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THE VARIANCE OF PURE TONE REVERBERANT SOUND POWER MEASUREMENTS

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ABSTRACT

The 1996 version of the draft international standard ISO/DIS 3741, "Acoustics - Determination of sound power levels of noise sources using sound pressure - Precision methods for reverberation rooms" deleted the room qualification procedure for the measurement of discrete frequency components. The alternative multiple source position method has been retained. This paper shows that there is an error in the constant in the equation for determining the number of source positions in the retained alternative method. It also shows that the multiple source position method is not sufficient at low modal overlap. Thus the room qualification procedure should be reinstated.

The measurement variance can be split into source position, receiver position and room variance. The room variance depends on the distribution of modal spacings. Earlier theoretical and numerical calculations used the Poisson or "nearest neighbour" distributions. Both these distributions produce non-zero room variance. The Gaussian Orthogonal Ensemble (GOE) distribution, which is currently believed to be correct, produces zero room variance at high modal overlap. At low modal overlap, the GOE and "nearest neighbour" distributions produce room variance values which tend towards the non-zero values produced by the Poisson distribution.

THE MULTIPLE SOURCE POSITION METHOD

Equation (3) of ANSI (1980) is used to compute the number of source positions to be used in the multiple source position method for measurement of sound power in a reverberant room. This equation also appears as equation (4) in ISO (1996). Baade (personal communication) has asked for clarification of the statement in Davy (1989) that "It was also shown that the value of the constant 0.79 in equation (3) of ANSI (1980) is wrong because of an error in Lyon's (1969) paper". It is shown in the following that the constant should be approximately one. The first paragraph of Davy (1990) reads as follows.

"The transmission function of a reverberation room is defined to be the square of the modulus of the ratio of the reverberant field sound pressure at a point in the room to the volume velocity of the sound source. Theoretical work by Lyon (1969), Davy (1981a) and Weaver (1989) has shown that if the transmission function is averaged over an array of N source positions and L receiver positions, the relative covariance of the averaged transmission function at two angular frequencies which differ by θ is given by

$$rel\,\mathrm{cov} = \varphi(\theta)\left\{\frac{1}{LN}+\frac{1}{M}\left[\left(\frac{K-1}{N}+1\right)\left(\frac{K-1}{L}+1\right)-C\left(\frac{2}{LN}+1\right)\right]\right\}. \tag{1}$$

where $\varphi(\theta)$ equals $1/[1 + (\theta/2\gamma)^2]$, M equals $2\pi n\gamma$, K equals $(\langle p^4(x)\rangle/\langle p^2(x)\rangle^2)$, γ is the decay rate of the modal amplitudes in nepers per unit of time, n is the modal density in number of modes per unit of angular frequency, p(x) is the modal amplitude as a function of position x in the room and C is a function of the distribution of the modal frequency spacings. φ is Schroeder's frequency autocorrelation function with angular frequency as the argument and M is the statistical modal overlap which is the product of the modal density with the statistical bandwidth of the modes. The statistical bandwidth of a mode is twice the effective or noise bandwidth of the mode and π times the half power or 3 dB bandwidth of the mode. For a rectangular parallelepiped room with rigid walls K is equal to $(3/2)^3$, $(3/2)^2$ or $(3/2)$ for oblique, tangential or axial modes respectively. C is equal to 0, 1/2 or 1 for Poisson, nearest neighbour or Gaussian orthogonal ensemble (GOE) distributions of modal frequency spacings respectively."

Using the notation of equation (1) above, equation (3) of ANSI (1980) can be reorganised to read

$$\frac{1}{B} \geq \frac{1}{LN} + \frac{1}{M}\frac{Ka}{N}, \tag{2}$$

where B is the constant K of equation (3) of ANSI (1980), K = 27/8 and a =1/2. In other words, the relative variance of the averaged transmission function of the reverberation room must be less than 1/B. Comparison of the right hand side of equation (2) with the right hand side of equation (1) shows that equation (2) cannot be theoretically correct. This is because the term which multiplies 1/M does depend on the number of receiver positions L, and cannot be expressed as a constant divided by N, the number of source locations.

However, it will be assumed that L is large enough so that it can be set to infinity in the term which multiplies 1/M. Setting θ to zero in equation (1) gives

$$rel\,\mathrm{var} = \frac{1}{LN} + \frac{1}{M}\left(\frac{K-1}{N} + 1 - C\right). \quad (3)$$

For the Gaussian orthogonal ensemble (GOE) distributions of modal frequency spacings, which is now believed to be correct, equation (3) becomes

$$rel\,\mathrm{var} = \frac{1}{LN} + \frac{1}{M}\frac{K-1}{N}, \quad (4)$$

since C = 1 for this case. The right side of equation (2) agrees with the right hand side of equation (4) except for the fact that K should have one subtracted from it instead of being multiplied by a = 1/2.

Averaging over all possible receiver positions enables a true estimate of the sound power actually injected into the room. Setting the number of receiver positions L to infinity, the number of source positions N to one and the angular frequency difference θ to zero in equation (1) gives the relative variance of the real part of the input impedance of a reverberation room,

$$rel\,\mathrm{var} = \frac{K-C}{M}. \quad (5)$$

Lyon (1969) obtained this equation with the correct value C equals zero in the Poisson case. In the "nearest neighbour" case, he obtained this equation with Ka instead of K-C where C equals 1/2. For high modal overlap Lyon's a is equal to 1/2, and this is the value used in equation (3) of ANSI (1980).

Assuming a rectangular parallelepiped room with rigid walls, and ignoring tangential and axial modes, K is equal to $(3/2)^3$=27/8. Lyon's value for Ka, as used in the standard, is then 27/16. In the nearest neighbour case K-C=27/8-1/2=23/8 and the constant needs to be multiplied by (23/8)/(27/16)=46/27=1.70. In the now accepted Gaussian orthogonal ensemble (GOE) case K-C=27/8-1=19/8 and the constant needs to be multiplied by (19/8)/(27/16)=38/27=1.41. Weaver (1989) stated that "This author is inclined somewhat to K = 3.0 which is appropriate for a Gaussian distribution of amplitudes and based on vague arguments invoking the central limit theorem." For K=3 and GOE case, the constant needs to be multiplied by 2/(27/16)=32/27=1.19.

For the Poisson case, Lyon (1969) derived formulae for the relative covariance of the real part of the input impedance and for the relative covariance of the transmission function. For the "nearest neighbour" case, he derived an incorrect formula for the relative covariance of the real part of the input impedance. Waterhouse (1978) published a paper giving theoretical formulae which were very different from those derived by Lyon.

The main purpose of Davy (1981a) was to reject Waterhouse's paper and to support Lyon's paper both theoretically and experimentally. While doing so, Davy found and corrected

Lyon's error in the formula for the relative covariance of the real part of the input impedance in the "nearest neighbour" case. One of the puzzles of Lyon's paper was that it should have been possible to combine his formulae for the covariance of the real part of the input impedance and the covariance of the transmission function by deriving the covariance of the transmission averaged over a number of source and receiver positions. It was not obvious from Lyon's paper how to do this. In fact equation (3) of ANSI (1980) is based on a reasonable but incorrect guess of how to combine the formulae. The main contribution of Davy (1981a) was to show how to combine these formula in the Poisson case. Like Lyon, Davy was unable to derive a formula for the relative covariance of the transmission function in the "nearest neighbour" case. Davy guessed that the equation was obtained from the Poisson case by replacing K with K-1/2, which he had shown was true for the equation for the covariance of the real part of the input impedance.

Davy (1987) used the data from 7 experiments based on the pure tone qualification procedure, to calculate the value of K which gave his equation, for the covariance of the averaged transmission function, the best fit to the experimental data. In these experiments, the angular frequency difference was zero, the number of source positions averaged over was one and the number of independent receiver positions increased linearly over the frequency range from 100 to 630 Hz because of the use of a circular microphone traverse. Davy obtained the value K equals 2.16. If tangential and axial modes were ignored, Davy's theoretical estimates of K were 3.375 and 2.875 for the Poisson and "nearest neighbour" cases respectively. If tangential and axial modes were included, Davy's theoretical estimates of K were 3.10 and 2.60 for the Poisson and "nearest neighbour" cases respectively.

Weaver (1989) pointed out that the Gaussian orthogonal ensemble (GOE) distribution was more appropriate, and derived an equation for the covariance of the averaged transmission function in the GOE case. His method also applied to the "nearest neighbour" case, and showed that Davy's guess for the covariance of the average transmission function in this case was incorrect. Weaver's equation alters the form of the equation from Davy's equation and not just the value of K. However, if number of receiving positions is large, equation (4) shows that it replaces K with K - 1. Thus the theoretical estimates in Davy (1987) for the GOE case become 2.375 and 2.10 depending on whether tangential and axial modes are excluded or included. If Weaver's (1989) estimate of an uncorrected K equals 3.0 is accepted, then K -1 equals 2.0. Hence the GOE values agree well with the experimental result of 2.16 from the pure tone qualification procedure.

The 2.10 theoretical value and the 2.16 experimental value depend on the percentage of tangential and axial modes. Thus they depend on room volume and frequency. It must be borne in mind that the above results are for a 607 m^3 reverberation room. Reverberation rooms will normally be smaller than this volume. Thus these values would be expected to be slightly smaller in smaller reverberation rooms. On the other hand, equation (4) gives results which are slightly too small because the number of receiver positions has been set equal to infinity in the second term. To avoid the need to calculate the percentage of axial and tangential modes, the use of the 2.375=19/8 value for K-1 in equation (4) is suggested. As shown above this means that the 0.79 constant should be multiplied by 1.41 to give 1.11. It is further suggested that this value be rounded to 1. This makes K-1 equal to 2.16, which is equal to Davy's (1987) experimental value and close to the three theoretical GOE values of 2.375, 2.10 and 2.0 which were calculated above.

THE PURE TONE QUALIFICATION PROCEDURE

The room qualification procedure for the measurement of discrete frequency components has been deleted from ISO (1996). The alternative multiple source position method has been retained. Baade (personal communication) has asked for clarification of the statement "It is now known that multiple source positions will not necessarily solve all the problems, and hence it is desirable that all reverberation rooms which are to be used for sound power measurements should pass the qualification procedure." This statement appears in Davy (1981b), and Davy (1981b) is appendix C of Davy (1989). The following appears in Davy (1981a).

"Maling (1973) compared Lyon's theory with the relative spatial variance of the real part of the input impedance and concluded that Lyon's theory overestimated the relative variance at low and medium frequencies. Unfortunately it is necessary to vary either the frequency of excitation, the speed of sound in the room, or the room geometry to excite a random selection of modal frequencies before one obtains the full relative variance that Lyon's theory is attempting to predict (see the paper by Bodlund (1977)). In fact, if the selection of excited modes is varied, the source position can be left fixed because the different modes have different spatial distributions."

"To obtain the required variance it is necessary to take averages over the ensemble of all possible values of modal frequencies and all possible modal functions. In theory this requires one to average over different room shapes. This can be achieved by changing the geometry of a single room or by making measurements in different rooms."

"In practice, because the quantities to be measured depend only on the modal frequencies (and their associated modal functions) which are very close to the excitation frequency, changing the excitation frequency will change the selection of modal frequencies and modal functions which determine the value of the measured quantities. Thus averaging over the excitation frequency will give an appropriate average.

The same effect can be obtained by leaving the excitation frequency fixed and varying the room temperature and hence the speed of sound in the room. This will also change the selection of modal frequencies and modal functions which determine the value of the measured quantities."

The author's analysis of the 125 to 1000 Hz experimental values of source position variance in figure 3 of Maling (1973) produces an experimental value of K-C in equation (5) equal to 0.68. This is much less than Davy's (1987) experimental K-C estimate of 2.16 for the total variance case, which was obtained using the pure tone qualification procedure's frequency variation method. This shows experimentally that source position variation does not produce the total variance that exists in pure tone measurements In turn, this suggests that the pure tone qualification procedure should be included in ISO (1996).

Bodlund (1977) and Jacobsen (1979) separate the total variance into a room variance and a source position variance. Using numerical procedures, Bodlund obtains K-C equals 1.42 for the room variance and K-C equals 2.84 for the source position variance. Using theoretical techniques, and the Poisson assumption for the room variance case, Jacobsen obtains K-C equals 1 for the room variance and K-C equals 2.375 for the source position variance.

Note that Jacobsen's results sum to produce K-C equals 3.375, which is the correct result for the total variance in the Poisson case, providing that tangential and axial modes are ignored. Also note that Jacobsen's equations do include the effects of tangential and axial modes, but these terms have been ignored in this analysis. Bodlund's results sum to produce K-C equals 4.26 for the total variance. Both Jacobsen's and Bodlund's results are much higher than Maling's (1973) experimental result for the source position variance. Nevertheless, they both show that the room variance is significant. Since this room variance cannot be reduced by source position averaging, these results again suggest that the pure tone qualification procedure should be included in ISO (1996).

Setting the number of source positions N and the number of receiver positions L equal to infinity and the angular frequency difference θ to zero in equation (1) gives

$$rel\,\mathrm{var} = \frac{1-C}{M}, \tag{6}$$

for the room variance. This means that K-C is equal to 1-C for the room variance. Thus for the Poisson distribution, K-C is equal to 1 for the room variance. This agrees with Jacobsen's theoretical result. It is also the result obtained for Davy's (1981a) incorrect guess of the form of equation (1) for the "nearest neighbour" distribution. (Davy effectively guessed that C was equal to zero and that K was replaced by K-1/2.) The correct result for the "nearest neighbour" distribution is K-C equal to 1/2 for the room variance.

For the Gaussian orthogonal ensemble (GOE) distribution, K-C is equal to zero for the room variance. This surprising result suggests that the multiple source position method is equivalent to the pure tone qualification procedure. However it will soon be seen that this result is not valid at low frequencies.

If the room variance and source position variance are uncorrelated, subtracting equation (6) from equation (5) gives the source position variance of the real part of the input impedance,

$$rel\,\mathrm{var} = \frac{K-1}{M}. \tag{7}$$

Ignoring tangential and axial modes, this agrees with Jacobsen's (1979) theoretical value of K-C=K-1=23/8=2.375 for the source position variance. It is interesting to note that this is independent of the modal frequency spacing distribution as Jacobsen showed.

Equation (1) is not valid for the "nearest neighbour" and Gaussian orthogonal ensemble (GOE) distributions of modal spacings at low values of the statistical modal overlap M. For low values of M, the relative covariance for these distributions tend to that for the Poisson distribution (see figure 1 of Weaver (1989), figure 13 of Lyon (1969) and appendix B of Davy (1981a)). This trend doesn't have a great effect on the total variance because it is offset by the increasing percentages of tangential and axial modes as the frequency reduces and the increasing variance of decay rate at low frequencies.

However equations (6) and (7) show that the choice of distribution only effects the room variance (via C), while the percentages of tangential and axial modes only effect the source position variance (via K). Also the room variance is less than half the total variance. This means that all the increase due to low modal overlap occurs in the smaller room variance, which is not decreased by the increasing percentages of tangential and axial modes. Thus this effect is very significant for room variance. This means that the Gaussian orthogonal ensemble (GOE) distribution of modal spacings predicts significant room variance at low frequencies. This is the frequency region where the variances are most significant because they are largest. Again, since this room variance cannot be reduced by source position averaging, this result suggests that the pure tone qualification procedure should be included in ISO (1996). The use of three or more source positions will make the source position variance less than the low frequency limit of the room variance. Further calculations are needed to determine the exact frequency below which the multiple source position method is unacceptable.

CONCLUSION

All the experimental, theoretical and numerical research results suggest that the pure tone qualification procedure should be included in ISO (1996). The value of the constant 0.79, should be increased to 1 in the equation used to calculate the number of source positions in the multiple source method in ISO (1996).

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