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# FREQUENCY DOMAIN DYNAMIC ANALYSIS OF SYSTEMS WITH VISCOUS AND HYSTERETIC DAMPING

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## ABSTRACT

This paper introduces initially a general formulation for frequency-domain dynamic and vibration analysis using discrete Fourier transforms through the Implicit Fourier transform concept. The issues of treatment of initial conditions in the frequency-domain and convergence are analysed. An efficient iterative method for the frequency domain dynamic analysis of MDOF systems and a method for analysis of nonlinear systems are introduced.

## 1. INTRODUCTION

Frequency-domain (FD) methods for dynamic analysis of structural and vibrating systems pertain to the class of superposition methods like the mode superposition method which nevertheless is a time-domain one. FD methods have been well developed in recent years and became competitive with methods in the time domain due to the possibility of their computational implementation through the well known FFT algorithm. Clough and Penzien, in the first edition of the excellent text on structural dynamics [1], presented some elements of dynamic structural analysis in the FD and, *en passsant*, mentioned that the FFT technique is "so efficient and powerful that it has made the FD approach computationally competitive with traditional time-domain analysis and thus revolutionizing the field of structural dynamics". 18 years latter, in the second edition [2], that authors gave a very thorough treatment of FD dynamic analysis of single and multi-degree-of-fredom (SDOF and MDOF) systems.

The application of FD methods is mandatory for a rigorous analysis when the system properties are frequency dependent and when hysteretic (structural) damping is present. Interaction forces in structural systems with soil - or fluid-structure interaction can be frequency dependent and, consequently, the properties of these systems (stiffness and damping) can depend on the frequency spectrum of the excitation. On the other hand, in dynamic structural analysis, hysteretic damping can only be rigorously considered by means of a FD method. These two situations emphasize the importance of FD methods. Furthermore, they can adequately account for nonproportional damping in structural systems.

The mathematical background to develop FD methods for analysis of dynamic and vibrating systems stems from Fourier transforms in their discrete form - Discrete Fourier Transforms (DFT's) [1], [2]. The computational implementation is, succesfully, performed through FFT's algorithms. Venancio-Filho and Claret [3] presented a formulation for the FD dynamic analysis of SDOF systems - the Implicit Fourier Transform (ImFT) formulation- [4], [5] by which the DFT's and the inverse DFT's are implicitly incorporated in one single matrix expression. Moreover, with this formulation, the number of sampling intervals in the DFT's can be arbitrarily selected, the analyst having thus more flexibility. Venancio-Filho and Claret [6] introduced, subsequently, a general formulation for the FD dynamic analysis of MDOF systems in terms of nodal and modal (generalized) coordinates. Jangid and Datha [7] developed an iterative process for the calculation of the modal complex frequency response matrix for the nonproportional damping case. Green and Cebon [8] used a FD approach to calculate the dynamic response of highway bridges to heavy vehicle loads.

Although FD methods are of the class of superposition methods and, therefore, are essentially linear they have been applied in the analysis of nonlinear systems through appropriate linearization techniques. Kawamoto [9] developed a hybrid frequency-time-domain method for nonlinear analysis in the FD. Aprile, Benedetti, and Trombetti [10] extended Kawamoto's method with the consideration of hysteretic and nonproportional damping, proposing the Generalized-Alternate Frequency Time (G-AFT) method. Venancio-Filho and Claret [3] introduced the Step-by-step Incremental Linearization in the Frequency Domain (SILFD) method which considers the dependence of damping with frequency. Darbre and Wolf [11] analysed problems of stability and implementation of a hybrid frequency-time-domain approach for nonlinear dynamic analysis.

In this paper a presentation of the ImFT formulation is initially given and two new developments - treatment of inital conditions in the FD and convergence analysis - are introduced. A general formulation for the FD dynamic analysis of MDOF systems with viscous and hysteretic damping is subsequently developed. A new iterative process for the treatment of MDOF systems with nonproportionl damping is developed. Finally, the SILFD method for FD nonlinear analysis is presented. Numerical experiments covering the subjects of treatment of initial conditons and convergence are displayed. There are given results of the analysis of a MDOF system by the above mentioned iterative process and of a nonlinear system by the SILFD method.

## 2. RESPONSE OF SDOF SYSTEMS

The dynamic equilibrium equation of a SDOF system

$$m\ddot{v} + c\dot{v} + kv = p(t) \tag{2.1}$$

where $v, \dot{v}$, and $\ddot{v}$ are, respectively, the displacement, the velocity, and the aceleration; m, c, and k are, respectively, the mass, viscous damping, and stiffness; and p(t) is the external load, is to be solved in the FD. c and k can be frequency dependent, the usual case being c frequency dependent.

The total time in which the response is to be calculated, $T_p$, is divided in N sample intervals $\Delta t = T_p/N$. The external load is defined, along these N sample intervals, $\Delta t$, as the $(N \times 1)$ vector

$$\mathbf{p} = \{p(t_0), p(t_1), \cdots, p(t_n), \cdots, p(t_{N-1})\} \tag{2.2}$$

with $t_n = n\,\Delta t$, $(n = 0, 1, 2, \ldots, N-1)$. Likewise the response is searched as

$$\mathbf{v} = \{v(t_0), v(t_1), \cdots, v(t_n), \cdots, v(t_{N-1})\}. \tag{2.3}$$

**Response for a given external load.** The solution of Eq 2.1, as in Eq. 2.3, for a given external load is expressed by the following pair of DFT's [2]:

$$v(t_n) = \frac{\Delta\Omega}{2\pi}\sum_{m=0}^{N-1} H(\Omega_m)\,P(\Omega_m)\,e^{mn\left(i\frac{2\pi}{N}\right)} \tag{2.4}$$

$$P(\Omega_m) = \Delta t\sum_{n=0}^{N-1} p(t_n)\;e^{-mn\left(\frac{i\,2\pi}{N}\right)}. \tag{2.5}$$

where $\Omega_m = m\,\Delta\Omega\;(m = 0, 1, 2, \cdots, N-1)$; $P(\Omega_m)$ is the DFT of the load at the frequency $\Omega_m$; and

$$H(\Omega_m) = \left\{k\left[\left(1 - \beta_m^2\right) + i(2\,\xi\,\beta_m + \lambda)\right]\right\}^{-1} \tag{2.6}$$

is the complex frequency response function at the same frequency. In this equation $\beta_m = \Omega_m/\Omega$; $\Omega = \sqrt{k/m}$ is the natural frequency; $\xi$ is the damping ratio; and $\lambda$ is the hysteretic damping factor.

The pair of Eqs. 2.4 and 2.5 can now be cast in a single matrix equation. For this let, initially,

$$\mathbf{P} = \{P(\Omega_0), P(\Omega_1), \cdots, P(\Omega_m), \cdots, P(\Omega_{N-1})\} \tag{2.7}$$

be the vector of the DFT's of the load, defined at the discretes frequencies $\Omega_m = m\,\Delta\Omega$. With the definitions of Eqs. 2.2 and 2.7 Eq.2.5 can be expressed in matrix form as

$$\mathbf{P} = \Delta t\,\mathbf{E}^*\,\mathbf{p} \tag{2.8}$$

where the $(N \times N)$ matrix $\mathbf{E}^*$ is defined as the matrix whose generic term is

$$E^*_{mn} = e^{mn\left(-i\frac{2\pi}{N}\right)}. \tag{2.9}$$

By the same token, the response from Eq. 2.4 is written in matrix form as

$$\mathbf{v} = \frac{\Delta\Omega}{2\pi}\,\mathbf{E\,H\,P} \tag{2.10}$$

where **E** is the matrix defined by Eq 2.9 with positive signs in the exponentials, instead of negative ones, and **H** is the diagonal matrix formed with the complex frequency response functions calculated at the discrete frequencies, Eq. 2.6. Introducing now **P** from Eq. 2.8 into Eq. 2.10 and considering that $\Delta\Omega\,\Delta t/2\pi = 1/N$, the following equation is obtained:

$$\mathbf{v} = \frac{1}{N}\,\mathbf{E\,H\,E^*\,p}. \tag{2.11}$$

Considering now the matrix

$$\mathbf{e} = \mathbf{E\,H\,E^*} \tag{2.12}$$

Eq. 2.11 transforms into

$$\mathbf{v} = \frac{1}{N}\,\mathbf{e\ p}. \tag{2.13}$$

Eq. 2.11 expresses the matrix formulation of the response of a SDOF system in the FD. The DFT of the load, $\mathbf{E^*}$ **p**, the DFT of the response, $\mathbf{H\,E^*\,p}$ - the response in the FD - and the inverse DFT of the response - the response in the time domain - are implicitly embodied in that equation. For this reason it was coined as ImFT. It can be conveniently used for the solution of the uncoupled modal equations in a mode superposition analysis of a MDOF system.

**Response due to initial displacement** $v_0$. This response is calculated, in the FD, as the sum of the initial displacement $v_0$ plus the response due to a step force -k v(0). Taking into acount Eq. 2.13, the response due to $v_0$ is

$$\mathbf{v}_d = \frac{1}{N}\,\mathbf{e}\left[-k\,v_0\,\mathbf{1}\right] + v_0\,\mathbf{1}. \tag{2.14}$$

In this equation **1** is a (N × 1) vector with 1's in every position.

**Response due to initial velocity** $\dot{v}_0$. An initial velocity $\dot{v}_0$ produces, in the time domain, a response given by

$$v(t) = m\ \dot{v}_0\,h(t) \tag{2.15}$$

where h(t) is the unit-impulse response function. Bearing in mind that h(t) is the inverse DFT of H(Ω) according to

$$h(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} H(\Omega)\, e^{i\Omega t}\, d\Omega, \tag{2.16}$$

v(t) can be obtained, alternatively, in the frequency domain, as

$$v(t) = \frac{m\dot{v}(0)}{2\pi} \int_{-\infty}^{+\infty} H(\Omega)\, e^{i\Omega t}\, d\Omega. \tag{2.17}$$

Equation 2.17 transforms into discrete form as

$$v(t_n) = \frac{\Delta\Omega}{2\pi}\, m\dot{v}_0 \sum_{m=0}^{N-1} H(\Omega_m)\, e^{mn\left(i\frac{2\pi}{N}\right)} \tag{2.18}$$

or, taking into account that $\Delta\Omega\, \Delta t/2\pi = 1/N$, as

$$v(t_n) = \frac{1}{N\Delta t}\, m\dot{v}_0 \sum_{m=0}^{N-1} H(\Omega_m)\, e^{mn\left(i\frac{2\pi}{N}\right)}. \tag{2.19}$$

The response due to $v_o$ follows therefore from Eq. 2.19 according to

$$\mathbf{v}_v = \frac{1}{N\Delta t}\, m\dot{v}_0\, \mathbf{e}\, \boldsymbol{\delta} \tag{2.20}$$

where $\delta$ is a $(N \times 1)$ vector with 1 in the first position and zero elsewhere.

**Total response due to external load and initial conditions.** This response is obtained by the superposition of the responses given by Eqs. 2.13, 2.14 and 2.20 according to

$$\mathbf{v} = \frac{1}{N}\, \mathbf{e}\left[\mathbf{p} - k\, v_0\, \mathbf{1} + \frac{m\dot{v}_0}{\Delta t}\, \boldsymbol{\delta}\right] + v_0\, \mathbf{1} \tag{2.21}$$

Fig. 1 compares the response of a SDOF to an initial displacement, calculated with the application of Eqs. 2.14 and 2.20 in the begining of each period, with the exact (time-domain) solution. Fig. 2a displays the response of a SDOF, with the damping variation with frequency indicated in Fig. 2b and with constant damping. These results validate the use of Eqs. 2.14 and 2.20 in obtaining the response in the FD of a SDOF system submitted to initial conditions

**Period extension.** A crucial point in the FD response calculation is the period extension. In order that the DFT's from Eqs 2.4 and 2.5 be valid the time of actuation of the load must be extended to infinity. Actually, if the time of duration of the load is $t_d$, the load must be extended with a trail of zeros until a time $T_p$ ($T_p >> t_d$). A rule of thumb for this extension, which provides fairly good results for short duration loads, is given in [12]. According to this

$$T_p = \frac{4.605}{\xi\, \omega} \tag{2.22}$$

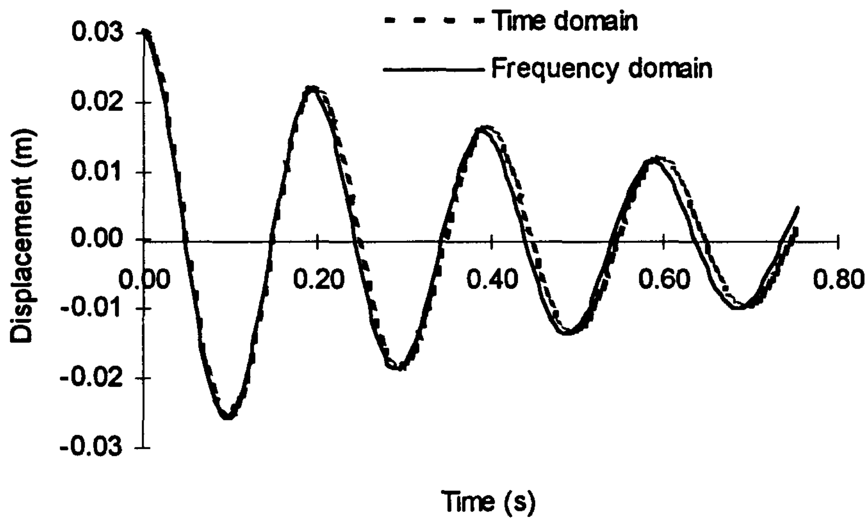


*Fig. 1 - Response for initial displacement*

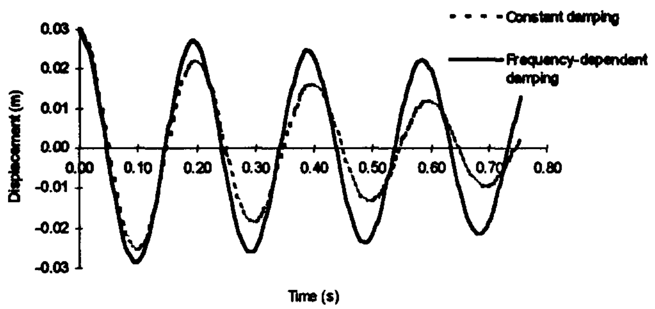


*Fig. 2a - Response for initial displacement with frequency-dependent damping*

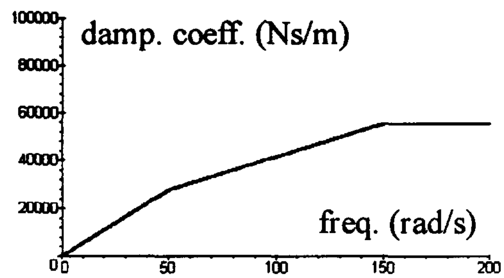


*Fig. 2b - Dependence of damping with frequency*

where $\xi$ is the damping ratio and $\omega$ is the system natural frequency. $T_p$ given by this equation stems from the time interval in which the free vibration of a damped system reduces to 1% of its initial value.

**Convergence.** It was proven in [4] that when N is even there is a complex term in the response which corresponds to the Nyquist frequency ($\Omega_{N/2}$). In [13] it was proven that the modulus of that complex term tends to zero with increasing N. Fig. 3 indicates this trend.

One advantage of the response calculation through the ImFT formulation is that the number of sampling intervals N can be arbitrarily selected, conversely to the selection in the FFT algorithm where N must be a power of 2. The analyst has, therefore, more flexibility in the selection of N. Figs. 4a and b display the convergence of the response of a SDOF system, with viscous and hysterestic damping, respectively, submitted to a short duration impulsive load.

## 3. RESPONSE OF MDOF SYSTEM

The dynamic equilibrium equation of a MDOF system with I DOF's submitted to an harmonic excitation with frequency $\Omega$ is

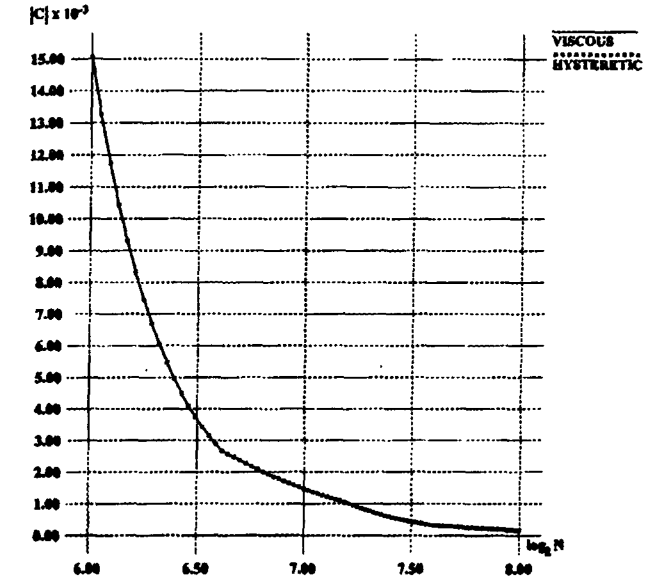


*Fig. 3 - Convergence of the modulus of the complex term*

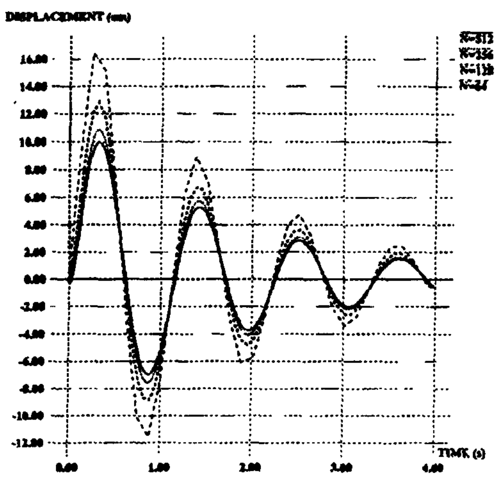


*Fig. 4a - Convergence of the response (viscous damping)*

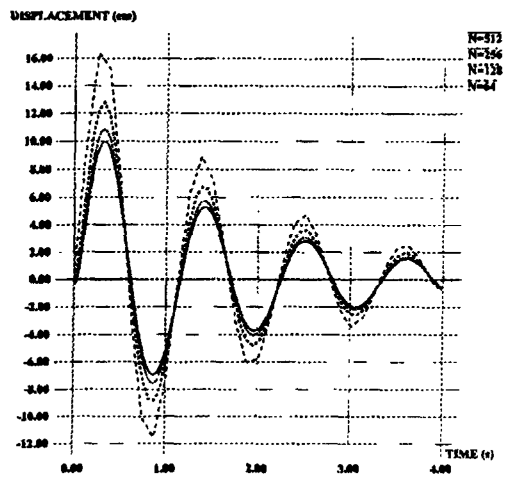


*Fig.4b - Convergence of the response (hysteretic damping)*

$$\mathbf{m}\,\ddot{\mathbf{v}} + \mathbf{c}\,\dot{\mathbf{v}} + \left(\mathbf{k} + i\,\mathbf{k}_H\right)\mathbf{v} = \mathbf{p}_0\,e^{i\Omega t}. \tag{3.1}$$

In this equation $\mathbf{v}$, $\dot{\mathbf{v}}$, and $\ddot{\mathbf{v}}$ are, respectively, the (I × 1) vectors of nodal displacements, velocities, and accelerations; **m**, **c**, and **k** are, respectively, the (I × 1) mass, viscous damping, and stiffness matrices; $\mathbf{k}_H$ is the (I × 1) hysteretic damping matrix composed by the superposition of the stiffness matrices of the system elements, factored by the respective damping factors; and $\mathbf{p}_0$ is the (I × 1) vector of the amplitude of the harmonic loading.

Consider at this point the modal transformation

$$\mathbf{v} = \mathbf{\Phi}\,\mathbf{w} \tag{3.2}$$

where $\mathbf{\Phi}$ is the (J × J), (J << I), matrix of the normal modes and **w** is the (J × 1) vector of modal coordinates. The introduction of Eq. 3.2 into Eq. 3.1, subsequent premultiplication of both sides by $\mathbf{\Phi}^T$, and consideration of FT's of both sides lead to

$$\left[-\Omega^2 \mathbf{I} + i(\Omega \mathbf{C} + \mathbf{K}_H) + \Lambda\right] \mathbf{W}(\Omega) = \mathbf{Q}(\Omega) \tag{3.3}$$

where $\mathbf{W}(\Omega)$ and $\mathbf{Q}(\Omega)$ are, respectively, the $(J \times 1)$ vectors of the FT's of the modal coordinates $\mathbf{w}$ and the modal loading $\mathbf{q} = \Phi\, \mathbf{p}$, at one of the discrete frequencies $\Omega_m$.

The normal modes introduced in Eq. 3.2 are normalized in such a way that $\Phi^T \mathbf{m}\, \Phi = \mathbf{I}$, $\mathbf{I}$ being the $(J \times J)$ unit matrix, and that $\Phi^T \mathbf{k}\, \Phi = \Lambda$, $\Lambda$ being the diagonal matrix formed by the J natural frequencies squared. Moreover, in Eq. 3.3, $\mathbf{C} = \Phi^T \mathbf{c}\, \Phi$ is the modal viscous damping matrix and $\mathbf{K}_H$ is the modal hysteretic damping matrix.

The inversion of Eq. 3.3 produces

$$\mathbf{W}(\Omega) = \mathbf{H}_1(\Omega)\, \mathbf{Q}(\Omega) \tag{3.4}$$

$$\mathbf{H}_1(\Omega) = \left[-\Omega^2 \mathbf{I} + i(\omega \mathbf{C} + \mathbf{K}_H) + \Lambda\right]^{-1}. \tag{3.5}$$

$\mathbf{H}_1(\Omega)$is the modal complex frequency response matrix.

In the absence of hysteretic damping and if the system is classically damped - or the damping is proportional - the modal viscous damping matrix $\mathbf{C}$ is diagonal and, therefore, $\mathbf{H}_1$ from Eq. 3.5 is also diagonal, it turning out that the individual equations from Eq. 3.3 are uncoupled. The solution for the $j^{th}$ modal coordinate, $w_j$, can now be obtained by Eq. 2.10 as

$$\mathbf{w}_j = \frac{\Delta\Omega}{2\pi} \mathbf{E}\, \mathbf{W}_j \tag{3.6}$$

where $\mathbf{W}_j = \left\{W_j(\Omega_0), W_j(\Omega_1), \cdots, W_j(\Omega_n), \cdots, W_j(\Omega_{N-1})\right\}$, and likewise in Eq. 2.3, $\mathbf{w}_j = \left\{w_j(t_o), w_j(t_1), \cdots, w_j(t_n), \cdots, w_j(t_{N-1})\right\}$ is the vector of the response of the $j^{th}$ modal coordinate.

When the system is not classically damped - or the damping is nonproportional - $\mathbf{H}_1$ from Eq. 3.5 is not diagonal. An iterative process developed by Venancio Filho and Claret [14] for the treatment of nonproportional damping in the time domain and applied by Jangid and Datha [7]to find $\mathbf{H}_1$, Eq. 3.5, when damping is nonproportional, is herein extended for the solution of Eq. 3.3. Consider, initially, matrices $\mathbf{C}$ and $\mathbf{K}_H$ split, respectively, as

$$\mathbf{C} = \mathbf{C}_d + \mathbf{C}_f \tag{3.7}$$

and

$$\mathbf{K}_H = \mathbf{K}_{H_d} + \mathbf{K}_{H_f} \tag{3.8}$$

where $\mathbf{C}_d$ is a diagonal matrix which has the diagonal elements of $\mathbf{C}$ and $\mathbf{C}_f$ has zero diagonal elements and the corresponding off-diagonal ones of $\mathbf{C}$, the same definitions being considered for $\mathbf{K}_H$. Substituting $\mathbf{C}$ from Eq. 3.7 and $\mathbf{K}_H$ from 3.8 into Eq. 3.3 and transferring the terms containing $\mathbf{C}_f$ and $\mathbf{K}_{H_f}$, for the RHS, one obtains

$$\left[-\Omega^2 \mathbf{I} + i\left(\Omega \mathbf{C}_d + \mathbf{K}_{H_d}\right) + \Lambda\right] \mathbf{W}(\Omega) = \mathbf{Q}(\Omega) - i\left(\Omega \mathbf{C}_f + \mathbf{K}_{H_f}\right) \mathbf{W}(\Omega). \qquad (3.9)$$

The matrix in the LHS of Eq. 3.13 is diagonal. Its inversion is trivial and produces

$$\mathbf{H}_2(\Omega) = \left[-\Omega^2 \mathbf{I} + i\left(\Omega \mathbf{C}_d + \mathbf{K}_{H_d}\right) + \Lambda\right]^{-1}. \qquad (3.10)$$

From Eq. 3.9 and 3.10 it follows then

$$\mathbf{W}(\Omega) = \mathbf{H}_2(\Omega)\left[\mathbf{Q}(\Omega) - i\left(\Omega \mathbf{C}_f + \mathbf{K}_{H_f}\right) \mathbf{W}(\Omega)\right]. \qquad (3.11)$$

Eq. 3.11 is treated by an iterative process as in [7]. The $k^{th}$ iterative step is given by the following equation:

$$\mathbf{W}^{(k)}(\Omega) = \mathbf{H}_2(\Omega)\left[\mathbf{Q}(\Omega) - i\left(\Omega \mathbf{C}_f + \mathbf{K}_{H_f}\right) \mathbf{W}^{(k-1)}(\Omega)\right]. \qquad (3.12)$$

In the first step: $i(\Omega \mathbf{C}_f + \mathbf{K}_{Hf}) \mathbf{W}^{(0)}(\Omega) = \mathbf{0}$

Convergence is obtained when

$$\left| \frac{W_j^{(k)}(\Omega_m) - W_j^{(k-1)}(\Omega_m)}{W_j^{(k-1)}(\Omega_m)} \right| < \varepsilon \qquad (3.13)$$

for all $j = 1, 2, \ldots, J$ and all $m = 0, 1, 2, \ldots, N-1$, $\varepsilon$ being a small threshold parameter. To arrive, finally, at the solution in the time domain the final iterate $\mathbf{W}_j^{(k)} = \mathbf{W}_j$ is introduced into Eq. 3.6.

## 4. THE SILFD METHOD FOR NONLINEAR ANALYSIS

In this proposed method of FD nonlinear analysis the total time interval in which the response is to be calculated, is divided in small sub-intervals. The assumption is made that, in each sub-interval, the system stiffness is constant. A linear analysis, in modal coordinates, in the FD is therefore performed, whith initial condition taken as the final ones in the previous sub-interval, by means of Eqs. 2.14 and 2.20.

Consider a general sub-interval s where the system stiffness $\mathbf{k}_s$, is assumed constant and is obtained from the final configuration in the previous sub-interval (secant stiffness). The system eigenproperties in s are, consequently, constant and are obtained from the eigenvalue equation

$$\mathbf{k}_s \, \boldsymbol{\phi}_s = \mathbf{m} \, \boldsymbol{\phi}_s \, \Lambda_s \qquad (4.1)$$

where $\boldsymbol{\phi}_s$ and $\Lambda_s$ are the eigenproperties in the considered sub-interval.

From now on, all the groundwork for the linear analysis in the sub-interval has been previously developed. The response is obtained using Eq. 2.21 in terms of modal coordinates.

## 5. EXAMPLES

The first example is the response of a two-story shear building submitted to a short duration impulsive load. Damping is frequency dependent according to Fig. 5 ($c_1$ in the upper story and $c_2$ in the lower). The response of the upper story by the iterative process and the exact response are compared in Fig. 6. In the iterative process the threshold parameter $\varepsilon = 0.001$ was adopted and the number of iteration for each frequency was around 4. The second example is a 2 DOF system which is a simplified mathematical model of a NPP building. The spring which models the soil has a bilinear stiffness. The linear and nonlinear responses due to a seismic excitation are displayed in Fig. 7

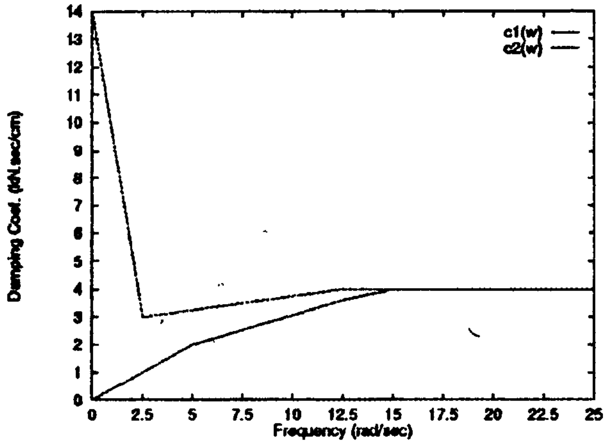


*Fig. 5 - Dependence of damping with frequency*

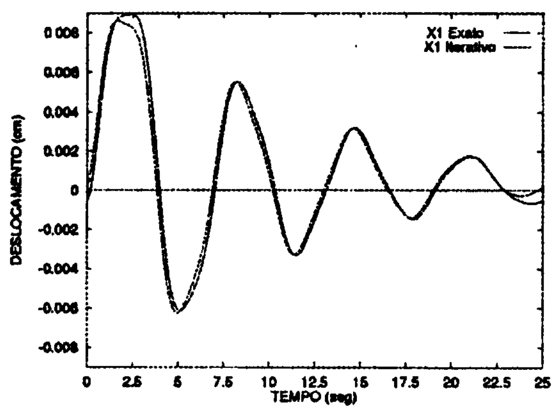


*Fig. 6 - Response of the upper story*

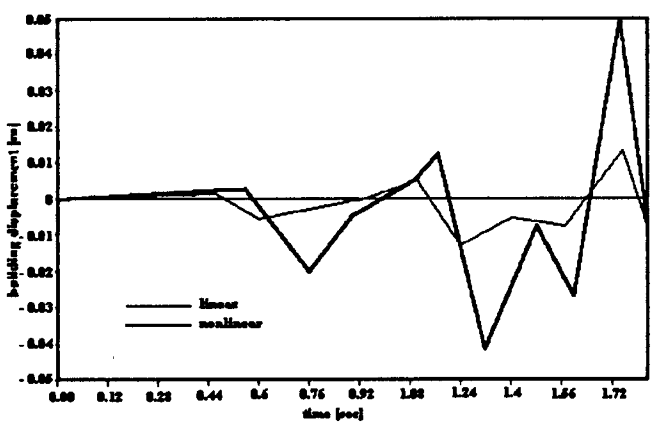

*Fig. 7 - Response of nonlinear system*

## 6. CONCLUDING REMARKS

FD methods are proven to be very suitable for analysis of structural and vibrating systems with frequency-dependent damping and with hysteretic and nonproportional damping.

The topics of treatment of initial conditions and convergence in FD dynamic analysis were covered and numerical experiments validated the obtained conclusions. A general method for FD dynamic analysis of MDOF systems with the referred damping characteristics and a proposed method for nonlinear analysis were presented.


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## REFERENCES


[1] Clough, R.W. and Penzien, J., "Dynamics of Structures", 1st Edition, McGraw Hill, Inc. New York, 1975.

[2] Clough, R.W. and Penzien, J., "Dynamics of Structures", 2nd Edition, McGraw Hill, Inc. New York, 1993.

[3] Venancio-Filho, F. and Claret, A.M., "An Implicit Fourier Transform Method for Nonlinear Dynamic Analysis with Frequency Dependent Damping", DAMPING'91 Conference, Paper BCB, San Diego, CA, 1991.

[4] Venancio-Filho, F. and Claret, A.M., "Matrix Formulation of the Dynamic Analysis of SDOF in the Frequency Domain", Computers & Structures, Vol. 42, nº 5, 835-855, 1992.

[5] Venancio-Filho, F. and Claret, A.M., "Frequency Domain Dynamic Analysis with Application to Soil-structure Interation: Linear and Nonlinear Systems", Advances in Computational Methods for Simulation ", edited by B.H.V. Topping, Edinburgh, UK, 1996.

[6] Venancio-Filho, F. and Claret, A.M., "Frequency Domain Analysis of MDOF Systems: Nodal and Modal Coordinates Formulations", Computers & Structures, Vol. 5, nº 1, 189-1981, 1995.

[7] Jangid, R.S. and Datta, T.K., "Spectral Analysis of Systems wth Non-classical Damping Using Classical Mode Superposition Technique", Earthquake Engineering and Structural Dynamics, Vol. 22, nº 8, 823-735, 1993.

[8] Green, M.F. and Cebon, D., "Dynamic Response of Highway Bridges to Heavy Vehicle Loads: Theory and Experimental Validation", Journal of Sound and Vibration, Vol. 170, nº 1, 51-78, 1994.

[9] Kawamoto, J.D., "Solution of Non-linear Dynamic Structural Systems by a Hybrid Frequency-time Domain Approach", Ph.D. Thesis, Department of Civil Engineering, MIT, 1983.

[10] Aprile, A., Benedetti, A. and Trombetti, T., "On Nonlinear Dynamic Analysis in the Frequency Domain", Earthquake Engineering and Structural Dynamics, Vol. 3, nº 4, 363-388, 1994.

[11] Darbre, G.R. and Wolf, J.P., "Criterion of Stability and Implementation Issues of Hybrid Frequency-time-domain Procedure for Nonlinear Dynamic Analysis", Transactions of 9th International Conference on Structural Mechanics in Reactor Technology, Lausanne, 1987.

[12] Venancio-Filho, F. and Graça Ferreira, W., "Frequency Domain Analysis for Short Duration Impulsive Loads", Applied Mechanics in America, Vol. 4, edited by L.A. Godoy, M. Rysz, and L.E. Suarez, Iowa City, IA, 1997.

[13] Barbosa, F.S. and Venancio-Filho, F., "Convergence Analysis of the Response of SDOF systems in the Frequency Domain", submitted to Computers & Structures, 1997.

[14] Claret, A.M. and Venancio-Filho, F., "A Modal Superposition Pseudo-force Method for Dynamic Analysis of Structural Systems with Non-proportional Damping", Earthquake Engineering and Structural Dynamics, Vol. 20, nº 20, 303-315, 1991.