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A FILTERED-X ADAPTIVE NOTCH FILTER WITH ON-LINE CANCELLATION PATH ESTIMATION

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ABSTRACT.

This paper presents a simple adaptive algorithm based on the adaptive notch two-taps filter [1] and the filtered-X LMS algorithm [2], to be used in active noise control applications. The system consists of two adaptive algorithms working together. One algorithm estimates on-line the error path transfer function as proposed in [3] and the other cancels a frequency narrowband of the residual signal. The convergence factors of the two adaptive algorithms determine the spectrum width of the cancellation and the stability of the control system. The adaptive filter used for estimating the error path has also two taps; it is only necessary to estimate the module and the phase at the center frequency of the cancellation bandwidth. The system can be generalized to a configuration of multiple cancellation frequency narrowbands and a multiple notch filter control system. This adaptive algorithm can be used in active noise control applications to cancel only frequency narrowbands and when it is not possible to get any reference signal.

INTRODUCTION.

In some active noise control configurations it is not possible to get a *clean* reference signal of the acoustic noise because: a) of the working environmental conditions of the noise source, b) there are many changing noise sources or c) because there is an acoustic feedback between the control signal and the reference signal. The classical adaptive algorithms, the filtered-X LMS algorithm [2] and the filtered-U LMS algorithm [4], need a reference signal to cancel the acoustic noise around the error microphone. The electro-acoustic transfer function between the control output signal and the error input signal of the algorithm has to be on-line estimated with other adaptive algorithm, to get a stable convergence of the main adaptive control algorithm.

It is possible to design a very simple active noise control, without any external reference signal, to cancel some specific spectrum narrowbands of the external acoustic noise

[5, 6]. This active noise control is based on the adaptive notch filter proposed by Widrow [1] and on the filtered-X LMS algorithm [2] as shown in figure 1. The algorithm is inside the dashed square. When the electro-acoustic transfer function between the control signal and the error signal, represented by $H_e(z)$, does not exist ($H_e(z)=1$), the algorithm is the adaptive notch filter proposed by Widrow.

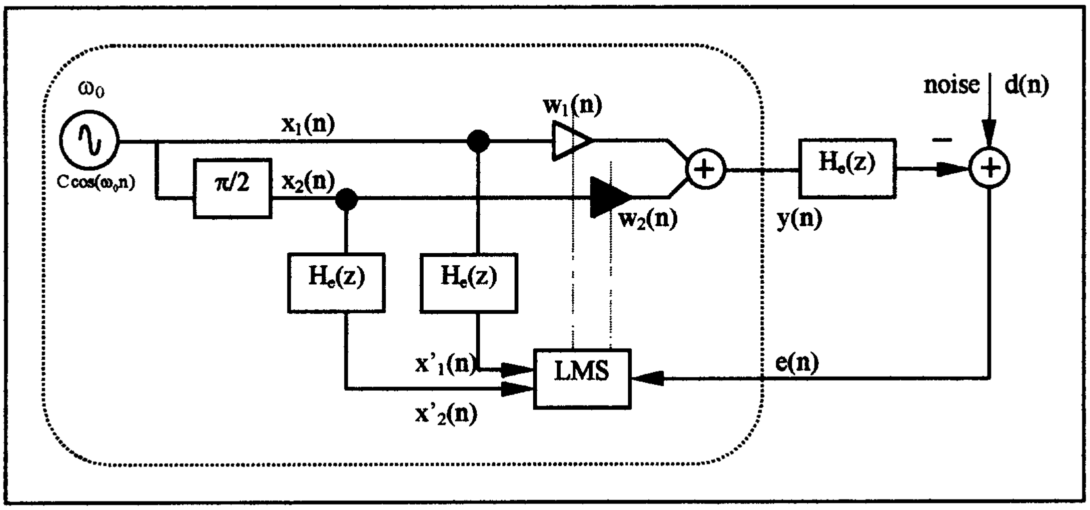

Figure 1. Filtered-X adaptive notch filter.

FILTERED-X ADAPTIVE NOTCH FILTER ANALYSIS.

The transfer function $H_e(z)$ has to be evaluated only at the frequency f_0 because the output signal is a sinusoidal signal. The only $H_e(z)$ influence over this control signal y(n) is only an amplitude and phase change.

$$H_e(z) = ke^{+j\phi} \tag{1}$$

In the block diagram of figure 1 the $H_e(z)$ situation can be modified without changing the global system response, in order to do a simpler analysis of the adaptive algorithm, as shown in figure 2.

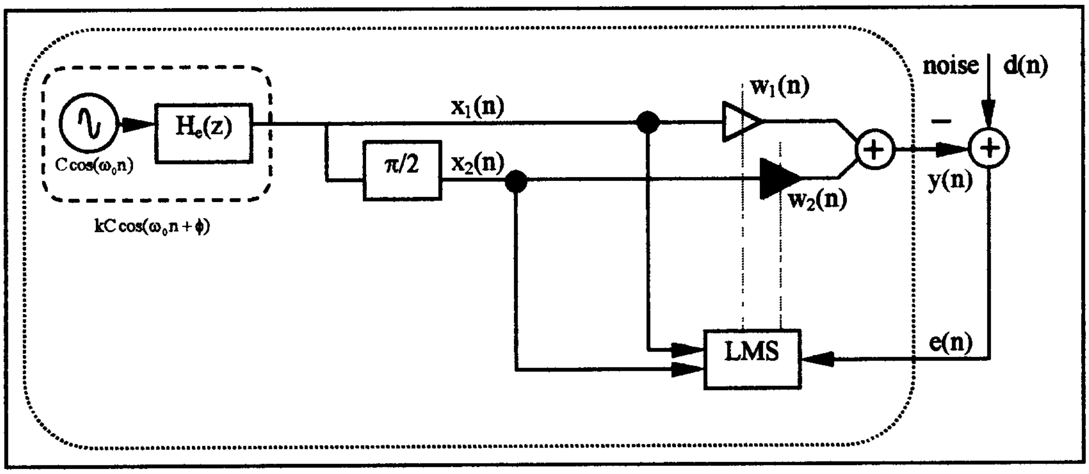

Figure 2. Simplified block diagram of the filtered-X adaptive notch filter.

The transfer function $H_e(z)$ effect on the control system is an amplitude and phase change on the sinusoidal reference signal. The frequency narrowband to cancel is centered at the reference frequency ω_0 and the bandwidth depends on the convergence factor of the adaptive algorithm and the reference amplitude [1]. The higher these parameters are, the higher the bandwidth is. Figure 2 shows a single-frequency noise canceller with two adaptive weights. The noise input is assumed to be any kind of signal –stochastic, deterministic, periodic, transient, and so on- or any combination of signal. From a pure cosine wave (the reference signal) are formed $x_1(n)$ and $x_2(n)$; $x_1(n)$ is directly the reference and $x_2(n)$ is the same signal delayed $\pi/2$ radians.

$$x_1(n) = kC\cos(\omega_0 n + \phi) \tag{2}$$
$$x_2(n) = kC\sin(\omega_0 n + \phi) \tag{3}$$

The adaptive algorithm minimizes the instantaneous squared error updating the two weights with the equation (4):

$$w_i(n+1) = w_i(n) + 2\mu e(n) x_i(n) \tag{4}$$

where μ is the convergence factor of the LMS adaptive algorithm that determines the stability and convergence of the algorithm and x_i is the reference signal for each weight w_i. The output signal y(n) can be calculated from (4), as shown in figure 2:

$$y(n) = w_1(n)x_1(n) + w_2(n)x_2(n) \tag{5}$$

The steady state transfer function of the control system can be obtained analyzing the signal path from d(n) to e(n). As shown in [1] the impulse response from e(n) to y(n) is:

$$G(z) = 2\mu k^2 C^2 \left[\frac{z\cos\omega_0 - 1}{z^2 - 2z\cos\omega_0 + 1} \right] \tag{6}$$

A simplified block diagram of the figure 2, as it is shown in figure 3, lets to get easily the transfer function between d(n) and e(n).

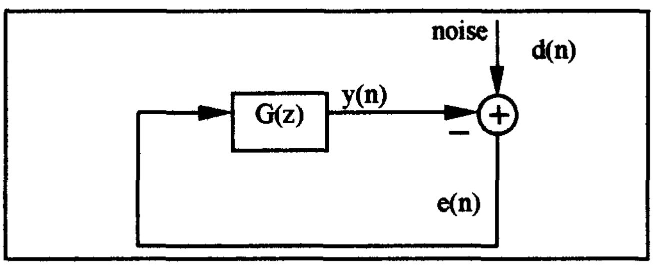

Figure 3. Feedback loop of the adaptive notch filter.

$$\frac{E(z)}{D(z)} = \frac{1}{1+G(z)} = \frac{1 - 2\cos\omega_0 z^{-1} + z^{-2}}{1 - 2(1-\mu k^2 C^2)\cos\omega_0 z^{-1} + (1 - 2\mu k^2 C^2) z^{-2}} \tag{7}$$

The above equation has a conjugated complex pair of zeros (numerator) and another pair of poles (denominator). The location of these zeros and poles on the Z-plane determines the notch filter response. Taking into account that a conjugated complex pair of zeros or poles has the next expression:

$$1 - 2r\cos\omega z^{-1} + r^2 z^{-2} \tag{8}$$

where r is the radius from the Z-plane origin and ω is the angle in radians where the poles or zeros are located, it can be determined their location from the equation (7). The conjugated complex zeros are located at the unit circle with angles $\pm\omega_0$ radians. Therefore, the system produces a null response at the frequency ω_0 radians. The poles are located at the radius:

$$r = \sqrt{1 - 2\mu k^2 C^2} \approx 1 - \mu k^2 C^2 \tag{9}$$

and with angles:

$$\pm\omega = \pm\cos^{-1}\left(\frac{1 - \mu k^2 C^2}{\sqrt{1 - 2\mu k^2 C^2}}\cos\omega_0\right) \approx \omega_0 \tag{10}$$

The above approximations are only true for small values of $\mu k^2 C^2$. The zero-pole plot on the Z-plane and frequency response (module) of the adaptive notch filter is shown below in figure 4.

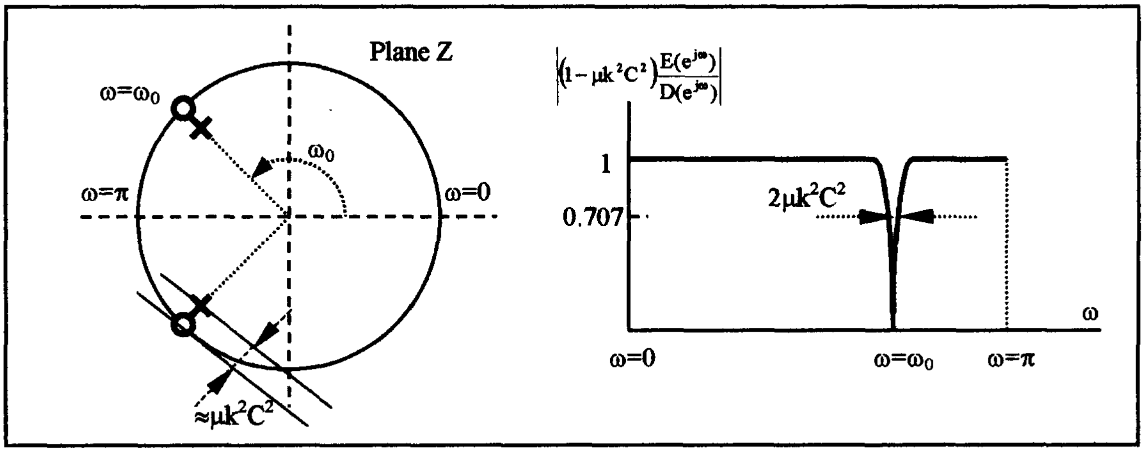

Figure 4. Zero-pole plot on the Z-plane and frequency response (module) of the filtered-X LMS adaptive notch filter.

When the poles are very close to the zeros the filter bandwidth is twice the pole-unit circle distance, and the quality factor, Q, is given by:

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{\omega_0}{2\mu k^2 C^2} \tag{11}$$

For small values of $\mu k^2 C^2$ the poles are very close to the zeros and the notch filter is very narrow. The greater the zero-pole distance is, the greater the notch filter bandwidth is, but the approximations in equations (9) and (10) are not already true and the poles' angles are not the same that the zeros' angles. The effect is a reinforcement of the spectrum around the frequency ω_0.

ON-LINE CANCELLATION PATH ESTIMATION.

A very simple model is used to estimate the electro-acoustic transfer function $H_e(z)$, based on the on-line estimation proposed by H. Fan [3]. It is only necessary to estimate the transfer function $H_e(z)$ at the frequency ω_0, so a two-taps adaptive filter as the adaptive notch filter can be used. Figure 5 shows the global active noise control system proposed in this paper. It is a very simple active noise control because of low complexity and few operations.

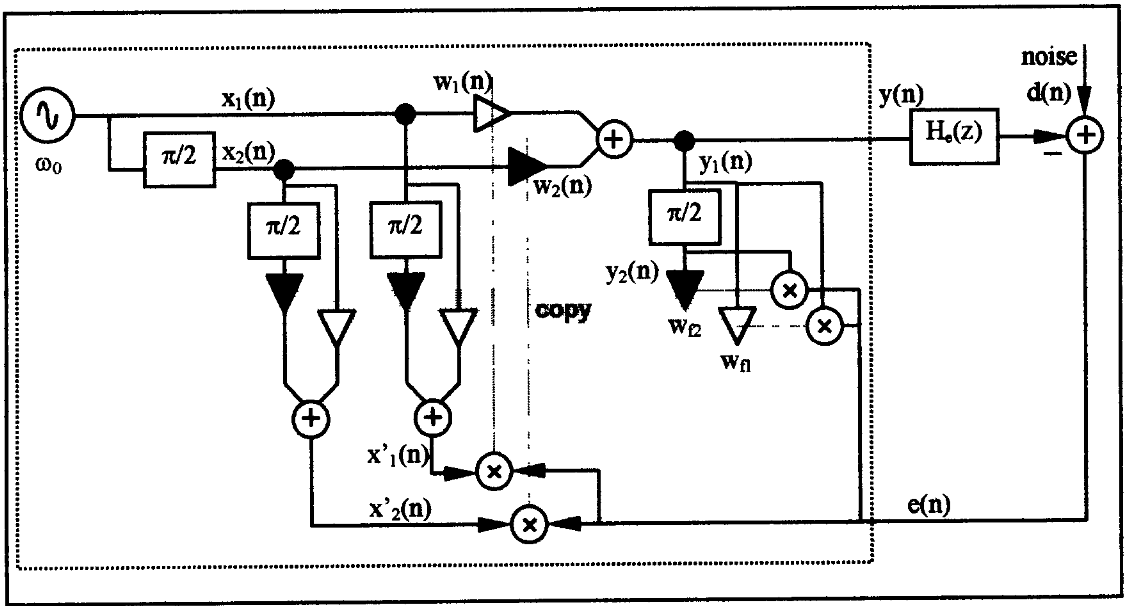

Figure 5. Filtered-X adaptive notch filter with on-line cancellation path estimation.

The notch filter bandwidth $\Delta\omega$ (equation 11) depends on: the convergence factor, the reference signal amplitude and the gain/attenuation of the transfer function $H_e(z)$ at the frequency ω_0. The maximum value of the convergence factor, in order to get the maximum bandwidth, can be obtained from the equation (9). Taking into account the radius range:

$$0 \leq r < 1 \tag{12}$$

then

$$0 < \mu \leq \frac{1}{2k^2C^2} \tag{13}$$

The adaptive algorithm, used to estimate the transfer function $H_e(z)$, updates the weights with the next equation:

$$w_{fi}(n+1) = w_{fi}(n) - 2\mu_f e(n) y_i(n) \tag{14}$$

where μ_f is the convergence factor of the adaptive algorithm and $y_i(n)$ are the output signals used, $y_2(n)$ is delayed $\pi/2$ radians from $y_1(n)$ as it is shown by the equation (16):

$$y_1(n) = y(n) \tag{15}$$

$$y_2(n) = -\frac{\partial}{\partial(n\omega_0)} y_1(n) \tag{16}$$

Taking into account the equations (2), (3), (4) and (5) the equation (16) can be expressed by:

$$y_2(n) = w_1(n)x_2(n) - w_2(n)x_1(n) \tag{17}$$

The weights w_{fi} are used to calculate the two filtered reference signals, $x'_1(n)$ and $x'_2(n)$, as shown in the next equations:

$$x_1'(n) = w_{f2}(n)\left(-\frac{\partial}{\partial(n\omega_0)}x_1(n)\right) + w_{f1}(n)x_1(n) = w_{f2}(n)x_2(n) + w_{f1}(n)x_1(n) \tag{18}$$

$$x_2'(n) = w_{f2}(n)\left(-\frac{\partial}{\partial(n\omega_0)}x_2(n)\right) + w_{f1}(n)x_2(n) = -w_{f2}(n)x_1(n) + w_{f1}(n)x_2(n) \tag{19}$$

and with these signals to obtain the weights of the adaptive notch filter:

$$w_i(n+1) = w_i(n) + 2\mu e(n)x_i'(n) \tag{20}$$

SIMULATION RESULTS.

The global active control system has been tested in order to get the maximum convergence factors. The values obtained are shown in tables below. These tables show the maximum values of the main convergence factor (μ) obtained for different frequencies of the reference signal (rows) and for different delays (columns) of the transfer function $H_e(z)$.

$H_e(z)$										$H_e(z)$								
z^{-0}	z^{-4}	z^{-8}	z^{-12}	z^{-16}	z^{-20}	z^{-24}	z^{-28}	z^{-32}	n	z^{-0}	z^{-4}	z^{-8}	z^{-12}	z^{-16}	z^{-20}	z^{-24}	z^{-28}	z^{-32}
0.51	0.27	0.17	0.13	0.10	0.08	0.07	0.06	0.06	4	0.22	0.14	0.09	0.06	0.05	0.04	0.03	0.03	0.02
0.42	0.16	0.17	0.09	0.10	0.06	0.07	0.05	0.06	8	0.20	0.09	0.10	0.05	0.04	0.04	0.03	0.03	0.02
0.34	0.23	0.11	0.11	0.10	0.08	0.05	0.05	0.06	16	0.20	0.12	0.07	0.06	0.04	0.04	0.03	0.03	0.02
0.45	0.16	0.13	0.10	0.07	0.06	0.06	0.06	0.05	32	0.24	0.08	0.07	0.06	0.04	0.03	0.03	0.03	0.02
0.65	0.14	0.10	0.08	0.06	0.07	0.06	0.05	0.04	64	0.24	0.11	0.05	0.04	0.04	0.04	0.04	0.03	0.02
0.75	0.21	0.09	0.07	0.05	0.05	0.04	0.04	0.03	128	0.16	0.07	0.05	0.03	0.03	0.03	0.03	0.03	0.03
maximum values of μ with $\mu/\mu_f=10$									n frequency = *fs*/n	maximum values of μ with $\mu/\mu_f = 1$								

Table 1. Maximum values of the main convergence factor for different $H_e(z)$ delays and for different frequencies of the reference signal.

The left side table shows values for a relationship between the convergence factors of $\mu/\mu_f=10$, and the right side table shows values for a relationship of $\mu/\mu_f=1$. The noise signal has also been a sinusoidal signal with the same frequency that the reference, added with a low level white noise ($\sigma^2=0.001$). It can be observed in the previous tables that the convergence factor decreases when the function $H_e(z)$ delay increases, so the cancellation bandwidth of the filtered-X LMS adaptive notch filter also decreases when this delay increases. It is also noted that the higher the frequency is, the smaller the convergence factor is obtained.

When ω is near but not at ω_0 the weights do not converge to stable values but tumble at the difference frequency, and the adaptive filter behaves like a modulator, converting the reference frequency into a noise input frequency. This effect can be observed in figure 6, where the error signal convergence and the frequency module response of the adaptive notch filter are shown, with a 1% reference frequency deviation with respect to the noise frequency (left side curves). This modulation can be eliminated if the reference signal amplitude is at least five times the noise amplitude. Right side curves show this, the reference signal amplitude is 1 and the sinusoidal noise amplitude is 0.2 (tone added with a low level white noise, σ^2=0.001).

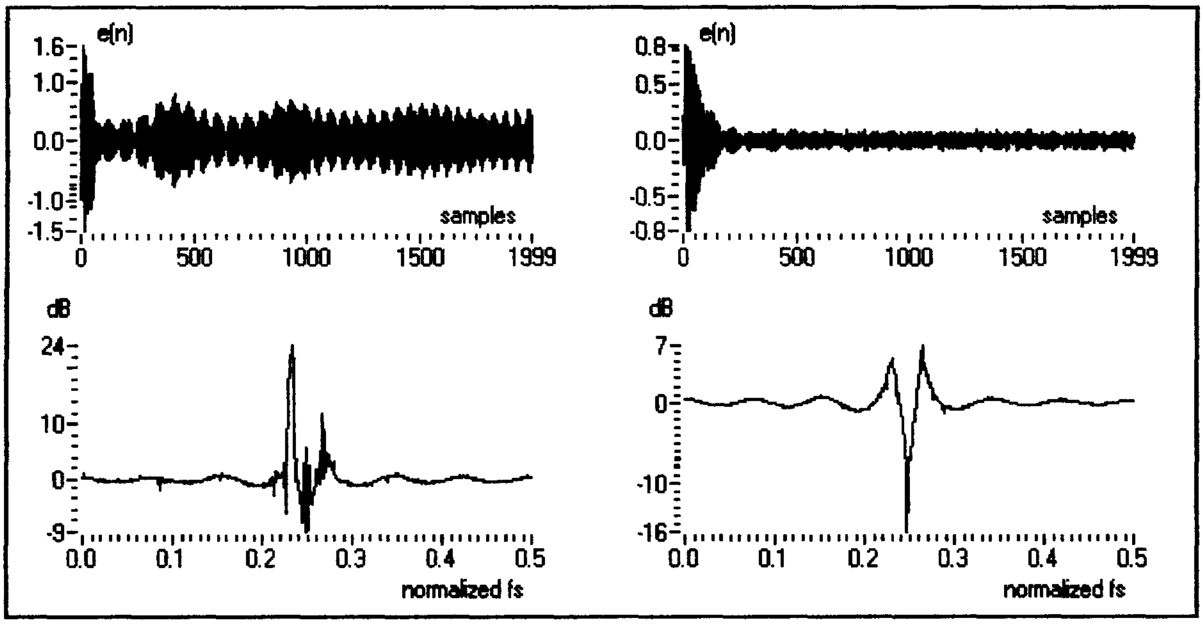

Figure 6. Modulation effect of the filtered-X LMS adaptive notch filter. Error signal convergence and frequency module response for a 1% frequency deviation between the reference signal and noise signal. Right side curves with signals' amplitude relationship $A_{x(n)}/A_{d(n)}=1$, left side curves with signals' amplitude relationship $A_{x(n)}/A_{d(n)}=5$.

The filtered-X LMS adaptive notch algorithm with on-line estimation shown in figure 5 can be generalized to a multiple configuration to cancel multiple frequency narrowbands.

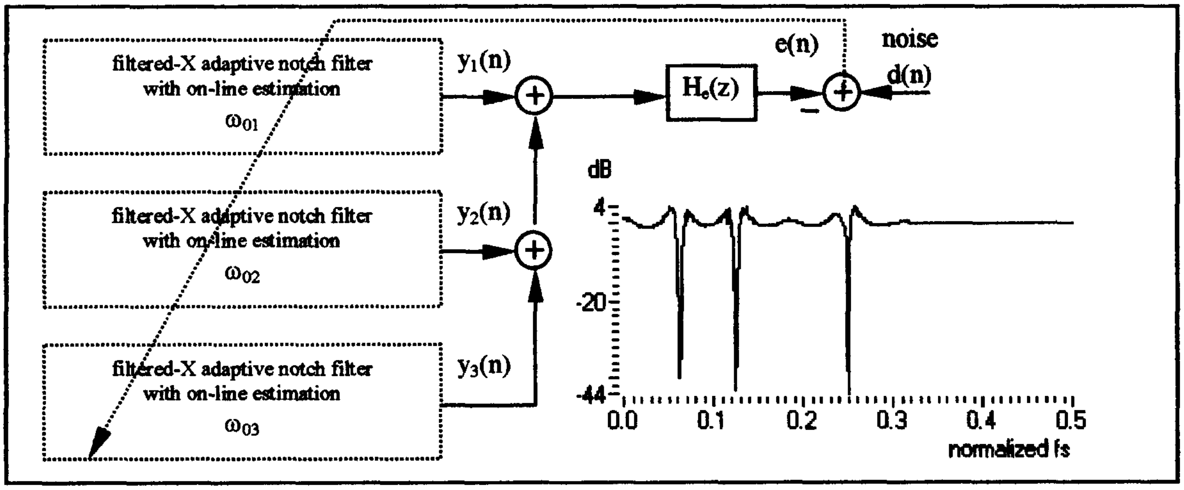

Figure 7. Multiple filtered-X adaptive notch filter with on-line cancellation path estimation.

Figure 7 shows three algorithms working together, all of them with the same convergence factors μ and μ_f, to cancel three spectrum bandwidths. The system has been tested with a noise signal that consists of three different frequency sinusoidal signals with the same amplitude, added with a low level white noise (σ^2=0.001). Table 2 shows the maximum values obtained of the main convergence factor (μ), for different frequencies and for different transfer function $H_e(z)$ delays. The convergence factors relationship used has been $\mu/\mu_f = 10$. It can be observed in this table that the convergence factors are smaller than the ones obtained for a single algorithm (table 1). It is also noted that the values tend to decrease when the transfer function delay increases.

frequency = sampling/n n	$H_e(z)$								
	z^{-0}	z^{-4}	z^{-8}	z^{-12}	z^{-16}	z^{-20}	z^{-24}	z^{-28}	z^{-32}
4, 8, 16	0.171	0.122	0.093	0.005	0.083	0.005	0.044	0.005	0.054
8, 16, 32	0.142	0.063	0.073	0.005	0.063	0.005	0.005	0.005	0.054
16, 32, 64	0.171	0.063	0.054	0.054	0.044	0.005	0.005	0.005	0.034
32, 64, 128	0.269	0.054	0.034	0.034	0.024	0.024	0.024	0.024	0.024

maximum values of μ with $\mu/\mu_f = 10$

Table 2. Maximum values of the main convergence factor for different $H_e(z)$ delays and for different frequencies of the reference signal in a triple adaptive notch filter.

CONCLUSION.

This paper has presented the analysis of a filtered-X adaptive notch filter with an on-line cancellation path estimation for active noise control applications. It is very simple, only two-taps two adaptive algorithms are necessary for cancelling a spectrum narrowband of the noise. With a multiple algorithm configuration it is possible to cancel multiple spectrum narrowbands. The bandwidth control is very easy with the convergence factor of the adaptive algorithm. The maximum values of the convergence factor have been obtained for different cancellation path delays and for different reference signal frequencies. The simulation results show that when the cancellation path delay increases, the bandwidth of the notch filter decreases.

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