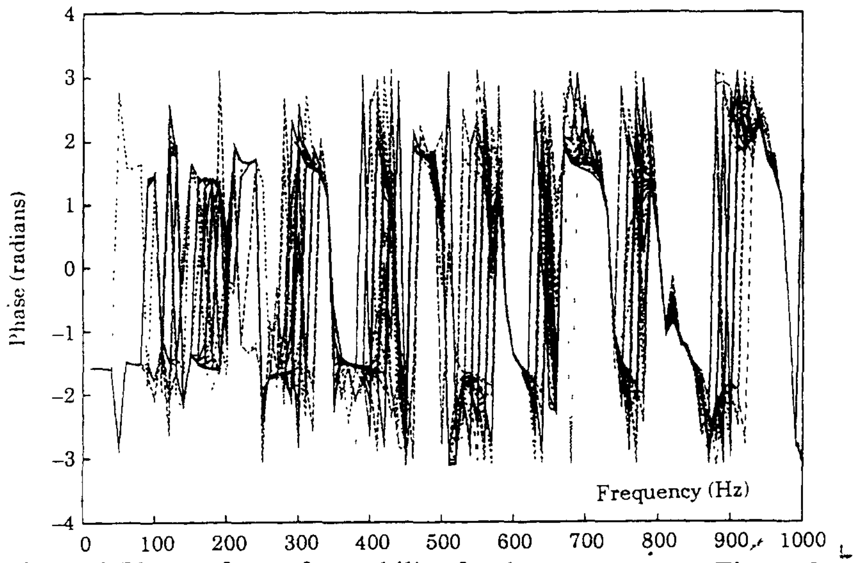


Figure 3 Phase of transfer mobility for the same case as Figure 2.

Let us now study a more complicated structure, namely eighteen coupled plates where the driving force is on the first plate for a complete information see REBILLARD and GUYADER.

The control parameter is the transfer mobility between the driving force point and the middle of each plate, however to make trends to appear we are going to average this quantity over frequency.

Let us consider two structures. The first one is the lattice without defect and the second one has a defect. For each plate of the altered structure is defined the relative error on the transfer mobility modulus at a fixed frequency.

$$Er_i(\omega) = \sqrt{\left(\frac{|\overline{Y_{ij}}(\omega)| - |Y_{ij}(\omega)|}{|\overline{Y_{ij}}(\omega)|}\right)}$$

where i is the receiving plate index and j the excited plate index, $\overline{Y_{ij}}(\omega)$ is the transfer mobility modulus for the reference structure without defect and $Y_{ij}(\omega)$ the transfer mobility modulus for altered one. To make trends to appear, the observation at particular frequency is not convenient, then we propose an average over frequency to smooth the phenomenon. We define a mean all over the angular frequency band Δ :

$$\langle Er_i \rangle_\Delta = \frac{1}{\Delta}\int_{\omega_c - \frac{\Delta}{2}}^{\omega_c + \frac{\Delta}{2}} Er_i(\omega) d\omega$$

Where ω_c is the center of the frequency band.

As one can see in figure (4) each time an hypersensitive junction appears in the transmission path the transfer mobility presents more and more dispersion, as indicated by increasing values of $\langle E_{r_i} \rangle_\Delta$.

Thus, one important thing in the hypersensitivity phenomenon, is associated to the propagation of hypersensitivity in coupled plates. When several plates are coupled, if one junction is hypersensitive, the transfer mobility display hypersensitivity if the transmission path from excitation point to measurement one include the hypersensitive junction. Thus, in complicated structures, the possibility of hypersensitive behaviour is strong, as it is sufficient that one hypersensitive substructure is involved in the transmission path.

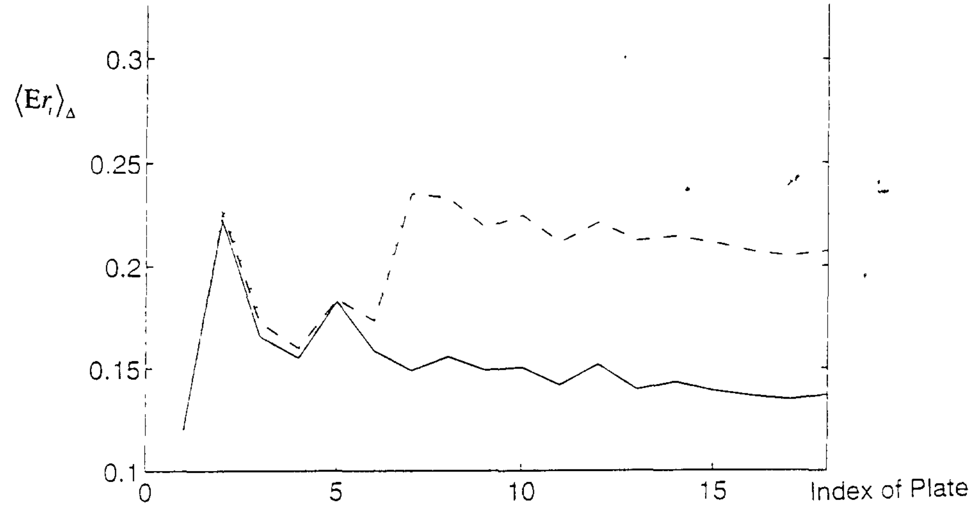


Figure 4. Effects of some angular defects of 1 degree applied to a lattice of eighteen plates coupled with an angle of 4 degrees. One angular defect : on the first connection (solid line), two : on the first and on the sixth connections (dashed line), three : on the first, on the sixth and on the twelfth connection (dooted line).

EXPERIMENTAL RESULTS

In order to verify how small geometrical defects can introduce vibrations differences. An experiment was made on real automobile parts see FRADET. 12 heating walls and 10 fire walls were first collected from the production line in order to see how simple elements can behave differently. Figure (5) presents the amplitude of transfer mobility measured for ''same'' excitation and measurements points on the 12 different heating walls. Figure (6) presents similar results for the 10 different fire walls.

As one can see some differences appear, they are mainly due to structural defects, even if the measurement process it self, is also responsible for differences in the amplitude of mobility. Uncertainty due to the measurement was studied making at several different time the ''same'' experiment on one structure. The result not presented here demonstrate that the spectra of the amplitude of mobility have differences much smaller than in figure (6).

The coupling between the two set of substructures was then realised. in a way looking like the real procedure using robots. The two parts where maintained with a longitudinal force, during the spot welding operation.

During the real spot welding process the longitudinal force is not controlled and thus it results in coupled substructures having geometrical small defects. This result in a big difference of the vibroacoustic behaviour of the population of coupled structures as demonstrated in figure (7), where the transfer mobility from a point of a substructure to one point of the other substructure is presented.

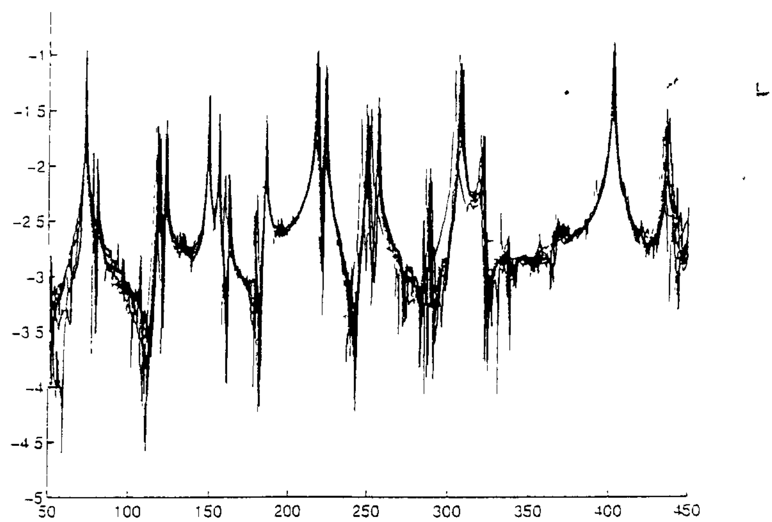


Figure 5. Transfer mobility of 12 heating walls measured in same conditions.

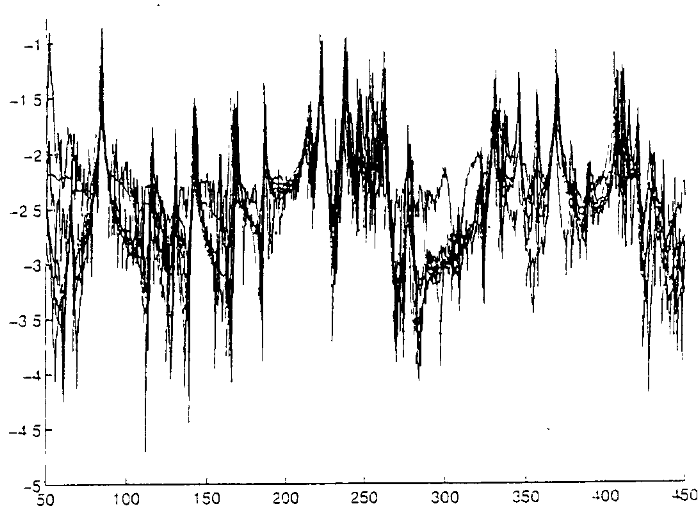


Figure 6. Transfer mobility of 10 fire walls measured in same conditions.

As conclusion, one can say that the uncertainty of vibroacoustic behaviour of real car structures are for one part, related to the uncertainty in the spot welding process. Even if the coupled structures have very close geometrical shapes. they behave differently. The reason for this is probably related to the coupling of transverse and in plane motions, as it was theoretically demonstrated for plate coupling.

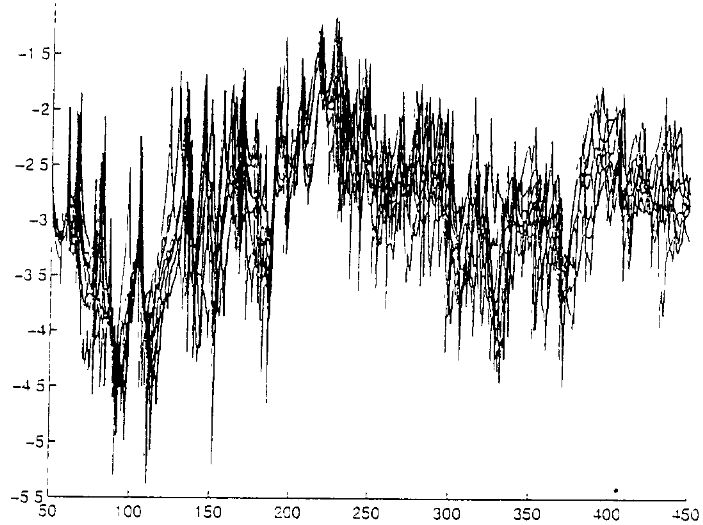


Figure 7. Transfer mobility from one point on the heating wall to a point on the fire wall.

However, other causes are involved in the different behaviours of population of cars structures. A more simple phenomenon can arise due to the heterogeneity of cars structures. In particular, on the location where the engine is connected to the car structure, the input mobility vary strongly when the input point location is slightly changed. The figure (8) presents the amplitude of the input mobility when the imput point is located in a circle of one centimeter diameter. This is obviously due to the presence of stiffeners, near the location where the engine is connected. Let us also remember that in this put mobility is important because it controls the power injection from the engine to the structure that finally produce the noise in the car.

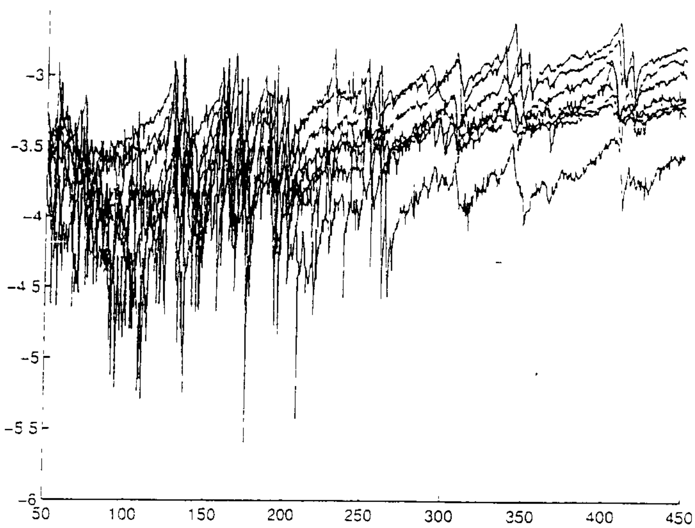
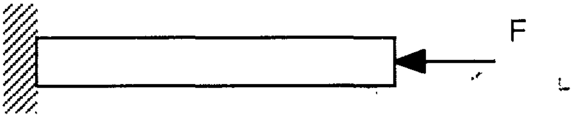


Figure 8. Amplitude of in put mobilities measured on the coupling point with the engine. Influence of small variation of the impact point.

A THEORETICAL MODELISATION OF THE EFFECT OF UNCERTAINTY

In order to make predictions one needs a mathematical model that represent the theoretical object under study, in general it is the mean structure of the population . Let us, to simplify, consider the case of the longitudinal vibration of a rod clamped at one end and driven by a harmonic force of angular frequency Ω, at the other end.



The mean structure is taken as an homogeneous rod, that has a unique solution $\left(\overline{\mathrm{U}}(x), \overline{\mathrm{N}}(x)\right)$ satisfying the following equations.

$$+\Omega^2\bar{\mu}\,\overline{\mathrm{U}}(x)+\frac{d\overline{\mathrm{N}}}{dx}(x)=0 \tag{1}$$

$$\overline{\mathrm{N}}(x)=\overline{\mathrm{K}}(1+j\eta)\,\frac{d\overline{\mathrm{U}}}{dx}(x) \tag{2}$$

$$\mathrm{U}(0)=0 \tag{3}$$

$$\overline{\mathrm{N}}(\mathrm{L})=\mathrm{F} \tag{4}$$

Where $\overline{\mathrm{U}}(x)$ is the amplitude of the longitudinal dispacement and $\overline{\mathrm{N}}(x)$ is the amplitude of the longitudinal force, $\bar{\mu}$ and $\overline{\mathrm{K}}$ are respectively the mass per unit length and the longitudinal stiffness of the rod, F is the driving force amplitude and η is the damping loss factor.

Let us assume, the real structures, of the population have imperfections that introduce heterogeneities on $\mu(x)$ and $\mathrm{T}(x)$. In that case the actual solution $(\mathrm{U}(x), \mathrm{N}(x))$ satisfy the equations :

$$\Omega^2\mu(x)\,\mathrm{U}(x)+\frac{d\mathrm{N}}{dx}(x)=0 \tag{5}$$

$$\mathrm{N}(x)=\mathrm{K}(x)(1+j\eta)\,\frac{d\mathrm{U}}{dx}(x) \tag{6}$$

$$\mathrm{U}(0)=0 \tag{7}$$

$$\mathrm{N}(\mathrm{L})=\mathrm{F} \tag{8}$$

The predictions are made on the mean structure and one calculate $(\overline{U}(x), \overline{N}(x))$: from the knowledge of this solution, is it possible to have some indications on the behaviour of all the actual solutions $(U(x), N(x))$?.

Of course one can make a lot of calculations for different distributions of mass and stiffness uncertainty but this is quite impossible in real case due to the amount of calculation. There is another possibility based on residual energy calculation, that is presented in the following.

Let us first introduce the kinetic energies of the mean problem solution $T(\overline{U}(x))$ and of the actual solution, $T(U(x))$:

$$T(\overline{U}(x)) = \int_0^L \frac{\Omega^2}{2} \mu(x)|\overline{U}(x)|^2 dx$$

$$T(U(x)) = \int_0^L \frac{\Omega^2}{2} \mu(x)|U(x)|^2 dx$$

The first energy has no reality because it takes into account the solution of the mean problem but, the mass density of the actual rod, however it can be easily calculated because $\overline{U}(x)$ is unique.

The second energy is the real one for a given rod of the population, but it is difficult to calculate, as $U(x)$ is unknown.

Is there any relation between these quantities $T(U(x))$ and $T(\overline{U}(x))$? To answer this question let us remark that $(\overline{U}(x), \overline{N}(x))$ is an a approximate solution of problem (5)- (8) (because of heterogeneity) and thus;

$$+\Omega^2\mu(x)\ \overline{U}(x) + d(\overline{N}(x)) = \varepsilon(x) \qquad x \in]0,L[\tag{9}$$

$$\overline{N}(x) - K(x)(1+j\eta)\frac{d\overline{U}}{dx}(x) = \theta(x) \qquad x \in]0,L[\tag{10}$$

For sake of simplicity we assume that the same loss factor η and boundary conditions remains verified by the mean structure problem and the actual problems.

The interpretation of (9), (10), is that the equations are not exactly verified and thus the second member is not equal to zero but to values $\varepsilon(x)$ and $\theta(x)$ that are small if $(\overline{U}(x),\overline{N}(x))$ is a good approximation of the actual solution $(U(x),N(x))$. Let us introduce the energy associated to the residual terms $\varepsilon(x)$ and $\theta(x)$ in the form :

$$\varnothing(\overline{U}(x),\overline{N}(x)) = \int_0^1 \frac{|\varepsilon(x)|^2}{\Omega^2\mu(x)} + \frac{|\theta(x)|^2}{K(x)(1+\eta^2)} dx \tag{11}$$

It can be interpreted as a residual energy due to the approximate verification of equations (5) and (6). It reduces to zero when they are exactly verified.

We are now ready to remember the result demonstrated in paper by GUYADER.

$$\sqrt{T(\overline{U}(x))} - \sqrt{\frac{\varnothing(\overline{U}(x),\overline{N}(x))}{\Gamma}} \leq \sqrt{T(U(x))} \leq \sqrt{T(\overline{U}(x))} + \sqrt{\frac{\varnothing(\overline{U}(x),\overline{N}(x))}{\Gamma}} \tag{12}$$

where

$$\Gamma = \beta^2\left(\left|\frac{1}{\beta} - \beta\alpha + |\alpha - 1 - j\eta|^2\right|^2\right) \quad (13)$$

$\beta = \omega/\Omega$, where ω is the nearest Eigen angular frequency (of the considered rod of the population) to Ω and :

$$\alpha = \frac{2 + j\eta + \eta^2}{\beta^2(1+\eta^2)+1}$$

This relation allows one to bound the actual kinetic energy of one rod of the population from the knowledge of the mean structure response $(\overline{\mathrm{U}}(x), \overline{\mathrm{N}}(x))$.

In fact the function Γ, amplifies the effect of the approximation. When the function Γ is small the bounds are large.

As presented in GUYADER one can see that Γ is minimum when $\Omega = \omega$ and is equal to η.

When the driving frequency is not located on a resonance frequency the bounds are considerably reduced, unfortunately, unfortunately the resonance angular frequencies of the considered rod are unknown and thus, one has to take into account the case worst to calculate the bounds, that is to say.

$$\sqrt{\mathrm{T}(\overline{\mathrm{U}}(x))} - \sqrt{\frac{\varnothing(\overline{\mathrm{U}}(x), \overline{\mathrm{N}}(x))}{\eta}} \le \sqrt{\mathrm{T}(\mathrm{U}(x))} \le \sqrt{\mathrm{T}(\overline{\mathrm{U}}(x))} + \sqrt{\frac{\varnothing(\overline{\mathrm{U}}(x), \overline{\mathrm{N}}(x))}{\eta}} \quad (14)$$

This relation is very realistic and means that the energies of industrially identical structures having light damping can have large fluctuations when different structures of the population are considered.

Let us now show an example. The solution for the mean structure problem is well known, and one can get.

$$\overline{\mathrm{U}}(x) = \frac{\mathrm{F}}{\overline{\mathrm{K}}(1+j\eta)} \frac{Sin\ kx}{Cos\ kL} \quad (15)$$

$$\overline{\mathrm{N}}(x) = \frac{\mathrm{F}}{(1+j\eta)} \cdot k \frac{Cos\ kx}{Cos\ kL} \quad (16)$$

where $k = \Omega\sqrt{\frac{\overline{\mu}}{\overline{\mathrm{K}}(1+j\eta)}}$

Let us suppose that the uncertainty come from a constant lineic mass defect , m, of length 2Δ located at point x_0.

$$\mu(x) = \overline{\mu} + m \qquad \text{if } x \in \left]x_0 - \Delta, x_0 + \Delta\right[$$

$$\mu(x) = \overline{\mu} \qquad \text{if } x \notin \left]x_0 - \Delta, x_0, \Delta\right[$$

Then using solutions (15) and (16), one can calculate $\varepsilon(x)$ and $\theta(x)$ with (9) and (10). After calculation one obtains :

$$\theta(x) = 0$$

$$\varepsilon(x) = \begin{cases} 0 & \text{if } x \notin \left]x_0 - \Delta, x_0 + \Delta\right[\\ \Omega^2 m \overline{\mathrm{U}}(x) & \text{if } x \in \left]x_0 - \Delta, x_0 + \Delta\right[\end{cases}$$

The residual energy (11) can then be easily calculated

$$\varnothing\left(\overline{\mathrm{U}}(x), \overline{\mathrm{N}}(x)\right) = \frac{\Omega^2 m}{\overline{\mu} + m} \int_{x_0 - \Delta}^{x_0 + \Delta} \left|\overline{\mathrm{U}}(x)\right|^2 \mathrm{d}x \tag{17}$$

On can also calculate the kinetic energy associated to.the solution $\overline{\mathrm{U}}(x)$:

$$\mathrm{T}\left(\overline{\mathrm{U}}(x)\right) = \Omega^2 \overline{\mu} \int_0^{\mathrm{L}} \left|\overline{\mathrm{U}}(x)\right|^2 \mathrm{d}x + \Omega^2 m \int_{x_0 - \Delta}^{x_0 + \Delta} \left|\overline{\mathrm{U}}(x)\right|^2 \mathrm{d}x \tag{18}$$

With these two expressions one can easily calculate the bounds of kinetic energy associated to mass defect, using (14).

CONCLUSION

The calculation of the vibroacoustics of a mean structure to represent the behaviour of a population of structures is not sufficient for a lot of problems where the dispersion is important.

In this paper the hypersensitivity phenomenon has been experimentally shown on real cars structures, and the spot welding process is responsible for one part of the dispersion in the vibraocoustic behaviour of cars.

In order the control the dispersion, bounds of energy related to structural defects can be established, using the concept of residual energy.

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