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SOME ASPECTS OF INTERFACIAL MOTION RELEVANT TO FORCED OSCILLATIONS

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ABSTRACT

Mechanisms comprising oscillating components have important engineering applications. These systems are often based on forced oscillatory motion of vital elements. Solid surfaces, involved in dynamic contact, almost always experience sliding and wear along the active interface. This effect becomes extremely significant in mechanisms designed to perform multiple automatic motions. The recursive motion, intended to follow a precise kinematic pattern, ultimately shows adverse deviations. Engineering technologies developed to combat these deviations, e.g. the hardfacing, need the appropriate criteria to evaluate the maintenance economy. In addition to geometric degradation, the disadvantageous vibrations occur as the result of growing dissipative forces. The number of material attributes, e.g. the elastic moduli, can significantly change with the sliding distance. Presently, the theory of kinematics does not provide complete models for sliding distance for rolling-sliding contact. Currently, the fundamental concepts of circular motion and oscillations are presented without addressing this important aspect of the motion gradient with respect to contacting surfaces. This paper presents the mathematical derivation of the general case of interfacial motion. The presented kinematic relations are important for large class of dynamic systems that comprise interfacial motion, e.g. rail-wheel contact. Reference is made to practical cases where the proposed model can be applied.

1. INTRODUCTION

Motion can be observed only with respect to certain reference. For the large family of systems, involving dynamic contact of solids, the convenient reference for motion of some observed point is the opposing contacting surface. As long as the contacting solids mutually slide, there is a coherence within the observed system. Both surfaces maintain the general courses of their motion by sacrificing relatively thin surface layers. To ease this perception, we highlight a collision of solids as an opposite situation. Collision causes the elastic repulsion (ie a significant change in motion tensor), plastic deformation, fracture, or decay, depending on the time - space scale of the observation. Yet another option is joining the aggregates into completely modified consolidations (eg by friction welding).

The most harmonic case - sliding of solid surfaces - relies on interfacial motion. Ideally, interfacial motion of solids can be realised via pure rolling or pure sliding, involving only elastic deformation. However growing evidence shows that rotary contact of solid systems always involves sliding, which in turn involves permanent deformation (deterioration).

We observe one familiar example of oscillatory motion - somewhat idealised - rolling the wheels on horizontal rails. If there was no external force, the wheel would cease rolling, following the law of damping oscillations. Dissipative forces will damp the harmonic oscillations (rotations) of the wheel, as time goes on. However, if the oscillations are forced to continue with a constant amplitude due to a contributing power engine, the resulting process is an antagonism between the damping and driving forces that significantly affects the surface layers. Degradation of surface material occurs causing the changes in material attributes. Familiar mechanisms in this situation are material fatigue and wear.

Figures 1 - 3 present a magnification of a contact zone at the intermediate stage of wear history of a solid of revolution engaged in rolling-sliding contact:

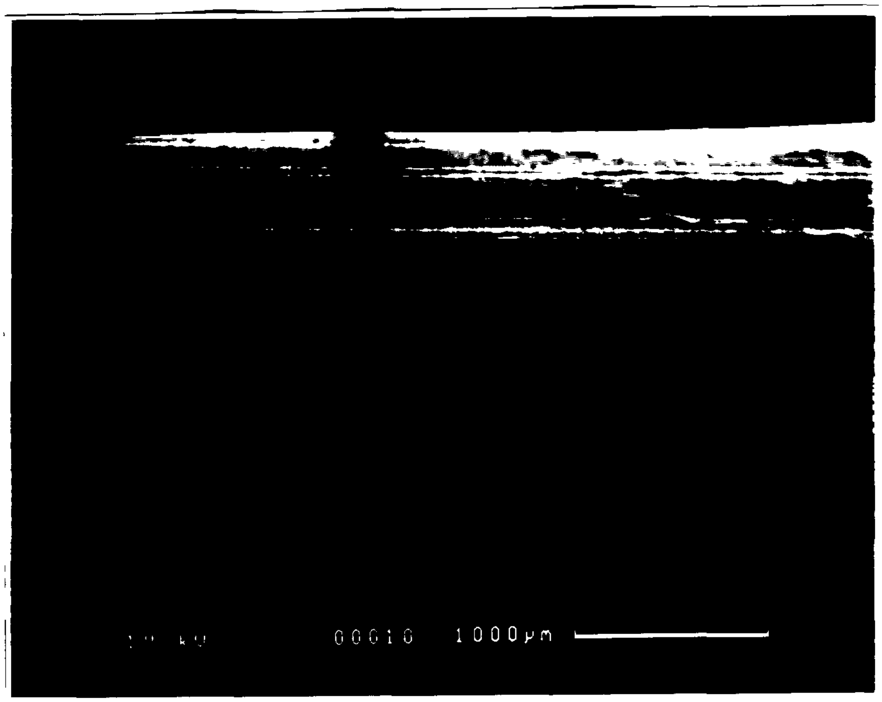

Fig 1: Surface of an Adamite roll sample exposed for 3 minutes to rolling-sliding contact at normal force F = 200 N and sliding velocity U = 0.25 m/s; surface temperature: 800 ^{o}C; [1]

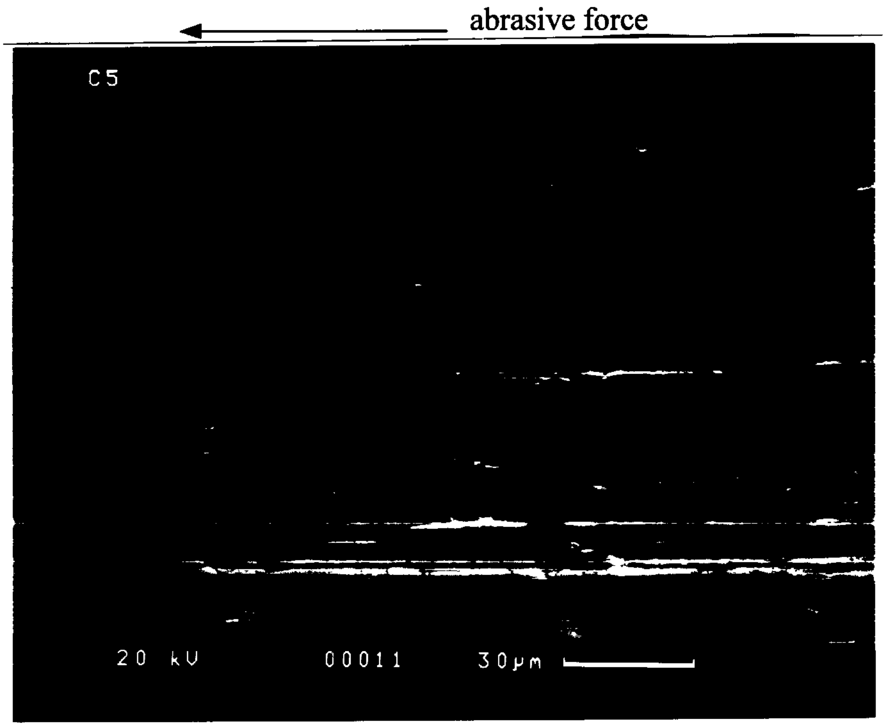

Fig 2: The same sample as in Fig. 1: SEM micrograph of wear track topography - a normal view on the worn surface; hard carbide presents obvious barrier to sliding [1]

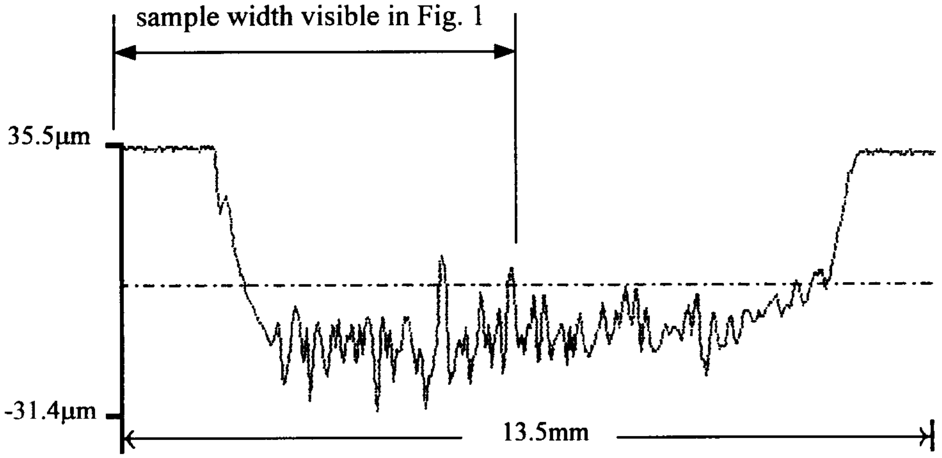

Fig. 3: The same sample as in Fig. 1; wear track profile exhibiting the surface texture [1]

The above figures 1-3 indicate that, during the rolling-sliding, the contact layers significantly depart from an ideal of a homogeneous, isotropic and continuous solid surface.

Archard's law, one of the fundamental relations in selection of engineering materials, is formulated by Eq (1) describing several principal factors affecting wear of solid surfaces:

$$WA_a^{-1} = K\,p\,H^{-1} \quad \text{.........(1)}$$

where W = wear rate, ie lost volume per unit distance slid; A_a = area of the instantaneous contact; K= Archard wear constant; p = normal pressure; H = hardness of the worn material

The intuitive perceptions and engineering definitions for all the above variables seem to be commonly shared. Yet, a more detailed analysis of the term "distance slid" will show that there is insufficient understanding of this notion for the case of rolling-sliding contact.

Mechanisms comprising oscillating components are amongst the important engineering applications, eg gears, shafts, cams, ball joints, rail-wheel systems and rolling mill rolls. These configurations often exhibit increasingly forced vibrations during the exploitation. Solid surfaces involved in dynamic contact almost inevitably experience sliding and wear. This effect becomes extremely significant in mechanisms designed to perform multiple automatic motions, eg in robots. The recursive motion intended to follow a precise kinematic pattern ultimately shows adverse deviation. The advanced technologies developed to combat these deviations, eg hardfacing, need the appropriate criteria. In addition to geometric degradation, the disadvantageous vibrations readily occur due to dissipative forces caused by friction. The whole range of material attributes eg damping capacity and elastic moduli can significantly change with the sliding distance, due to degradation of surface layers.

Presently, the stock of knowledge describing the kinematics, does not provide models for sliding distance even for most basic cases of interfacial motion, eg in rolling-sliding contact. The fundamental concepts of circular motion and oscillations are presented without addressing important aspects of the motion gradient with respect to contacting surfaces. Models of oscillatory motion should be complemented by interfacial kinematics to enable analysis of the changes in tangential force, material attributes and contact geometry, with sliding distance. The changes in the above listed variables will affect the dynamic stability of oscillating system causing further nuisances, eg the noise.

This paper presents the mathematical derivation of the general case of interfacial motion. A reference is made to selected practical cases where the proposed model can be applied using an interdisciplinary strategy. The new presented kinematic relations are important for the large class of dynamic systems that comprise significant components of interfacial motion.

Rail-wheel contact can be observed as an exemplary oscillatory system. The problems of vibrations and wear in rail-wheel contact are the objects of vigorous research [2-7]. Serious nuisances occur when this system is disturbed from ideal harmonic oscillations. To analyse and control the wheel-rail contact mechanics, the known models should be complemented with relations that define further significant relations. It has been widely recognised that the phenomena that are pertinent to oscillatory motion, eg material fatigue and acoustic noise, depend on the attributes of contacting layers. Relations involving effects of mechanical energy, problems involving elastic response, vibrations, and further aspects of the mechanics of solids, were derived under the assumptions of isotropic, continuous and homogeneous material. These assumptions do not satisfy even the static systems, ie solids are intrinsically anisotropic and heterogeneous objects, yet, more importantly, the attributes of solids change as the contact processes advance.

Apart from nominal stress and its amplitude, the typical measures for oscillatory systems are frequency and the cumulative number of the oscillations. For example, the resistance to fatigue of a system undergoing apparently harmonic oscillations, is usually measured with respect to the number of oscillations. However it has been shown [5,8] that resistance to fatigue depends also on the sliding distance. Elastic moduli and Poisson's ratio change not only with the contact temperature, but also with contact pressure [9] and with the anisotropy developed in the surface layers. All the above listed features depend on contact kinematics. Our ability to predict the behaviour of materials under dynamic stress, by means of theories of elasticity, decreases with the viability of the assumption that we are dealing with continuos, isotropic and homogeneous solids [9]. In the above highlighted rail-wheel system, undergoing the forced oscillations, we can observe how the equilibrium, maintained by driving forces, decays with the sliding distance. One of the basic equations of oscillatory motion is

$$\mathcal{E} = 0.5\ k\ A^2 \quad \text{.......(2)}$$

where $\mathcal{E}$ = energy of simple harmonic motion; k = force constant of the spring, that depends on the stiffens of the oscillating solid; A = amplitude.

Numerous papers treat the equations of motion involved in rail-wheel dynamics [2-7]. However, both experimental runs and real systems show many unsolved problems. Rail and wheel bear the effects of a multitude of the material attributes. In addition, the oscillatory motion of the wheel is affected by geometric properties of the contacting surfaces, eg roughness. Parameter **k** changes with surface temperature, ie with both the number of oscillations and sliding distance, because materials in surface layers experience physical and chemical changes as rolling-sliding contact continues. Competent engineering analyses of oscillatory systems that involve rolling-sliding contact require taking into account the above described phenomena. Testing and development of materials intended for rotary components can be futile, and corresponding inferences can be misleading if these important aspects of rolling-sliding contact are ignored.

2. DERIVATION

We focus on a special class of motion coplanar to a three-dimensional surface, so called interfacial motion. It should be noted that motion, by definition, can be observed only with respect to a certain reference object. For the large family of systems, the most convenient reference is the contacting surface. Observe the interface between two consolidated matter aggregates $\mathcal{G}$ and $\mathcal{P}$ such that the difference in the surface motion tensors satisfies relation

$$|\boldsymbol{g}| + |\boldsymbol{p}| \neq 0 \quad \text{.......(3)}$$

$\boldsymbol{g}$ = motion tensor of points coplanar to contact surface delimiting the aggregate $\mathcal{G}$
$\boldsymbol{p}$ = motion tensor of points coplanar to contact surface delimiting the aggregate $\mathcal{P}$

Assume that only the extreme points $\mathbf{G_i}$ on the surface of the aggregate $\mathcal{G}$ collide with the extreme points on the picks of the counter-surface. Furthermore, only one point, denoted by **P**, belonging to $\mathcal{P}$, will be observed during its collision with a series of counter-points $\mathbf{G_i}$. The whole dynamic system can be simplified as follows [1]: Assume that the point **P** moves from a position X_1 via X_2 and X_3 to a position X_4 with the constant velocity $\mathbf{v_p}$, and that points $G_1, G_2, ..., G_i, ..., G_K$ move along a line X_aX_b with the constant velocity $\mathbf{v_G}$ see Fig. 4:

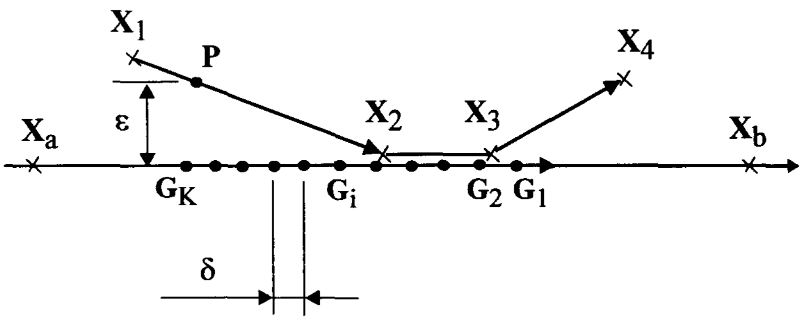

Fig. 4: Geometrical representation of the configurations of the points P and G_i

Note that in Fig. 4, the following conventions are assumed: the fixed positions (locations) are denoted by the symbol X. The locations denoted by X do not change their relative positions, while the points P and G_i can move only once from one to a subsequent location. Then, the time τ_s needed for point P to move from the location X_2 to location X_3 is equal to

$$\tau_s = (X_3 - X_2)/v_p \qquad (4)$$

Assume that the total number K of the points G_i is infinite, the distance δ between two neighbouring points $\mathbf{G_i}$ is constant, and that the vertical distance ε between the point P and line X_aX_b is equal to zero when point P is travelling between the locations X_2 and X_3, while $\varepsilon>0$ when **P** is outside the interval X_2X_3. In addition, let the following conditions be satisfied: Denote by G_k and G_{k+1} the two points which are the closest to the point P at the moment when P is at X_2. If $\mathbf{v}_p = \mathbf{v}_G$, the distance between P and G_i will remain constant during the time τ_s. On the other hand, if $|\mathbf{v}_p - \mathbf{v}_G| > 0$, during the time $d\tau$, there are two possible situations (Fig. 5):

1) $\mathbf{v}_p > \mathbf{v}_G$: the horizontal distance between P and G_{k+1} increases by **dx**, or
2) $\mathbf{v}_p < \mathbf{v}_G$: the horizontal distance between P and G_k increases by **dx**.

For $\varepsilon = 0$ it holds:

$$\mathbf{dx} = |\mathbf{v}_p - \mathbf{v}_G|\, d\tau \qquad (5)$$

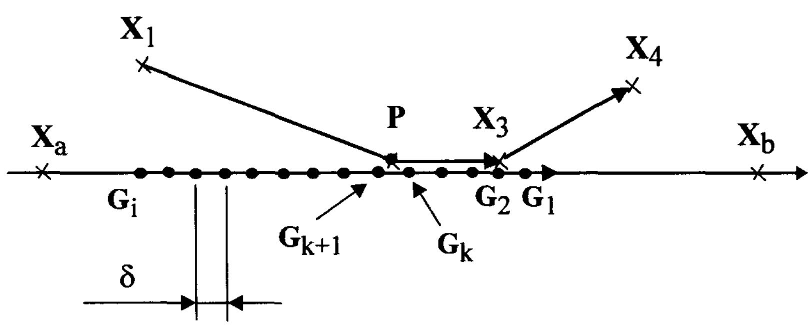

Fig. 5: Positions of the points P, G_k and G_{k+1}

By integrating Eq. (5) in the interval $\tau = 0$ .... $\tau = \tau_s$ it can be obtained:

$\Delta \mathbf{x} = |\mathbf{v}_P - \mathbf{v}_G|\ \tau_S$ ....(6)

If τ_S is substituted using Eq. (4), then Eq. (6) can be rewritten as follows:

$\Delta \mathbf{x} = |\mathbf{v}_P - \mathbf{v}_G|\ (X_3 - X_2)/\mathbf{v}_P$ ....(7)

Distance $\Delta \mathbf{x}$ will be called the sliding distance of the point P. In a more general case, the arguments of Eq. (7) can vary with a number of factors. Consider a series of points P_i belonging to a continuous surface, colliding subsequently with points G_i. A curvilinear contact is considered instead a linear contact. If we denote: $\mathbf{L}$ = sliding distance of point P_i, $\mathbf{s}$ = deformation zone (contact) curve length, eg, $\mathbf{s} = X_3 - X_2$, $\mathbf{U}$ = sliding velocity, eg, $\mathbf{U} = |\mathbf{v}_P - \mathbf{v}_G|$, then, Eq. (7) can be written as:

$$\frac{\partial \mathbf{L}}{\partial \mathbf{s}} + \mathbf{c}\frac{\partial \mathbf{U}}{\partial \mathbf{v}_p} = 0$$

.... (8)

where $\mathbf{c}$ is a component allowing for interdependence of attributes $\mathbf{L}$, $\mathbf{s}$, $\mathbf{U}$ and $\mathbf{v}_P$. Generally:

$\mathbf{c} = \mathrm{c}(\boldsymbol{g}, \boldsymbol{p}, \mathbf{f}, \tau_T\, \boldsymbol{y})$..(9)

where: $\boldsymbol{g}$ and $\boldsymbol{p}$ = motion tensors of surfaces G and P respectively; $\mathbf{f}$ = frequency of the oscillations, τ_T = total (cumulative) time of oscillations, $\boldsymbol{y}$ = stochastic component.

In real systems, factors $\mathbf{U}$, $\mathbf{v}_P$, $\mathbf{v}_G$ and $\mathbf{s}$ can vary significantly. For example, the diameters of solids of revolution change due to wear which affects their peripheral velocities; the contact length changes with mechanical properties, eg as influenced by temperature, etc.

A range of authors [1,10-12] have pointed out that micro-sliding phenomena occur along the contact zone. In some practical situations these phenomena can be neglected, but some cases of rolling-sliding contact are characterised by high variation of sliding velocity along the interface eg the deformation zone in the roll gap developed during plastic forming by rolling. By introducing propitious simplifications, we can modify Eq. (8) to enable approximate solution, ie:

$\mathrm{d}\mathbf{L}/\mathrm{d}\mathbf{s} + \mathbf{c}\ \mathrm{d}\mathbf{U}/\mathrm{d}\mathbf{v}_P = 0$ ....(10)

When a series of simplifications is assumed and taking into account that most systems can be observed within the closed routes, it can be written $\mathbf{c} = \mathbf{N}$ (ie each point P_i passes the interval X_2-X_3 $\mathbf{N}$ times under the same conditions), and a first approximate solution of Eq. (8) is:

$\mathbf{L} = \mathbf{N}\ \mathbf{s}\ \mathbf{U}/\mathbf{v}_P$ ....(11)

It seems that, in real applications, the above defined relations are subject to significant stochastic interactions, thus appropriate physical simulations are recommended before relying on the above deterministic models.

3. APPLICATION

The rolling-sliding phenomena are present in a wide range of engineering applications; Figure 6 presents a schematic of various rolling sliding configurations.

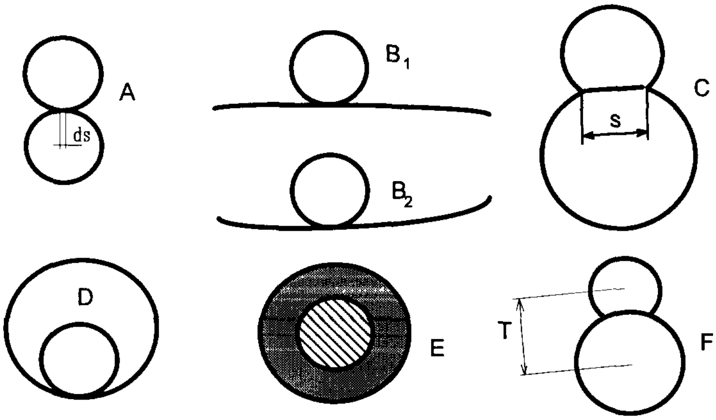

Figure 6: Schematic of various rolling-sliding configurations

The implications of the model defined by Eq. (8) can be projected into several fields of science and engineering eg:

- tribology: two-discs, configured as shown in case A in Fig 6, rub each other along the distances defined by the Eq (8); wear rate occurring during the simulation of rolling-sliding contact cannot be evaluated without understanding the kinematics of the interfacial motion. The presented concept of interfacial motion can eg be used to evaluate validity of Hertzian models in contact mechanics
- rail-wheel systems (case B shown in Fig 6): the oscillations and wear phenomena within the rail-wheel configurations are significantly affected by kinematics corresponding to Eq (8)
- solid mechanisms: cams, gears and bearings (cases C - F) operate in accordance to laws of interfacial (rotary) motion; without relations defined by Eq (8), only rough approximations can be made regarding the reliability of this family of mechanisms
- abrasive grinding of the solids of revolution (case F, Fig. 6): Equation (8) allows for derivation of more adequate norm for evaluating the performance of the abrasive disc, thus enabling more reliable optimisation of grinding operations; the control of vibrations during grinding is beneficial for process performance (eg quality of product, grinding wheel life)
- plastic forming by rolling: Equation (8) allows for derivation of the more adequate norm of roll life, and thus the better understanding of further factors influencing the process performance; the adverse vibrations during plastic forming by rolling are detrimental for both process and equipment; the corresponding noise is a serious health hazard
- vibrations: Eq (8) can be applied on various types of recurrent motion; the change of friction with sliding distance affects significantly whole range of phenomena, eg the noise generated by this class of systems.

More detailed algorithms describing effects and interactions within each of the above cases can be developed as innovative contributions to relevant technologies. The important aspect of the above class of systems is the contact (sliding) distance where the transfer of matter and energy takes place. For many of the above discussed systems, this aspect can be simulated using two-disc tribometers. From the viewpoint of tribology (and materials engineering), wear rate is a crucial norm that is used to compare various materials via two-disc simulation. A number of authors [14,15] agree that the wear rate $\mathbf{w_L}$ should be expressed as follows:

$$\mathbf{w_L} = \Delta\mathbf{R}/\mathbf{L} \quad (12)$$

where: $\Delta\mathbf{R}$ = radial wear (depth of the worn layer); $\mathbf{L}$ = rolling-sliding distance.

The recursive motion, designed to follow a precise kinematic model, ultimately shows disadvantageous deviations. In addition to geometric degradation, the distressing vibrations occur as the result of developing dissipative forces. These nuisances can be suppressed efficiently only if we are able to define their quantitative changes with sliding distance.

Generally, the external forces applied to oscillating systems to suppress damping oscillations, create a motion that does not decay with time. The energy lost to friction, deformation, heating, etc, is replaced, over the course of each cycle, by the driving force. The kinematics involved in compensating for the energy lost in dissipative processes, depends not only on the frequency and the total number of oscillations, but also on the interfacial motion. Wear of contact surfaces is an important factor in controlling forced vibrations. If this wear is not defined appropriately, the probability of undesired phenomena, eg resonance and growing noise, increases. This logic is widely used in machine fault diagnosis [16]. After an appropriate diagnosis has revealed the critical sub-system, an analysis can be performed and eventually a re-design could be required. The analysis of the fault sub-system, exhibiting too high vibrations, almost inevitably requires the understanding of the actual performance and service-life of the critical component. Often, this parameter is expressed using norms such as the number of revolutions (of a gear wheel) or the cumulative number of periods, N, performed since the last maintenance. A further norm - sliding distance - should be also involved to enable more efficient diagnosis of the problems and optimisation of the system.

4. CONCLUSIONS

The functionality and life of rotating elements, that are widespread phenomena in technical systems, are closely associated with the oscillation regime, but also with the kinematics of interfacial motion.

Presently, published works in kinematics do not provide satisfactory models even for the most basic cases of interfacial motion, eg rolling-sliding contact. Currently, the fundamental concepts of circular motion and oscillations are presented without addressing the motion gradient with respect to contacting surfaces. It should be noted that motion, by definition, can be observed only with respect to certain reference object. For the large family of systems, the most convenient reference is the contacting surface.

As long as the neighbouring aggregates mutually slide, there is a high probability that conditions for coexistence of both systems are met. Mutually sliding systems preserve the general courses of their motion, however the surface configurations are sacrificed. Otherwise, a collision causes elastic repulsion or permanent deformation (ie a significant change in motion tensor), fracture or decay, depending on the time - space scale of the observation. Yet further option is joining the aggregates into modified consolidations. Systems exhibiting the forced oscillations are, in addition, affected by the interfacial motion.

Bearing in mind the broad class of practical systems that are affected by sliding, some basic kinematic relations of the interfacial motion were analyzed more closely. This paper presents the mathematical derivation of the general case of the interfacial motion, with reference to selected mechanical systems characterized by the presence of the forced oscillations.

These examples emphasize an interdisciplinary approach, i.e. the mechanistic and materials engineering notions are utilized by means of stochastic and deterministic concepts.

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