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MATHEMATICAL AND COMPUTER MODELLING OF TRANSITIONAL PROCESSES IN TRANSMISSIONS INCORPORATING HIGH TORQUE HARMONIC DRIVES

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Abstract: It is of utmost importance, at the design stage, to obtain reliable data regarding the behaviour of harmonic drives during transition processes and to estimate its parameters (natural frequencies, magnification factor, torque variation in different parts of the drive). This enables the minimisation of stresses, reduction in overall dimensions and weight of the transmission, and increase in reliability. Equations of motion for these transmissions can be derived using the energy method or by considering the dynamic equilibrium of each link. For analysis of transition processes it is important to account for damping, which is normally considered viscous. This approach is suitable for the analysis of transition processes in conventional transmissions. Attempts to employ this method for the analysis of harmonic drives yields confusing results due to the presence of the stationary link. To overcome this discrepancy, the hypothesis of a "moving wall" was introduced, and equations of motion for power transmissions incorporating harmonic drives were developed. As a numerical example, a transmission for a walking excavator incorporating a harmonic drive was considered. Peculiarities of computer modelling of such transmissions are also discussed in this paper.

1. INTRODUCTION

The performance of mechanical transmissions incorporating harmonic drives to a large extent depends on proper selection of design parameters. Mathematical and computer modelling of transition processes in transmissions (start up and coast down) at the design stage becomes of utmost importance. It enables the evaluation of design parameters for different design concepts and helps save time and money on experimental testing. For example, on heavy metallurgical equipment with conventional transmissions the magnification factor at the start up mode often exceeds 3 [1], which results in overloading of the transmission elements and their failure. It is therefore important to design transmissions with higher damping.

However, methods of mathematical and computer modelling of conventional transmissions cannot be applied directly to the modelling of conventional transmissions because of the following peculiarities:
- Harmonic transmissions have very high speed reduction ratios in one stage (e.g. 300 : 1, 400 : 1). When equivalent mass moments of inertia and torsional stiffness of particular links are calculated (with respect to an input or an output link), their initial values are divided by the speed reduction

ratio squared, which often yields negligible value. Computer software in general do not accept numerical values of coefficients of differential equations which differ by a factor of 10^5 or 10^6.

- Harmonic drives have a fixed link (either a rigid or a flexible gear). When a conventional approach is used to calculate the torque T_{12} between links 1 and 2 connected by an elastic element with torsional stiffness C_{12}, the stiffness is multiplied by the difference in angular coordinates of these links

$$T_{12} = C_{12} \cdot (\varphi_1 - \varphi_2). \tag{1}$$

If one of the links is fixed, say link 2, $\varphi_2 = 0$, and with constantly increasing value of coordinate φ_1, formula (1) gives the torque value which tends to infinity. This is confusing because in reality it varies within a particular range, but does not increase infinitely.

Thus, a new approach to mathematical and computer modelling of transmissions incorporating harmonic drives has to be developed. In this paper the hypothesis of a "Moving Wall" is introduced, which enables the calculation of the torque between a fixed and rotating gears, and a special procedure is proposed which allows the modification of the mathematical model of the transmission when the numerical value of some of the parameters becomes negligible.

2. DETERMINATION OF THE MODEL PARAMETERS

For mathematical and computer modelling of transmissions, inertia, stiffness and damping parameters of the model parts have to be determined. Since different parts of a transmission rotate with different speed, these parameters' numerical values cannot be taken directly. Their equivalent values have to be determined with respect to an input or an output link (in most cases with respect to an input link).

An equivalent mass moment of inertia of each rotating part can be calculated using the following formula [2]

$$I_{ei} = \frac{I_i}{(u_i)^2} \tag{2}$$

where $u_i = n_1 / n_i$ = speed reduction ratio between the input link (1) and the i-th link;
n_1 and n_i = corresponding angular velocities of an input and the i-th link;
I_i = mass moment of inertia of the i-th link about its own axis of rotation.

An equivalent torsional stiffness of a link or a joint with the reference to an input link can be determined as [2]

$$C_{ei} = \frac{\eta}{e_i \cdot (u_i)^2} = \frac{C_i \cdot \eta}{u_i} \tag{3}$$

where η = efficiency of the part of the transmission between the i-th and the input links;
e_i = torsional compliance of the i-th link (or a joint);
C_i = torsional stiffness of the i-th link (or a joint).

A damping constant α_i for each link or a joint can be determined using the following relations [3]

$$\alpha_i = \frac{\varsigma_i}{\pi} \cdot \sqrt{C_{ij} \cdot \frac{I_i \cdot I_j}{I_i + I_j}} \tag{4}$$

where ς_i = damping factor which can be easily determined [3] if the logarithmic decrement is known;
C_{ij} = torsional stiffness between links i and j;
I_i and I_j = mass moment of inertia of adjoining links i and j.

3. HYPOTHESES OF THE "MOVING WALL"

Typical mechanical transmissions incorporating a harmonic drive consist of the following parts: a motor (normally electric motor); a coupling connecting the motor with an input shaft of a harmonic drive which includes a wave generator, a flexible gear, and a rigid gear. In most cases the rigid gear is fixed and the flexible gear is connected by a coupling to an output shaft which drives the particular mechanism. In some cases when the flexible gear is fixed, the rigid gear is connected to a driven mechanism.

As was discussed in the introductory paragraph, when one of the links is fixed (a rigid or a flexible gear) a computational problem arises - how to determine the correct value of a torque between the two parts, one of which is stationary. To overcome this problem the hypothesis of a "moving wall" is introduced. It is assumed that the fixed link rotates with the velocity of the input link (it becomes a "moving wall"). The torque between adjoining parts can be determined as the product of the torsional stiffness and the difference in angular coordinates of the moving part and the "moving wall". For the case of a fixed rigid gear the part of the transmission is shown schematically in the diagram (see Fig. 1).

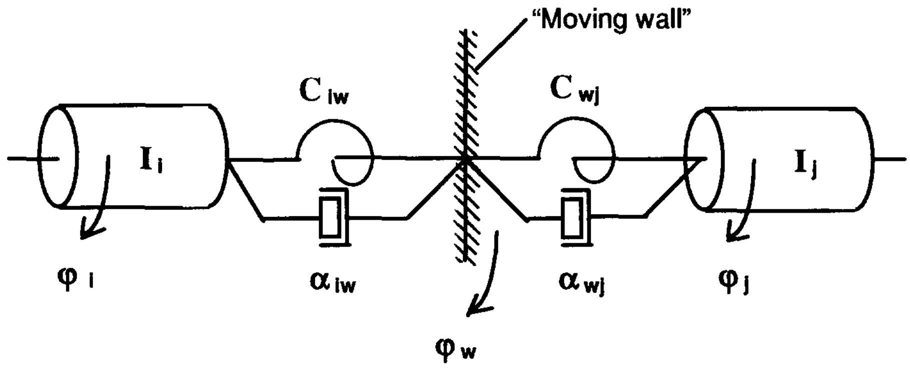

Fig. 1. Schematic diagram of the part of the transmission with the "moving wall".

If the wave generator with a mass moment of inertia I_i is attached to the rigid gear at the input side, and the flexible gear with a mass moment of inertia I_j - at the output side, the torque acting on the wave generator and on the flexible gear can be determined using the following expressions

$$T_{iw} = C_{iw} \cdot (\varphi_i - \varphi_w). \tag{5}$$

$$T_{wj} = C_{wj} \cdot (\varphi_w - \varphi_j). \tag{6}$$

4. ANALYSIS OF TRANSITION PROCESSES AND AN EXAMPLE

As an example for analysis of transition processes we will consider a transmission of a walking excavator with an output torque of 3000 kN·m, incorporating a harmonic drive. This transmission was developed at the Azov State Technical University (Mariupol city, Ukraine) in collaboration with NKMZ company (Kramatorsk city, Ukraine). The main parts of this transmission are shown in Fig. 2: a thyristor controlled electric motor (1); a coupling (2), connecting the motor with an input shaft of the preliminary gear box which includes gears (3; 4; 5); an output shaft of the preliminary gear box connected by a coupling 7 to an input shaft of a harmonic drive, which includes a wave generator (9; 10; 11), a flexible gear (13); a rigid gear (15), which is fixed. The flexible gear is connected by a coupling (19) to an output shaft (20), which drives the walking mechanism (21; 22; 23; 24).

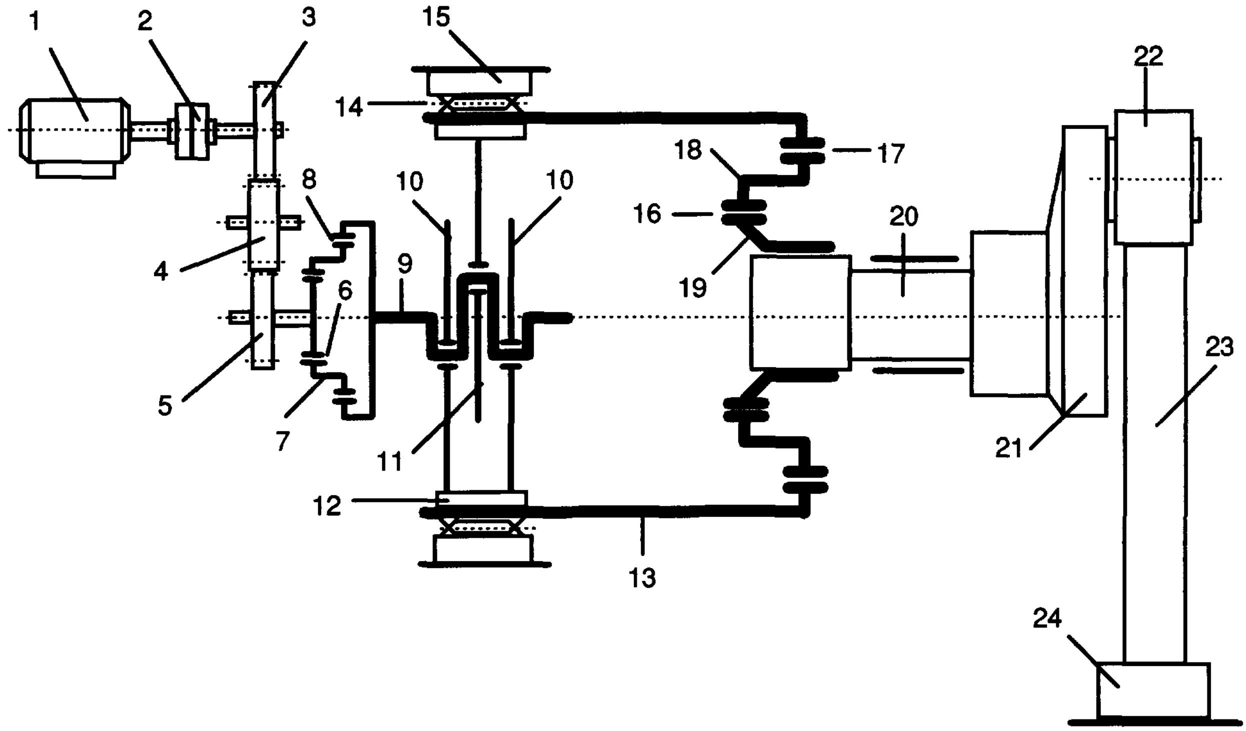

Fig. 2. Transmission of walking excavator incorporating high torque harmonic drive

For calculation of moments of inertia the main parts of the transmission can be represented as disks and cylinders. Some of these, such as the disks of the wave generator, perform epicyclic motion. To determine their moments of inertia the parallel axis principle was used. The torsional stiffness of transmission parts and joints were calculated according to recommendations [2]. An analysis of moments of inertia and torsional stiffnesses allows us to state the following:

- The highest inertial elements of the transmission can be divided in three groups: the rotor of electric motor $\mathbf{I}_1 = 70\,\mathrm{kg{\cdot}m^2}$; parts from the coupling of electric motor to the flexible gear, with a total moment of inertia $\mathbf{I}_2 = 7.89\,\mathrm{kg{\cdot}m^2}$; parts from the flexible gear to supporting shoe with total moment of inertia $\mathbf{I}_3 = 0.037\,\mathrm{kg \cdot m^2}$.
- There is a big difference between moments of inertia of these groups of parts, so that
$$\mathbf{I}_1 >> \mathbf{I}_2 >> \mathbf{I}_3.$$
- The most compliant elements of the transmission are the coupling of the electric motor; spline joints of the coupling and the output shaft. Groups of parts $\mathbf{I}_1$ and $\mathbf{I}_2$ are connected by a coupling with equivalent stiffness $\mathbf{C}_{12} = 7.78 \cdot 10^3$ N·m/rad. Groups of parts $\mathbf{I}_2$ and $\mathbf{I}_3$ are connected by spline joints with total equivalent stiffness $\mathbf{C}_{23}$ = 2.35 N·m/rad, which was determined by summing their compliances.

This allows us to develop the dynamic scheme of the transmission which is shown in Fig. 3.

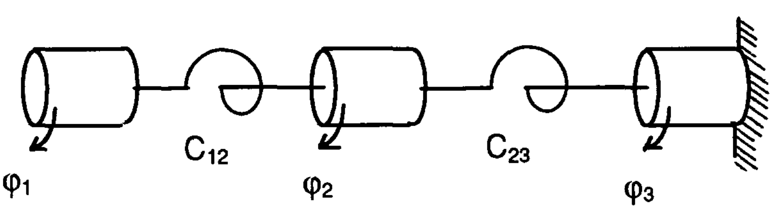

Fig. 3. Dynamic scheme of the transmission

In Fig. 3, the arrows indicate positive twist angle of parts in torsional vibration. As has been seen the third group of parts has negligible moment of inertia $\mathbf{I_1}$ compared to $\mathbf{I_2}$ and $\mathbf{I_3}$. When the walking mechanism works the supporting leg and shoe, prop against the ground and actually are stationary. Thus, it is possible to consider the transmission as two mass oscillating system with two degrees of freedom. Its natural frequencies can be determined using the following expression [4], p. 547

$$\mathbf{f_{1,2}} = (1/2\pi)\cdot\sqrt{\{0.5\cdot[C_{12}/I_1 + (C_{12}+C_{23})/I_2] \pm \sqrt{[C_{12}/I_1 + (C_{12}+C_{23})/I_2]^2 - 4\cdot C_{12}\cdot C_{23}/I_1\cdot I_2]\}} \tag{7}$$

Substituting numerical values into expression (7), we get natural frequencies of torsional vibrations

$$\mathbf{f_1} = 0.782\,\text{Hz}; \qquad \mathbf{f_2} = 5.896\,\text{Hz}.$$

We can calculate partial frequencies corresponding to groups of bodies $\mathbf{I_1}$ and $\mathbf{I_2}$

$$\begin{aligned} \mathbf{f_{1p}} &= (1/2\pi)\cdot\sqrt{(C_{12}/I_1)} = (1/2\pi)\cdot\sqrt{(7.78\cdot 10^3/70)} = 1.68\,\text{Hz}; \\ \mathbf{f_{2,p}} &= (1/2\pi)\cdot\sqrt{(C_{12}+C_{23})/I_2)} = (1/2\pi)\cdot\sqrt{(7.78\cdot 10^3 + 2.35\cdot 10^3)/7.89)} = 5.71\,\text{Hz}. \end{aligned} \tag{8}$$

As is seen, the second partial frequency $\mathbf{f_{2p}}$ is almost coincident with the second natural frequency $\mathbf{f_2}$. This happens because the second group of bodies has the moment of inertia $\mathbf{I_2}$ much smaller than $\mathbf{I_1}$. This enables further simplification of the dynamic scheme of the transmission according to the method [3], pp.78-82. The transmission can be considered as a single oscillating body with the mass moment of inertia $\mathbf{I} = \mathbf{I_1} + \mathbf{I_2}$. The equivalent torsional stiffness connecting this body with the "moving wall" has to provide the same natural frequency as the fundamental frequency of the original system.

$$C = 4\cdot\pi^2\cdot\mathbf{I}\cdot(\mathbf{f_1})^2 = 4\cdot\pi^2\cdot 77.89\cdot(0.782)^2 = 1.88\cdot 10^3\,\text{N}\cdot\text{m/rad}. \tag{9}$$

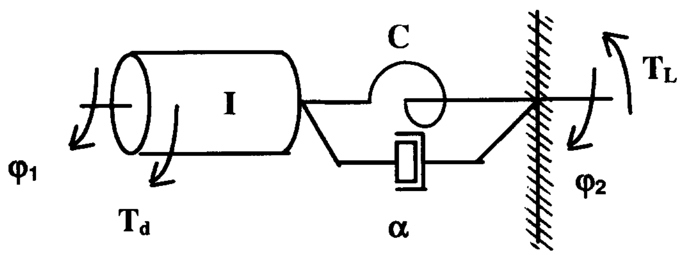

Fig. 4. Simplified dynamic scheme of the transmission

Where: φ_1 = angular coordinate of the rotor of electric motor;
φ_2 = angular coordinate of the driven part (supporting leg and shoe);
$\mathbf{T_d}$ = driving torque at electric motor;
$\mathbf{T_L}$ = equivalent loading torque;
α = 109.63 kg·m²/sec - damping constant calculated according to (4).

4.1 DERIVATION OF DIFFERENTIAL EQUATIONS OF TRANSITION PROCESSES IN THE TRANSMISSION

Differential equations of transition processes can be derived on the basis of D'Alembert's principle considering dynamic equilibrium of each rotating group of parts. For the group of parts **I** the equation of dynamic equilibrium can be written as

$$I \cdot \ddot{\varphi}_1 + C \cdot (\varphi_1 - \varphi_2) + \alpha \cdot (\dot{\varphi}_1 - \dot{\varphi}_2) - T_d = 0. \quad (10)$$

For the moving wall the equation of dynamic equilibrium is

$$C \cdot (\varphi_1 - \varphi_2) + \alpha \cdot (\dot{\varphi}_1 - \dot{\varphi}_2) - T_L = 0. \quad (11)$$

High torque transmissions are normally driven by thyristor controlled electric motors which allow provision of linear acceleration within a predefined period of time. In this case the acceleration time was 2 sec, and we can write

$$\begin{array}{llll} \text{if } t > 0, & \varphi_1 = \varphi_2 = 0; & \dot{\varphi}_1 = \dot{\varphi}_2 = 0 & \\ \text{if } 0 < t < 2\,\text{sec}, & \varphi_1 = \varepsilon \cdot t^2/2; & \dot{\varphi}_1 = \varepsilon \cdot t; & \ddot{\varphi}_1 = \varepsilon; \\ \text{if } t \geq 2\,\text{sec}, & \varphi_1 = 2 \cdot \varepsilon \cdot (t - 1); & \dot{\varphi}_1 = 2 \cdot \varepsilon; & \ddot{\varphi}_1 = 0, \end{array} \quad (12)$$

where $\varepsilon = 16.5\,\text{rad/sec}^2$ - predetermined angular acceleration.

The peculiarity of this model is that if kinematic parameters of the driving link are known (φ_1 and its derivatives), the unknown value is the driving torque T_d which can be determined as

$$T_d = I \cdot \varepsilon + M_L\ . \quad (13)$$

The general solution of the system of differential equations (10;11) is as follows

$$\varphi_2 = t^2 \cdot \varepsilon/2 - T_L \cdot (1 - e^{-C \cdot t/\alpha})/C. \quad (14)$$

Taking a derivative we get the angular velocity

$$\omega_2 = \dot{\varphi}_2 = t \cdot \varepsilon - T_L \cdot (e^{-C \cdot t/\alpha})/\alpha. \quad (15)$$

The torque between the driving and the driven links will be

$$T_{12} = T_L \cdot (1 - e^{-C \cdot t/\alpha}). \quad (16)$$

Substituting numerical values we can calculate angular coordinates and angular velocities for $t = 2$ sec. They are:

$$\varphi_1 = 33\,\text{rad};\ \dot{\varphi}_1 = 33\,\text{rad/sec};\ \varphi_2 = 33 - 5.319 \cdot 10^{-4} T_L;\ \varphi_2 = 33\,\text{rad/sec}. \quad (17)$$

When the motor reaches the nominal speed ($t > 2$ sec), the general solution to the system (10; 11) will be as follows

$$\varphi_2 = 2 \cdot t \cdot \varepsilon - 2 \cdot \varepsilon - T_L \cdot (1 - e^{-C \cdot t/\alpha})/C \quad (18)$$

The derivative (angular velocity)

$$\omega_2 = \dot{\varphi}_2 = 2 \cdot \varepsilon - T_L \cdot (e^{-C \cdot t/\alpha})/\alpha \quad (19)$$

And the torque T_{12} will be determined by the expression similar to (16) because for the point in time $t > 2\,\text{sec}$ acceleration is zero and $T_d = T_L$.

To plot resultant angular velocities and torque we need to specify the loading torque T_L. During the full walking cycle the loading torque varies according with the angular position of the eccentric on the supporting leg from zero to its maximum value, $T_L = 8$ kN. Thus, to investigate the toughest conditions we have to take the maximum value of the loading torque. Since the output shaft rotates very slowly, the loading torque variation during start up time (2 sec) is negligible, and we can assume it is constant.

4.2 DISCUSSION OF RESULTS

On Fig. 5, (a) angular velocities of the motor and the output shaft are plotted versus time. As it is seen from the graph, when brakes are released and the motor is switched on, the output shaft for approximately 0.1 sec rotates in the reverse direction, and then accelerates with the shift in angular velocities of 2.5 rad / sec. When the motor in 2 sec. reaches the nominal speed of 33 rad / sec, the output shaft with the delay of approximately 0.05 sec. reaches the same velocity. In the mean time, the driving torque $\mathbf{T_d}$ during the acceleration time is $116\,\%$ of its nominal value required to drive the transmission (see Fig. 5, (b)). The twisting torque in the transmission $\mathbf{T_{12}}$ steeply increases during 0.25 sec from zero to its nominal value and then remains constant. Thus, the magnification factor for the torque on the motor shaft is only 1.16, compared to $2...2.5$ on conventional transmissions. This is a significant advantage of transmissions incorporating harmonic drives over conventional ones.

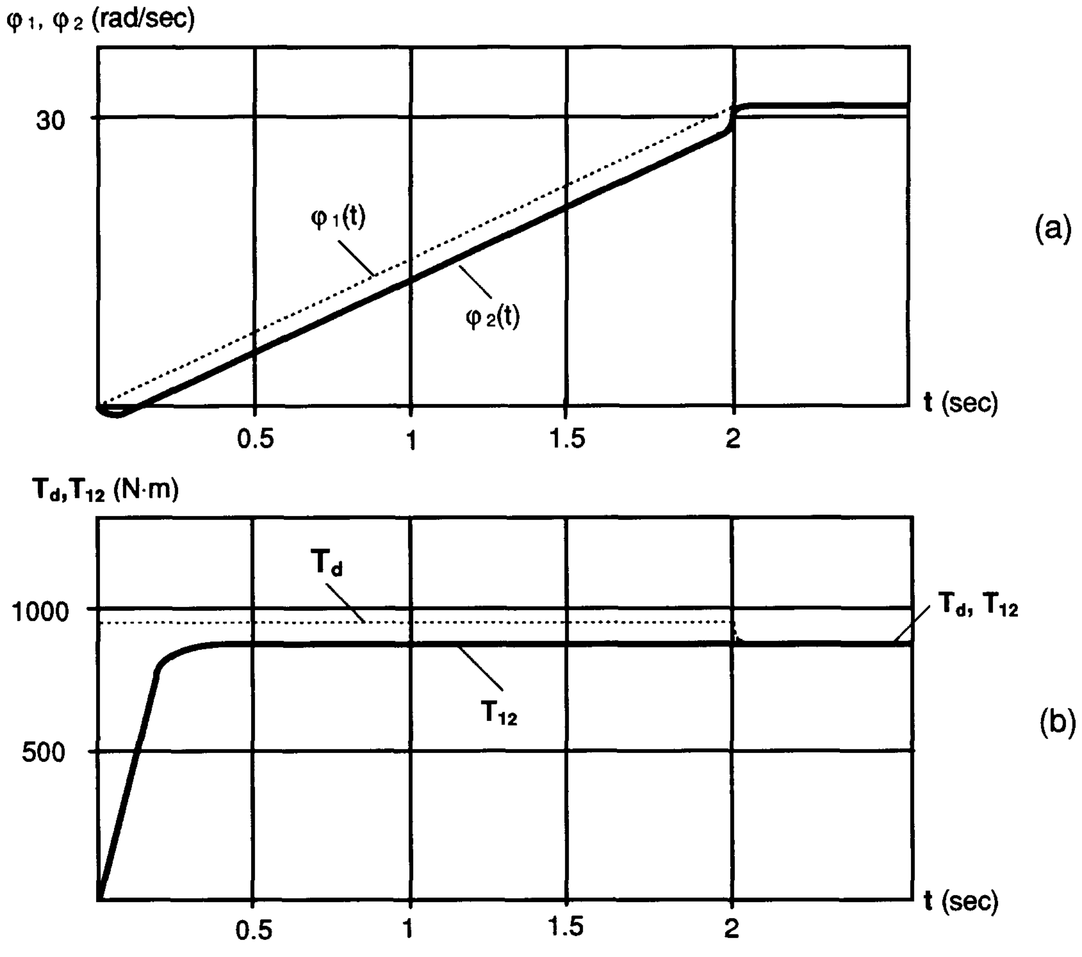

Fig. 5. Plot of results for the start up mode
(a) Variation of angular velocities of the driving and the driven links with time;
(b) Variation of the driving and the twisting torques with time.

5. PECULIARITIES OF COMPUTER MODELLING OF TRANSMISSIONS INCORPORATING HARMONIC DRIVES

For computer modelling of transition processes in transmissions the "Mechanica" software can be used. It allows the specification of torsional stiffnesses, damping constants, mass moments of inertia, and angular velocities of particular parts like electric motor and "moving wall" using so called "drivers". The limited space of this paper does not allow us to describe the modelling in all detail.

For modelling of transition processes the "Motion" analysis can be used. Angular velocities of particular parts are defined by "drivers", and driving torques are applied. They can be defined as a function of time (polynomial or ramp function), or user defined by a table. Variation of kinematic parameters of different links and joint reactions (torques) are plotted as functions of time. If the driving torque is activated during a short period of time, and then is given zero value, the "decay curves" can be plotted for velocities of different links which enable the determination of damping parameters of the transmission as a whole. The "Mechanica" allows the plotting of one variable parameter versus another variable parameter which makes it very useful in dynamic analysis.
To determine natural frequencies of torsional vibrations the "Holzer" method [5] can be used. At one end of the transmission, a sinusoidal torque is applied with variable circular frequency. At the other end the variation with time of the joint reaction (torque) is observed. On the plot of the torque versus circular frequency, points where the torque takes zero values (intersections with the horizontal axis) correspond to resonance states and give the values of circular frequencies which are coincident with the natural frequencies.

6. CONCLUSIONS

1. High speed reduction ratios in one stage of a harmonic drive results in negligible value of moments of inertia of many transmission components. The largest inertial part is the rotor of electric motor.
2. The most compliant parts of the transmission are the coupling of the electric motor and the gear coupling of the output shaft. Thus, the easiest way to influence the dynamic properties of the transmission is to vary the torsional stiffness of the motor coupling.
3. Harmonic drives provide high damping which together with high speed reduction ratios in one stage results in low magnification factor under 1.2 compared to 2...2.5 for conventional transmissions.
4. The hypothesis of a "moving wall" allows the determination of the angular velocity of parts adjoining stationary parts like the rigid gear, and the ground.
5. The partial frequency method enables the simplification of the model without the sacrificing accuracy of results.

7. REFERENCE

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