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AN ADAPTIVE SOUND INTENSITY CONTROL ALGORITHM FOR ACTIVE CONTROL OF TRANSFORMER NOISE

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ABSTRACT

A frequency domain filtered-x type least mean active sound intensity adaptive control algorithm based on waveform synthesis is developed for active control of transformer noise. The algorithm is useful for providing global control of transformer noise by minimising the mean active sound intensities for the fundamental frequency and its harmonics in the near acoustic field of the transformer. Mean active sound intensity has been found to be the most effective near field acoustic error sensing strategy for active control of free field sound radiation[1]. The algorithm adjusts the amplitude and phase of each tone to be controlled in the frequency domain and implements the waveform synthesis process in the time domain. The transfer function of the cancellation path is included in the algorithm so that it becomes a filtered-x type adaptive algorithm.

1. INTRODUCTION

In applications of active noise control, the measurement that is typically most readily available is sound pressure. However, for most applications, the control objective is often related to the energy in the sound field, rather than the sound pressure associated with the field. Therefore, energy based active control including sound energy density or sound intensity rather than direct sound pressure error sensing offers considerable performance benefits, albeit at the expense of implementation difficulties. For active control of transformer noise, the most effective near field acoustic error sensing strategy for providing global far field control has been shown to be minimisation of the mean active sound intensities for the fundamental frequency and its harmonics in the near acoustic field of the transformer[1].

Sommerfeldt and Nashif developed an adaptive filtered-x LMS algorithm for active control of energy density and intensity in acoustic or structural fields[2]. They found that by sensing the energy in the field, their method could overcome the observability problem associated with small localised zones of silence which occur when controlling the measured sound pressure. The convergence properties associated with their control scheme are similar to those associated with a standard filtered-x implementation, and only a modest increase in signal processing is required to implement the energy-based control scheme. However their intensity control update equation is only applicable to time domain signals. Kang and Kim developed a similar algorithm for active intensity control of radiated duct noise. They tried to avoid the complexities associated with real time control due to the FFT and a lengthy data averaging process associated with the frequency domain algorithm, and developed a fast and simple time domain adaptive algorithm based on the instantaneous sound intensity[3].

Although sound energy density and sound intensity may be measured in the time domain using analogue circuits, frequency domain approaches are more suitable for the sound intensity measurement of transformer noise. A frequency domain approach avoids the phase error problems associated with analogue circuitry and is becoming cost effective due to recent developments of digital signal processing hardware. For a multi channel sound intensity control system, each channel can be calibrated and its output adjusted accordingly with a look-up table so that all the channels can use the same digital system. While a time domain system, may be faster than the frequency domain system, it requires a band pass filter for each frequency and a calibration filter for each channel.

Frequency domain approaches offer more advantages for the adaptive sound intensity control system of transformer noise. For conventional time domain adaptive algorithms, instantaneous sound intensity may not be used directly as the error signal because it is not coherent with the primary noise and reference signal[4-6]. For example, if the primary noise signal variation with time is described by $\cos\omega t$, then the time domain instantaneous intensity signal is described by a constant plus a $\cos 2\omega t$ term, which is not correlated with the reference signal. However, in the frequency domain, the orthogonality of the Fourier transformer allows the spectral intensity to correlate well with the reference spectrum. With the adaptive algorithm in the frequency domain, each frequency may be adapted independently and with different weightings and convergence speeds, thus allowing simultaneous active attenuation of both strong and weaker frequencies and faster convergence. In addition, the Fourier transform operation suppresses random noise relative to the sinusoidal components of the signal which allows the adaptive system to concentrate on the signal's spectral peaks, which is especially suitable for active control of electrical transformer noise.

Perhaps the most widely employed methods of active noise cancellation are based on electronic filtering of a reference signal to produce the cancellation signal[6-7]. There are usually two acoustic paths which need to be modelled in a conventional filtered-x algorithm, one is the acoustic system transfer function from reference sensors to error sensors and another is the cancellation path transfer function from control sources to error sensors. Conventional filtered-x type adaptive noise controllers often employ a large number of filter taps in order to provide sufficient accuracy in modelling the transfer functions between reference sensors, error sensors and control sources over a wide frequency range. The filtering methods are necessary for broad band noise cancellation; however for periodic noise such as transformer noise, it is not necessary to determine the transfer function of the acoustic system for a wide frequency band. In addition, long filters for modelling the system to be controlled can result in slower adaptation and higher cost[8-9].

For an algorithm which employs a combination of time domain feedforward filtering and frequency domain adaptive optimisation, zero padding must be used to correct for the fact that spectral multiplication in the frequency domain corresponds to circular rather than linear convolution in the time domain[4-5]. The algorithm developed in this paper is a frequency domain filtered-x type least mean active sound intensity adaptive control algorithm based on waveform synthesis, which adapts only two parameters per frequency to be cancelled and synthesises the cancellation signal rather than filters a reference signal. There is no need for use of the zero padding correction in this case because no time domain filtering is involved in the algorithm.

2. DEVELOPMENT OF THE ADAPTIVE SOUND INTENSITY CONTROL ALGORITHM

Sound intensity is defined as a measure of the rate of local sound energy flow in an acoustic medium. It is a vector quantity characterised by a magnitude, a direction and a specific point location in the acoustic medium. The sound power transmitted through an imaginary surface can be obtained by integrating the real component of sound intensity normal to the surface over the area of the surface. As it is an energy based quantity, its measurement requires the determination of two independent quantities, the sound pressure and acoustic particle velocity, together with the relative phase between the two quantities[6][10].

For the p-p method, the determinations of sound pressure and acoustic particle velocity are both made using a pair of microphones. The microphones are generally separated by a fixed distance. A signal proportional to the particle velocity at a point mid-way between the two microphones and along the line joining their acoustic centres is obtained using the finite difference in measured pressure to approximate the pressure gradient while the mean is taken as the pressure at the mid-point. The assumed positive sense of the determined sound intensity is in the direction of the centre line from microphone 1 to microphone 2, and the mean active sound intensity in this direction decomposed in the frequency domain is[6][10]:

$$I(\omega) = -\frac{1}{\rho_0 \omega \Delta} \mathrm{Im}[G_{p_1 p_2}(\omega)] \qquad (1)$$

where G_{p1p2} is the cross spectrum of the pressure signals, Δ is the distance between the two microphones, ρ_0 is the mean air density and ω is angular frequency.

The usual gradient decent algorithm (LMS or Filtered-x LMS) used for the control filter weight coefficients update is

$$\mathbf{W}(n+1) = \mathbf{W}(n) - \mu e(n) \mathbf{R}_\mathbf{p}(n) \qquad (2)$$

where, **W**(n) is vector of weight coefficients, μ is a convergence coefficient chosen to maintain stability, *e(n)* is the error signal at the sensor location and $\mathbf{R}_\mathbf{p}$ is the reference signal or filtered reference signal. If the measured mean active sound intensity at the sensor location is directly used as the error signal *e(n)*, then the squared mean active sound intensity is used as the cost function. Although this approach guarantees the performance function to be positive definite, it introduces the problem of having a non-quadratic function and the result is that multiple minima occur[2]. Fortunately, the error sensing locations can usually be arranged so that the mean active sound intensity in a certain direction is positive definite and serves as a suitable quadratic cost function[1-2].

When the frequency domain mean active sound intensity is used as the cost function, the gradient decent algorithm used for the weight coefficients update at frequency ω becomes

$$\mathbf{W}_{n+1}(\omega) = \mathbf{W}_n(\omega) - \mu(\omega)\nabla J(\omega) \tag{3}$$

where $\nabla J(\omega)$ is the gradient of the cost function $J(\omega)$. Here it is mean active sound intensity $I(\omega)$. From Equation (1)

$$\begin{aligned} J(\omega) &= -\frac{1}{\rho_0\omega\Delta}\mathrm{Im}[P_1^*(\omega)P_2(\omega)] \\ &= \frac{1}{\rho_0\omega\Delta}\mathrm{Im}[P_1(\omega)P_2^*(\omega)] \\ &= \frac{1}{\rho_0\omega\Delta}\mathrm{Im}\{[P_{pl}(\omega)+W^T(\omega)R_{pl}(\omega)][P_{p2}(\omega)+W^T(\omega)R_{p2}(\omega)]^*\} \end{aligned} \tag{4}$$

where P_1 and P_2 are the total sound pressure of the primary and control sound field at locations of microphones 1 and 2, P_{p1} and P_{p2} are the sound pressure of the primary sound field, and R_{p1}, R_{p2} are the filtered reference signals if filtering methods were used.

Because waveform synthesis is to be used, Equation (4) take the form as follows:

$$J(\omega) = \frac{1}{\rho_0\omega\Delta}\mathrm{Im}\{[P_{pl}(\omega)+A(\omega)e^{j\phi(\omega)}R_{pl}(\omega)][P_{p2}(\omega)+A(\omega)e^{j\phi(\omega)}R_{p2}(\omega)]^*\} \tag{5}$$

where, $A(\omega)$ is amplitude of the synthesised wave at frequency ω and $\phi(\omega)$ is the phase. R_{p1}, R_{p2} represent the transfer functions relating the control output to the pressure at the error sensors (cancellation path transfer function). A number of effective on-line approaches have been developed to estimate the cancellation path transfer function[6] and it is assumed in this paper that these transfer functions have been obtained by one of those approaches and can be expressed as

$$R_{p1}(\omega) = A_1(\omega)e^{j\phi_1(\omega)} = G_1(\omega) \qquad R_{p2}(\omega) = A_2(\omega)e^{j\phi_2(\omega)} = G_2(\omega) \tag{6}$$

In order to simplify the derivation, $A(\omega)e^{j\phi(\omega)}$ is represented by a complex function $G(\omega)$ as

$$G(\omega) = G_R(\omega) + jG_I(\omega) \tag{7}$$

and Equation (5) becomes

$$J(\omega) = \frac{1}{\rho_0\omega\Delta}\mathrm{Im}\{[P_{pl}(\omega)+G(\omega)G_1(\omega)][P_{p2}(\omega)+G(\omega)G_2(\omega)]^*\} \tag{8}$$

Taking the gradient of the cost function yields

$$\nabla_{G_R} J(\omega) = \frac{1}{\rho_0\omega\Delta}\mathrm{Im}[P_1(\omega)G_2^*(\omega)+G_1(\omega)P_2^*(\omega)] \tag{9}$$

$$\nabla_{G_I} J(\omega) = \frac{-1}{\rho_0\omega\Delta}\mathrm{Re}[P_1(\omega)G_2^*(\omega)-G_1(\omega)P_2^*(\omega)] \tag{10}$$

Then the update equations for the synthesis coefficients G(ω) can be expressed as

$$G_{R,n+1}(\omega) = G_{R,n}(\omega) - \mu(\omega)\frac{1}{\rho_0\omega\Delta}\mathrm{Im}[P_1(\omega)G_2^*(\omega) + G_1(\omega)P_2^*(\omega)] \tag{11}$$

$$G_{I,n+1}(\omega) = G_{I,n}(\omega) - \mu(\omega)\frac{-1}{\rho_0\omega\Delta}\mathrm{Re}[P_1(\omega)G_2^*(\omega) - G_1(\omega)P_2^*(\omega)] \tag{12}$$

In the application, the convergence coefficient should be chosen small enough to maintain stability and the eventual synthesis coefficients are

$$A_n(\omega) = \sqrt{G_{R,n}(\omega)^2 + G_{I,n}(\omega)^2} \qquad \phi_n(\omega) = arctg\frac{G_{I,n}(\omega)}{G_{R,n}(\omega)} \tag{13}$$

As a comparison, when the squared pressure at the location of microphone 1 is selected as the cost function, the update equation for the synthesis coefficients G(ω) is

$$G_{R,n+1}(\omega) = G_{R,n}(\omega) - 2\mu(\omega)\mathrm{Re}[G_1^*(\omega)P_1(\omega)] \tag{14}$$

$$G_{I,n+1}(\omega) = G_{I,n}(\omega) - 2\mu(\omega)\mathrm{Im}[G_1^*(\omega)P_1(\omega)] \tag{15}$$

Putting these together, the update equation for the synthesis coefficients G(ω) becomes

$$G_{n+1}(\omega) = G_n(\omega) - 2\mu(\omega)G_1^*(\omega)P_1(\omega) \tag{16}$$

If the filtered reference signal is included in $G_I(\omega)$, the above algorithm becomes the ordinary frequency domain filtered-x LMS algorithm to minimise the squared pressure.

3. EXAMPLE AND DISCUSSION

The example considered here is the most simple system of all, where a single monopole control source located at (0.05λ,0) is used to attenuate the acoustic radiation from a single monopole primary source located at (-0.05λ,0) in 2-D rectangular coordinates as shown in Figure 1. The distance between them is 0.1λ, which places an absolute bound upon the level of acoustic power attenuation of 9.04dB. The acoustic power attenuation is defined as $10\log_{10}(W_{unc} / W_{cont})$. W_{unc} is the acoustic power of the primary source while W_{cont} is the total acoustic power of the whole system including the primary source and optimum control source. The error sensing point is selected at (0.15λ,0), microphone 1 is located at (0.14λ,0) and microphone 2 is located at (0.16λ,0).

The primary monopole source is oscillating at 343Hz with a volume velocity of 1.0m^3/s. So the sound pressure at the error sensing point is 1034Pa and the mean active sound intensity is 2585Wm^{-2} before active control[1]. After the optimum control using linear quadratic optimisation theory, when the mean active sound intensity is chosen as the cost function the acoustic power attenuation is 5.03dB with a sound pressure and mean active sound intensity at the error sensing point of 410Pa and –409.3Wm^{-2} respectively. However when the cost function is squared sound pressure at that point, the acoustic power attenuation is 3.07dB with a sound pressure and mean active sound intensity at that point of 0.0Pa and 0.0Wm^{-2} respectively. There is no reference sensor in the system because a waveform synthesis method rather than the filtering methods is used.

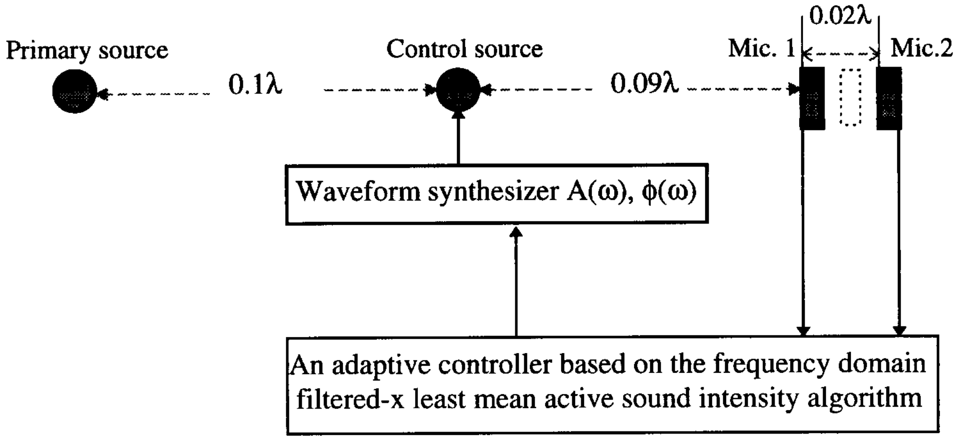

Figure 1. Single primary source, control source, and error sensor system with frequency domain filtered-x type adaptive control based on waveform synthesis.

Adaptive control using the frequency domain filtered-x type adaptive algorithm based on waveform synthesis gives the same results as optimum control using linear quadratic optimisation theory. The optimum synthesis coefficient for minimising mean active sound intensity is $0.54e^{2.89j}$ while for the sound pressure, it is $0.5e^{2.51j}$. In this example, the transfer functions relating the control output to the sound pressure at the error sensors are obtained theoretically as

$$G_1(\omega) = 2300e^{1.0j} \qquad G_2(\omega) = 1880e^{0.88j}$$

Figures 2 and 3 show the evolution of the mean active sound intensity and sound pressure at the error sensing point when each of them is chosen as the cost function respectively for a continuously adapting system using the above algorithm. The initial values for the synthesis coefficients are zero and the convergence coefficient is chosen small enough to maintain stability.

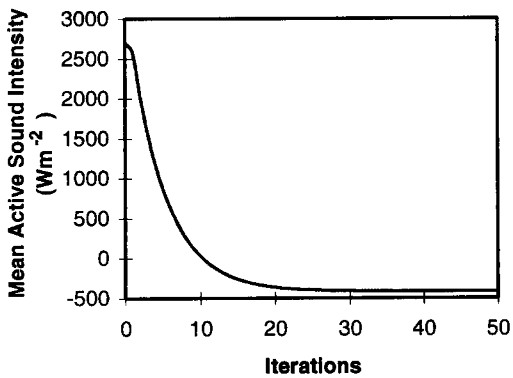

Figure 2. Evolution of mean active sound intensity at the error sensing point for a continuously adapting system to minimise mean active sound intensity.

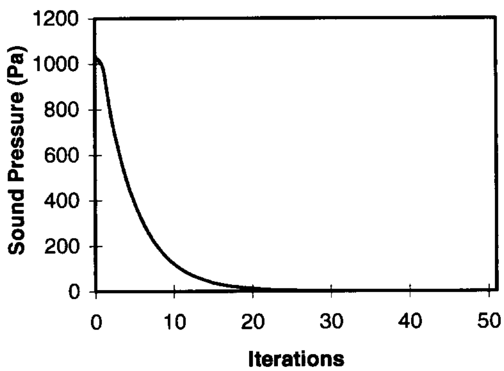

Figure 3. Evolution of sound pressure at the error sensing point for a continuously adapting system to minimise sound pressure.

There are several advantages and disadvantages associated with the frequency domain adaptive sound intensity control[2]. It is apparent that more signal processing is required to implement the frequency domain algorithm and the algorithm becomes a little more complex for minimising the mean active sound intensity rather than sound pressure. But the controller is only ready to control over the frequency to be cancelled, so it has a high rejection of the disturbances and system characteristics outside the range of frequencies being cancelled. Another issue associated with sound intensity control is that typically two highly phase-matched microphones are required if the p-p method is used, which can imply a relatively high cost for the sensors. However low cost microphones can be used to sense transformer noise because of the tonal characteristics of transformer noise, and the amplitude and phase of each stable microphone can be calibrated easily in the frequency domain.

Waveform synthesis adds more advantages to the algorithm[8]. No matter how long the noise propagation path is and how complicated the transfer functions of the noise propagation path are, the algorithm just tries to synthesise the wave according to the amplitude and phase at the frequencies of interest, so the controller will adapt more easily than it were based on time domain filtering methods which use a number of filter taps to model the transfer functions. Because only two coefficients, namely amplitude and phase of the synthesizer, are to be adapted every step for each frequency, the algorithm convergence speed or tracking capability will be faster than for the time domain filtering method, and different optimal convergence coefficients for each frequency can further improve the tracking capability.

The bounds on the convergence coefficient μ for stable operation of the algorithm are identical in form to the bounds placed on the convergence coefficient for stability in the implementation of the standard filtered-x LMS algorithm. The difference is that now the cancellation path transfer function has the major effect on the bound because there is no reference signal for the adaptive controller, which is a synthesizer here.

4. CONCLUSIONS

A frequency domain filtered-x type least mean active sound intensity adaptive control algorithm based on waveform synthesis is developed in this paper for active control of transformer noise. This algorithm is useful for providing global control of transformer noise by minimising the mean active sound intensities for the fundamental frequency and its harmonics in the near acoustic field of the transformer. Mean active sound intensity has been found to be the most effective near field acoustic error sensing strategy for active control of free field sound radiation. The algorithm adjusts the amplitude and phase of each tone to be controlled in the frequency domain and implements the waveform synthesis process in the time domain. The transfer function of the cancellation path is included in the algorithm so that it becomes a filtered-x type adaptive algorithm. In the algorithm, the mean active intensity rather than the time domain instantaneous intensity or squared intensity is used as the cost function.

The algorithm developed in this paper adapts only two parameters, namely amplitude and phase of synthesizer, for each frequency to be cancelled and synthesises the cancellation signal rather than filters a reference signal. The cancellation of each frequency can be optimised independently. The algorithm exhibits very rapid adaptation to change and may be implemented for a relatively low cost. Also the algorithm is set up to control only the frequencies of interest, so it has a high rejection of disturbances and system characteristics outside the range of the frequencies being cancelled. In addition, the Fourier transform operation suppresses random noise relative to the sinusoidal components of the signal which allows the adaptive system to concentrate on the signal's spectral peaks. All of these characteristics are very suitable for the active control of transformer noise.

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