FIFTH INTERNATIONAL CONGRESS ON SOUND AND VIBRATION
DECEMBER 15-18, 1997
ADELAIDE, SOUTH AUSTRALIA

ACTIVE CONTROL OF NONLINEAR VIBRATION IN A FLEXIBLE CANTILEVER BEAM

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ABSTRACT
The aim of this paper is firstly to address the consequences of the nonlinear interactions between higher order modes and the first order mode of a flexible cantilever beam. The tested cantilever beam exhibited numerous nonlinear phenomena that are commonly observed in many flexible structures. The experimental results have shown that active control of the nonlinear vibration in a flexible structure is effective. Secondly, the paper describes how the nonlinear vibration in the cantilever beam was cancelled using a digital feedback controller implemented in a dSpace TM Digital Signal Processing board. It has been demonstrated experimentally that the on-line feedback controller was capable of cancelling the low frequency vibration generated in the flexible cantilever beam due to nonlinear interaction between the modes of the beam. This control scheme has considerable potential for cancelling of nonlinear vibration in large structures such as aircraft, ships, etc. in order to reduce stress and fatigue of these structures.

1. INTRODUCTION
Many large structures are lightly damped due to the low internal damping of the materials used in manufacture. These structures can generate large low frequency resonant vibration when they are subject to a minimum level of excitation at one of the higher order resonance modes of the structure. A typical example is the deckhouse of a ship, which often has large vibrations at low order resonance modes, even though the engine of the ship only generates much higher frequency vibration. These large low frequency vibrations can in the long-term cause fatigue- induced failure of the deckhouse structure.

Vu *et al.* [3] have experimentally investigated nonlinear vibration in a flexible cantilever beam and observed the following phenomena: jump phenomenon, shifting of the natural

frequency of the resonance modes, energy transfer from higher order modes to lower order modes, modal coupling, and frequency modulation.
Among these nonlinear characteristics, modal coupling is the issue of greatest concern for flexible structures. Modal coupling occurs due to energy from the directly excited high frequency modes of the flexible structure being transferred to low frequency modes through nonlinear interactions. Anderson *et al.*[1] have also shown that when the directly excited high frequency modes became saturated, the energy from the higher mode cascaded into lower order modes.

The aim of this paper is firstly to address the consequences of the nonlinear interactions between higher order modes and the first order mode of the flexible cantilever beam (refer to section 2). The tested cantilever beam exhibited many nonlinear phenomena which are commonly observed in flexible structures. Secondly, the paper describes how the nonlinear vibration in a cantilever beam was cancelled using a digital feedback controller implemented in a dSpace TM Digital Signal Processing board. Details of the control system are described in section 4.

2. EXPERIMENTAL SETUPS

Figure 1 shows a complete experimental setup of the cantilever beam, with a Ling Dynamic Systems Model V406 electromagnetic shaker used as a excitation source. The electromagnetic shaker was driven by a signal generated from a Packard arbitrary function generator via a Ling Dynamic Systems power amplifier.

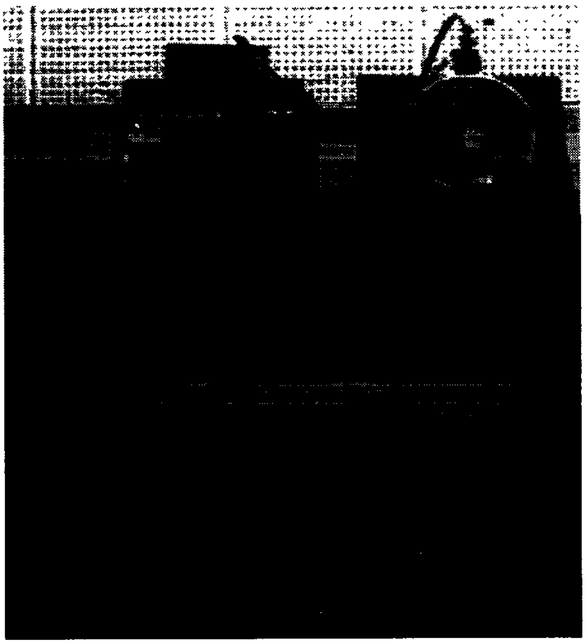

Figure 1. The experimental setup of the cantilever beam.

The system input was measured with an accelerometer mounted at the clamped end of the beam, while the beam response was picked up with an accelerometer attached at the free end (tip) of the beam. In order to avoid unnecessary weight loading on the beam, the cable associated with the accelerometer was glued on the surface of the beam using a thin double-sided adhesive tape. Since the beam is slender and flexible with a low fundamental response (as low as 4 Hz), an Entran EGAX-250 was chosen on account of low mass (0.5 gram), large operating range (250g), and sufficiently low frequency response. A PCB accelerometer was also used in the measurements. It has a mass of 1 gram, a sensitivity of 5mV/g, and a frequency range from 2 Hz to 20 kHz. The experimental results with the PCB accelerometer have shown similar nonlinear responses for the beam, except that the response had lower resonance frequencies. This was due to the larger mass of the PCB accelerometer.

The beam was excited in three different excitation orientations: i) the shaker pointing upwards, and the beam pointing horizontally; ii) the shaker pointing sideways, and the beam pointing horizontally; iii) the shaker pointing sideways, and the beam pointing vertically up. It was also observed that the three different beam orientations have similar nonlinear effects despite a slight variation in resonance frequencies, with a similar percentage change for all modes. The variation in resonance frequencies was assumed to be due to the gravitational force, the accelerometer mass and the associated cable loading on the beam. Since the weight of the accelerometer and the cable contributed to the decrease in resonance frequencies, the best setup arrangement was found to be with the shaker pointing sideways and the beam horizontal (see Fig. 1). With this arrangement, the measurement is less sensitive to gravity and the mass loading on the beam. In other words, the least difference in resonance frequencies with and without the tip accelerometer was observed with this configuration. This orientation was then chosen for further research.

3. MODAL COUPLING

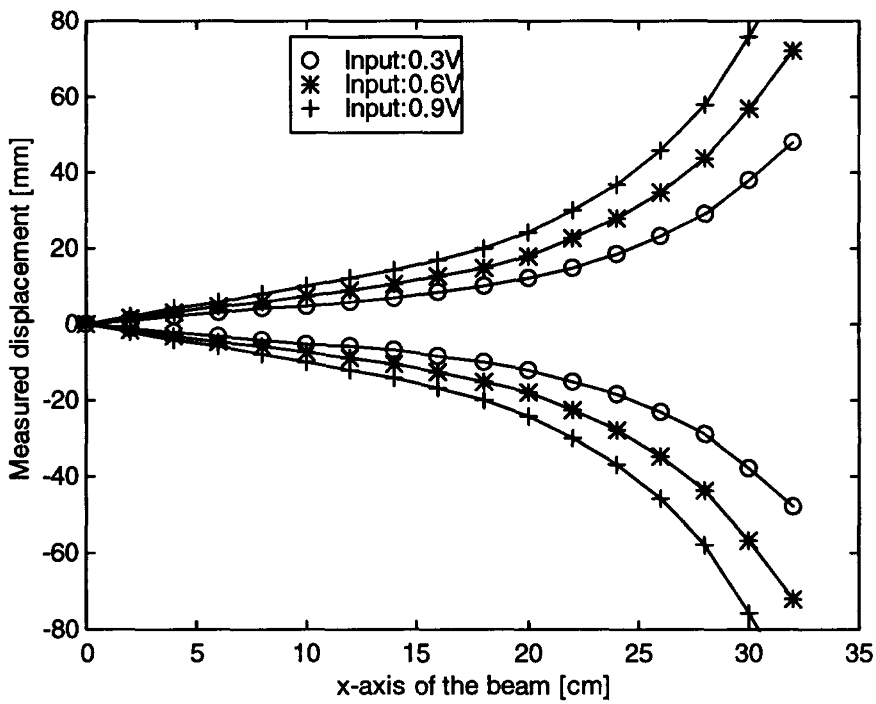

Figure 2. The first mode shape of the cantilever beam for different excitation amplitudes.

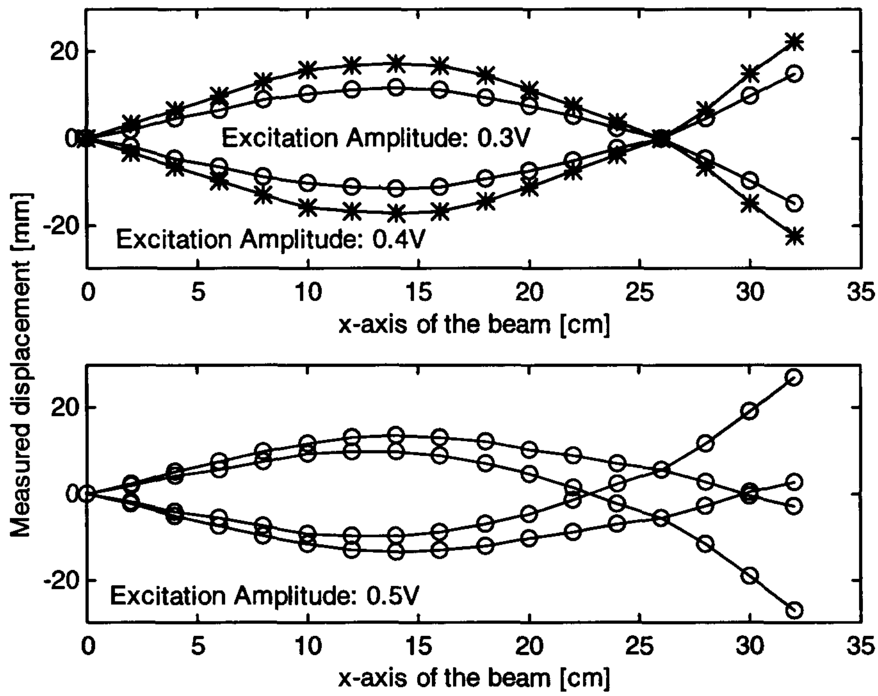

Figure 3. The second mode shapes of the cantilever beam for different excitation amplitudes.

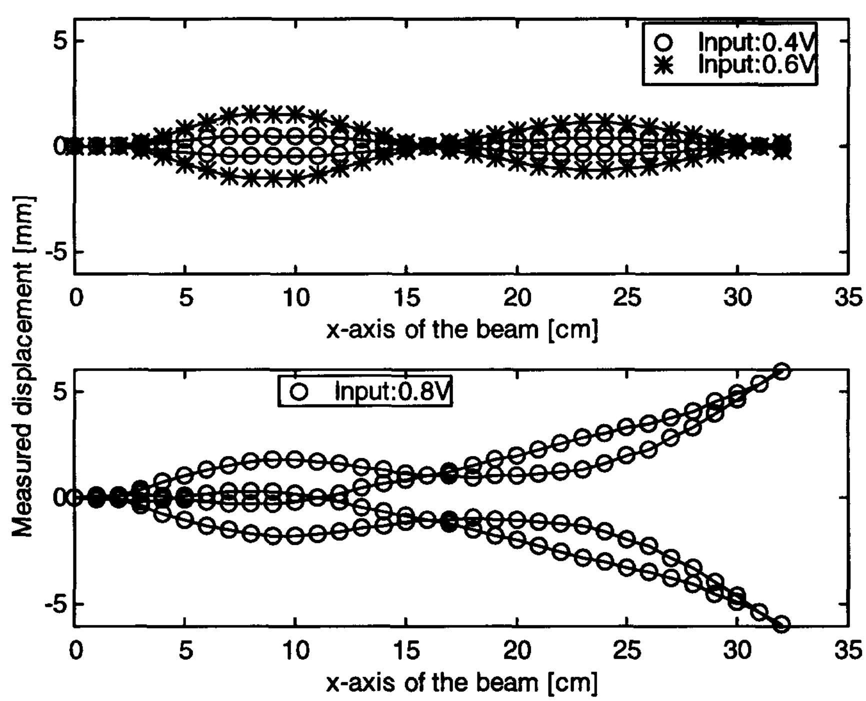

Figure 4. The third mode shapes of the cantilever beam for different excitation amplitudes.

The first, second, and third mode shapes of the beam for different excitation amplitudes are shown in Figures 2 to 4, respectively. The mode shapes were measured by a digital camera and plotted using Matlab when the beam was excited at its resonance frequencies.

According to Anderson *et al.* [1], the natural frequencies of the resonance modes shifted with increasing excitation amplitude. The excitation frequency was therefore changed corresponding to the excitation amplitude. It can be seen from the figures that the magnitude of the displacement of the beam for all three mode shapes increased proportionally to the increment in the excitation amplitude. However, when the beam reached its maximum deformation at the second or third order resonance, a further increase in the excitation amplitude no longer increased the deflection of the beam. The beam then started to couple to the next lower mode (see figures 3 and 4). For example, when the beam reached it's maximum deformation of the third order mode, a further increase in the excitation amplitude caused the beam to couple with the second order mode. Increasing the amplitude even more then caused coupling with the first order mode. As a result, the deflection of the beam became a summation of the bending due the excitation mode and the coupled modes.

Figures 5a and 6a show the displacement measured at the tip of the beam without nonlinear coupling when the beam was excited at the second and third order mode, respectively. When the excitation amplitude was increased to the level where the beam reached its maximum deformation at second and third mode, the beam then started to couple with the first order mode as shown in figures 5b and 6b.

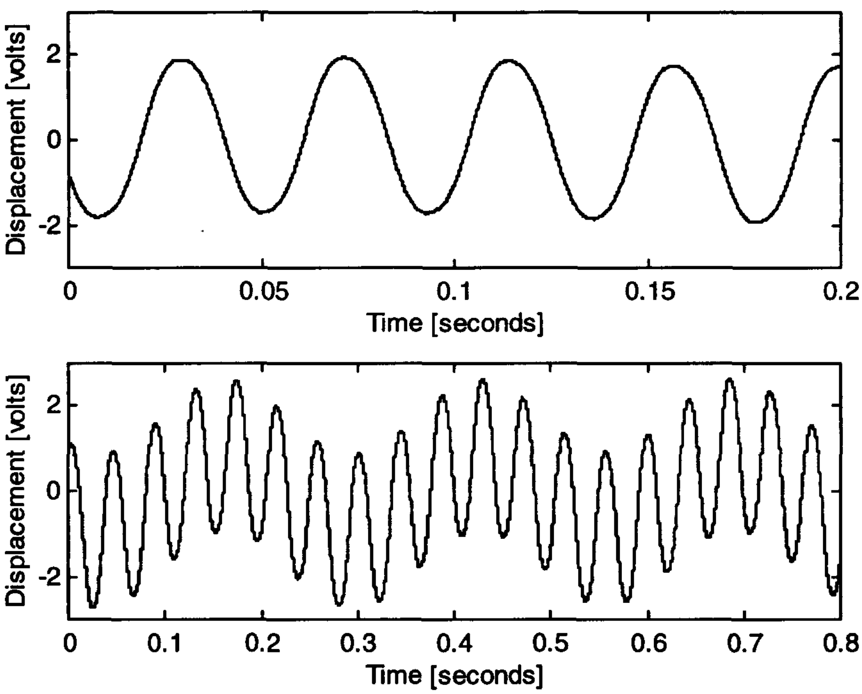

Figure 5. The displacement measured at the tip when the beam was excited at the second mode: a) without coupling with the first mode, b) with coupling with the first order mode.

In summary, when the excitation frequency is close or equal to one of the resonance frequencies of the higher order modes, and the excitation amplitude is sufficiently large, the beam starts to couple with the first order mode through nonlinear interaction. This nonlinear interaction is due to energy from the excited higher order mode dissipating to the next lower order modes when the excited order mode becomes saturated [1, 3]. The low frequency response due to nonlinear coupling can sometimes become quite large and unpredictable, even for only a slight increase in excitation amplitude.

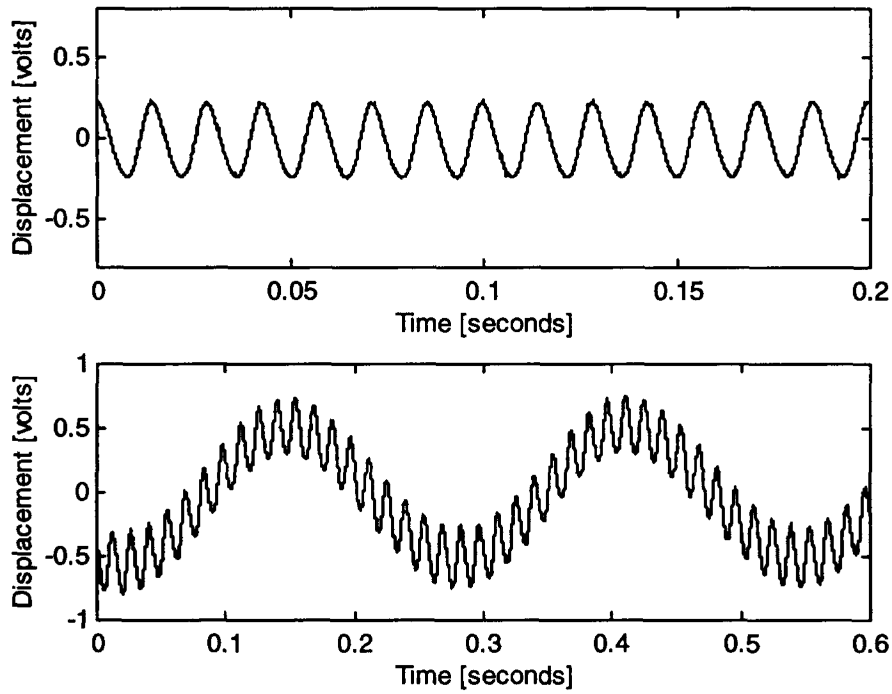

Figure 6. The displacement measured at the tip when the beam was excited at the third mode: a) without coupling with the first mode, b) with coupling with the first mode.

4. CONTROL SCHEME

As described in the previous section, the flexible cantilever beam could have a very large low frequency response when it was subject to a much higher frequency vibration. The large vibration response would cause more stress and fatigue of the beam. The aim of this control scheme was to cancel the low frequency vibration generated from the beam through nonlinear interaction when the beam was excited at the third order mode.

Figure 7 shows the feedback control scheme for the cancellation of nonlinear vibration in the cantilever beam. Initially, the beam was excited at 70.3 Hz (third mode) with the excitation amplitude at 0.5V. Figure 8 shows the response of the beam measured at the tip without control. The displacement at the tip of the beam was measured by the PCB accelerometer and fed to the DSP board via a conditioning amplifier and an A/D converter. The measured signal was filtered by a digital lowpass filter in order to select only the response of the fundamental mode. The filtered signal was then passed through a compensator. Both the lowpass filter and the compensator were implemented in the DSP board. The compensator was used to compensate for the frequency response of the beam, the accelerometer (including the conditioning amplifier), the power amplifier and the shaker. The control output was a summation of the output of the compensator and the reference signal. The control output was then fed to the power amplifier via a D/A converter in order to drive the shaker to cancel the low frequency vibration generated in the beam.

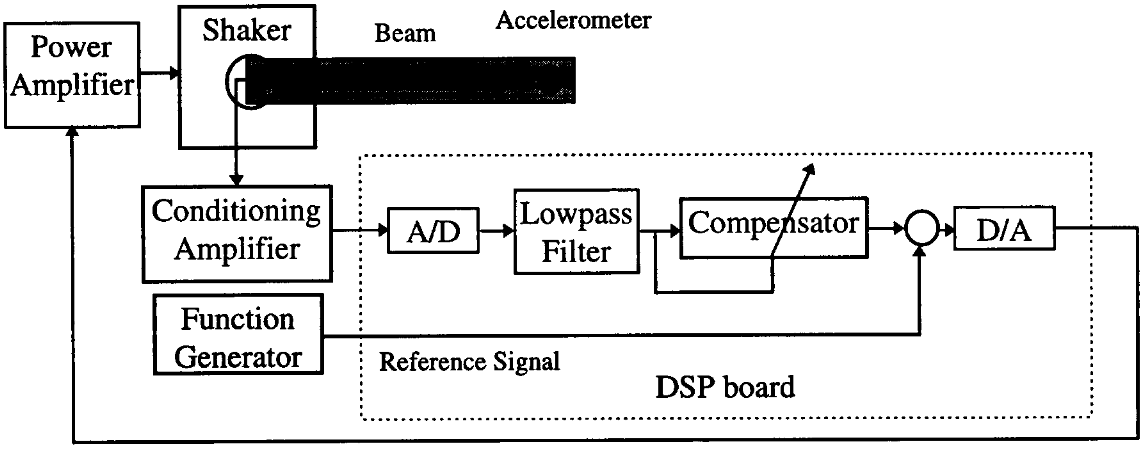

Figure 7. The feedback control scheme for the cancellation of nonlinear vibration.

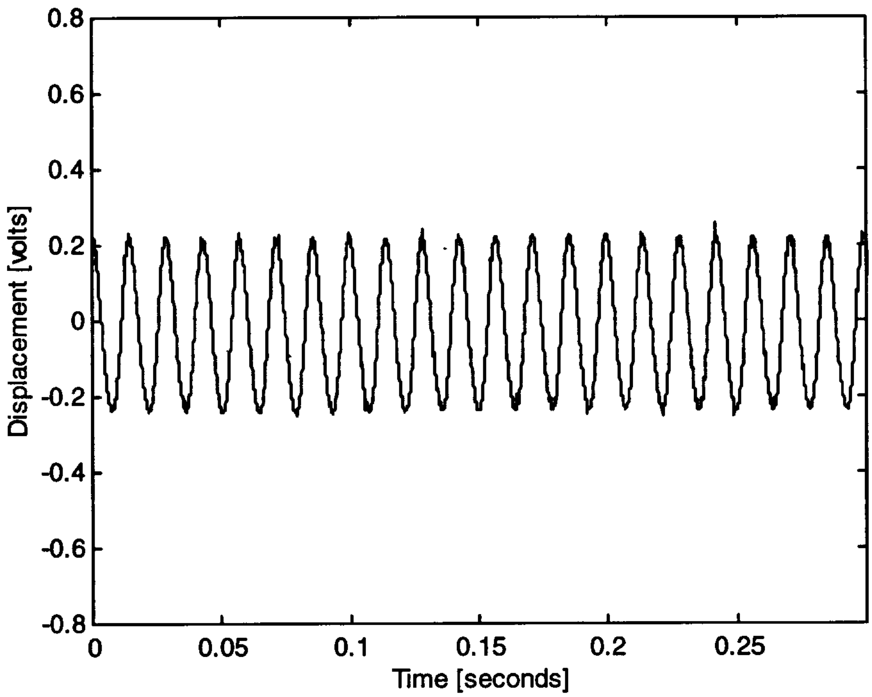

Figure 8. The measured displacement at the tip of the beam with feedback control.

Figure 8 shows the response measured at the tip of the beam with feedback control. It can be seen that the low frequency vibration was almost attenuated while the reference signal magnitude remained constant. For comparison, the auto spectrum of the responses with and without control were plotted as shown in Figure 9. The figure shows a significant reduction of the vibration of the first order mode (approximately 50 dB) as well as removing all the sub-harmonic frequency and coupling frequency components. The control scheme was also used to cancel the low frequency vibration generated when the beam was excited at its second order mode (23.5 Hz) as shown in figure 6.

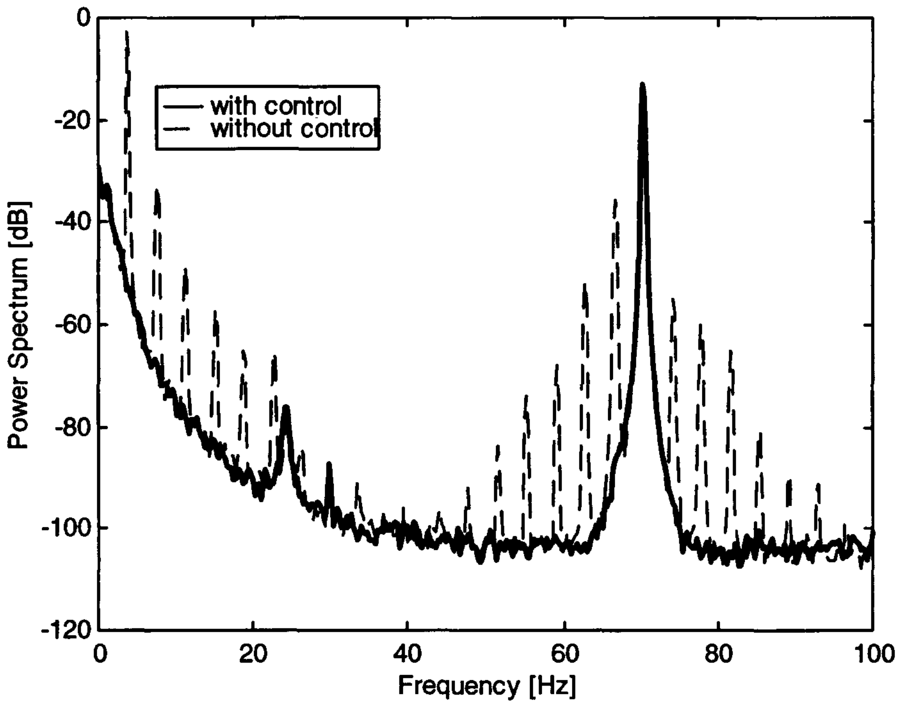

Figure 9. The response measured at the tip of the beam with and without control.

5. CONCLUSION

It has been demonstrated experimentally that the on-line feedback controller was capable of cancelling the low frequency vibration generated in the flexible cantilever beam due to nonlinear interaction between the modes of the beam. This control scheme has considerable potential for cancelling nonlinear vibration in large structures such as aircraft, ships, etc. in order to reduce stress and fatigue in these structures.

ACKNOWLEDGMENTS

The authors would like to thank Mr R. Greenhalgh for his technical advice and assistance, and Professor B. J. Stone and Mr S. Drew for use of the function generator, the power amplifier, and the shaker. The authors also acknowledge Mr S. Drew for his comments on the manuscript.

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