FIFTH INTERNATIONAL CONGRESS ON SOUND AND VIBRATION

DECEMBER 15-18, 1997
ADELAIDE, SOUTH AUSTRALIA

Invited Paper

WEAK SHOCK REFLECTION

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ABSTRACT

We present numerical solutions of an initial value problem for the two-dimensional Burgers equation which models the Mach reflection of weak shocks. These solutions provide evidence of a tiny supersonic bubble behind the triple point. Based on this observation, and on theoretical considerations, we propose that there is a centered expansion fan at the triple point, and that the waves in the supersonic bubble are generated by the reflection of incoming characteristics off an embedded sonic line.

1 INTRODUCTION

The two dimensional Burgers equation,

$$u_t + \left(\frac{1}{2}u^2\right)_x + v_y = 0, \qquad u_y - v_x = 0, \tag{1.1}$$

is a generalization of the familiar one-dimensional inviscid Burgers equation which provides an asymptotic description of the diffraction of weak shock waves in gas dynamics. The independent variables x and y in (1.1) are scaled longitudinal and transverse space variables. The variable u is proportional to the pressure variations and the x-velocity component while v is proportional to the y-velocity component. Equation (1.1) has been derived in several different physical contexts, including transonic flow, the diffraction of acoustic beams, and the focussing of weak shocks. Numerical solutions of this equation show the transition from linear to nonlinear focussing that is observed in weak shock focussing experiments [6].

Weak shock reflection is described by the solution of (1.1) in $y > 0$ with the initial data [5]

$$u(x, y, 0) = \begin{cases} 0 & x > ay \\ 1 & x < ay \end{cases}, \qquad v(x, y, 0) = \begin{cases} 0 & x > ay \\ -a & x < ay \end{cases}. \tag{1.2}$$

The boundary conditions are $v = 0$ on $y = 0$ and $v = 0$ for x sufficiently large and positive.

The incident shock is initially located at $x = ay$. The jump in u across the shock is normalized to one without loss of generality. For a weak gas dynamics shock with Mach number M, incident on a wedge with half-angle α measured in radians, the corresponding value of a is given by

$$a = \frac{1}{2}\left(\frac{\alpha}{\sqrt{M-1}}\right).$$

For example, the numerical solution of the Euler equations shown in Figure 7 of Colella and Henderson [3] has $M = 1.0483$ and $\alpha = 10°$, which corresponds to $a = 0.40$.

The detachment point for the asymptotic problem is $a = \sqrt{2}$, and regular reflection is impossible for values of a smaller than this [5]. The two-dimensional Burgers equation does not admit triple points in which three plane shocks meet at a point [1, 6], so the main problem is to understand the structure of the Mach reflection which appears when $a < \sqrt{2}$.

The shock-reflection problem is self-similar and the solution depends only on the two similarity variables $\xi = x/t$ and $\eta = y/t$. The self-similar equations,

$$-\xi u_\xi - \eta u_\eta + \left(\frac{1}{2}u^2\right)_\xi + v_\eta = 0, \qquad u_\eta - v_\xi = 0, \tag{1.3}$$

change type across the sonic line

$$u(\xi, \eta) = \xi + \frac{\eta^2}{4}. \tag{1.4}$$

In the supersonic, hyperbolic region, equation (1.3) has two families of plus or minus characteristics, whose slopes are given by

$$\frac{d\xi}{d\eta} = -\frac{1}{2}\eta \pm \sqrt{\xi + \frac{1}{4}\eta^2 - u}.$$

2 NUMERICAL SOLUTIONS

In Figures 1–2, we show a numerical solution of (1.1) and (1.2) which gives an overall picture of the Mach reflection for $a = 0.5$. A higher resolution solution of u near the triple point is shown in Figure 3. This solution was cut out from a solution on a uniform 3000×2400 grid. The sonic line is shown as a dotted line, and there is numerical evidence of a tiny supersonic bubble behind the triple point. The height, in y/t, of the bubble is roughly 2% of the height of the Mach stem. The width of the supersonic bubble is several times the width of the reflected shock, so it is not simply an artifact of numerical diffusion. The details of the flow inside the bubble are too small to resolve directly. Thus, we have to interpret the numerical solution in the light of theoretical considerations, which place strong constraints on the possible structure of the solution.

A schematic diagram of a proposed structure is shown in Figure 4. Essentially the same structure was proposed by Guderley [4] for the Mach reflection of weak shocks in steady flows. The incident and reflected shocks both belong to the plus wave family. When they merge, standard shock polar arguments show that they generate a plus Mach shock and a minus centered expansion wave. The characteristics of the waves in the supersonic bubble originate

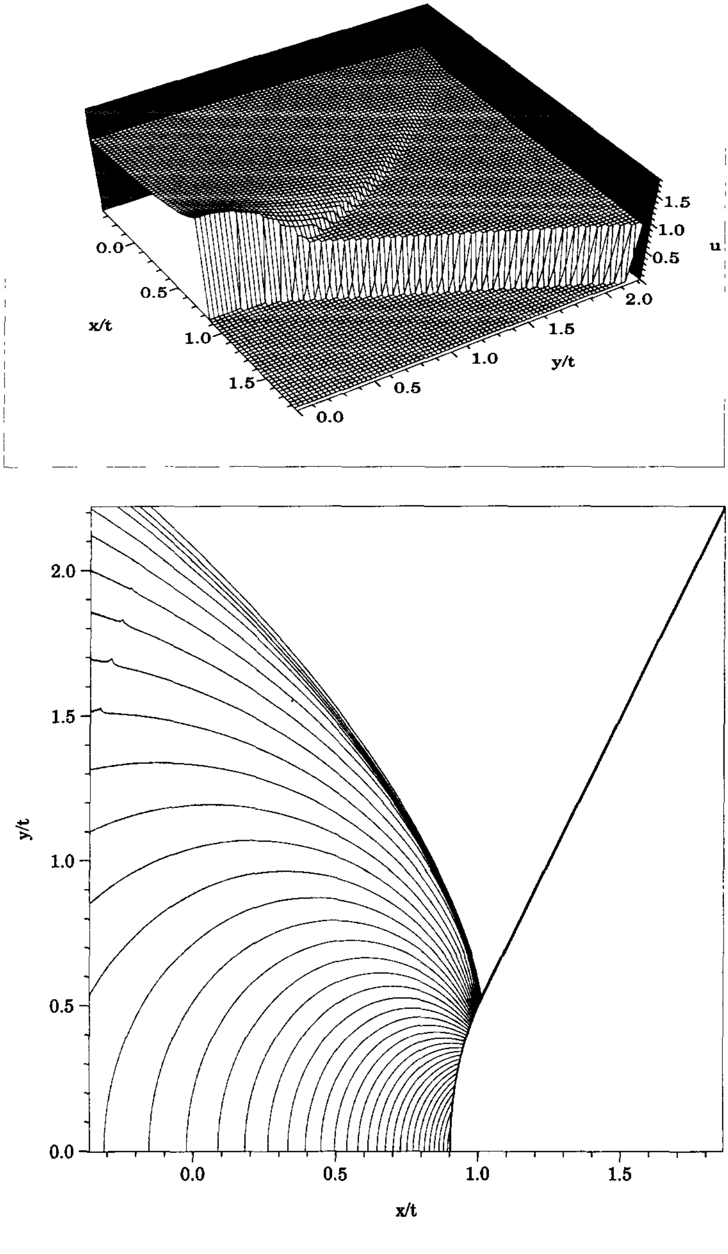

Figure 1: Solution for u when $a = 0.5$.

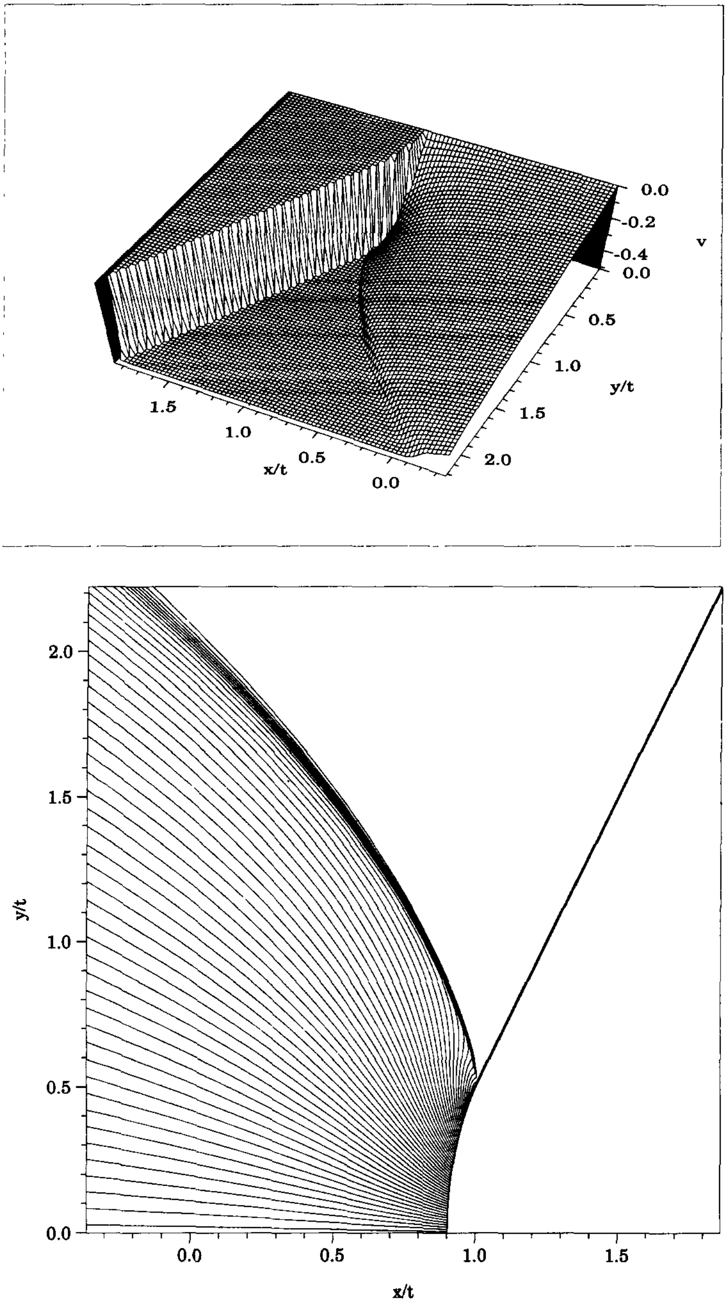

Figure 2: Solution for v when $a = 0.5$.

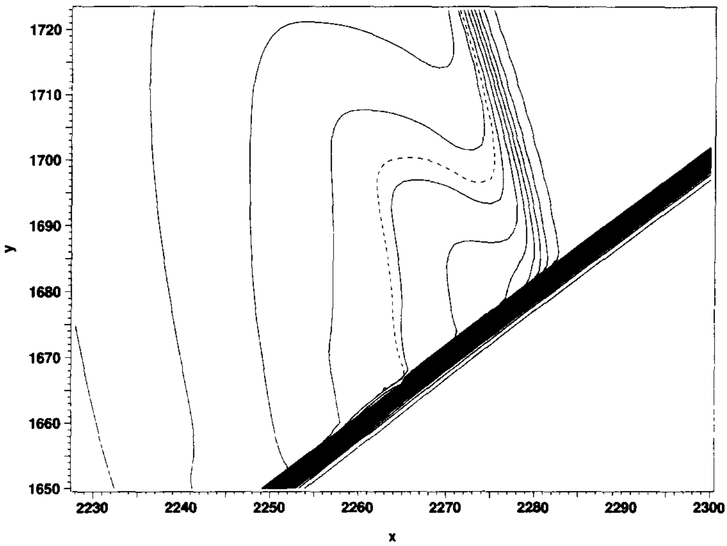

Figure 3: Solution for u near the triple point. The dotted line is the sonic line and the u-contour spacing is 0.01.

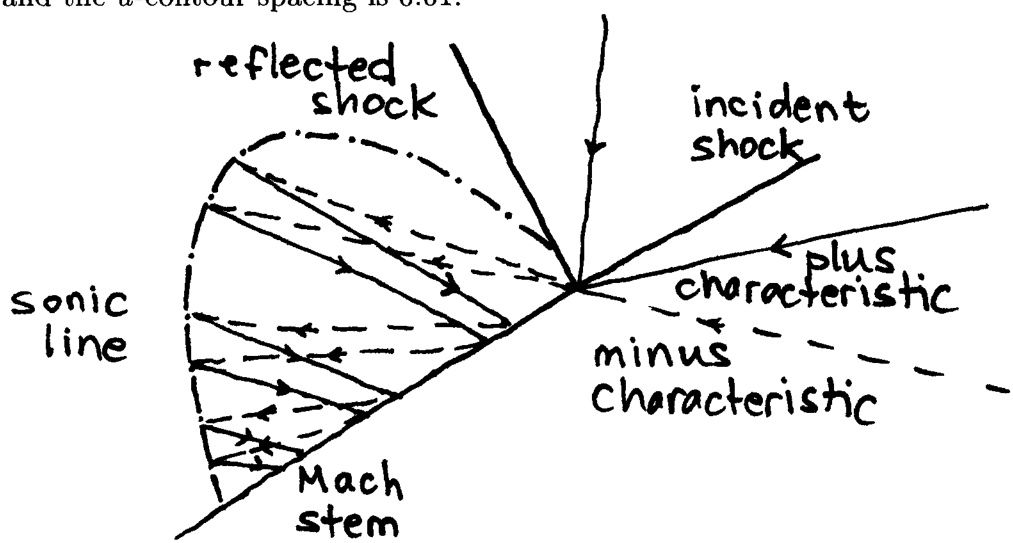

Figure 4: A schematic diagram of the proposed structure of weak shock irregular reflection.

at the sonic line. The structure is therefore consistent with domain of dependence arguments for the self-similar equations in the hyperbolic region, which imply that nonuniform waves or curves in the incident or reflected shocks cannot form in any region where both families of characteristics can be traced back through the hyperbolic region to infinity [6].

The minus expansion fan propagates away from the triple point and is reflected off the sonic line as a plus compression wave [4]. The plus compression wave hits the Mach stem and is reflected as a minus expansion wave. The supersonic bubble therefore contains plus compression waves and minus expansion waves which are multiply reflected between the Mach stem and the sonic line. The plus compression waves are partly absorbed by the Mach stem, so the strength of the stem shock increases as it moves away from the triple point. At the rear sonic point, the stem becomes strong enough that the state behind it changes from supersonic to subsonic.

The generation of a supersonic wave by the reflection of characteristics off a sonic line is well-known in steady transonic flows. However, there are several differences between steady flows and self-similar flows. When the sonic line propagates into a constant state, the characteristics of the self-similar equations are tangent to the sonic line. As a result, all information propagates towards the sonic line, and at first sight it appears that a nonuniform supersonic wave cannot form. However, the tangency of the characteristics to the sonic line is broken when the sonic line is embedded inside a nonuniform wave, so that a supersonic wave can be generated [2].

For the Mach reflection of weak shocks in steady flows, Guderley [4] gave an argument to show that the sonic line must pass exactly through the triple point. The argument does not apply to the self-similar equations, since it depends upon the existence of Riemann invariants, but the numerical results do not rule out this possibility. In fact, the "dip" of the sonic line towards the triple point becomes more pronounced with increasing numerical resolution.

In this structure, the flow behind the reflected shock and the Mach stem is assumed to be continuous. This assumption raises some subtle questions related to the transonic controversy. For steady transonic flows past airfoils, there exist special airfoil shapes which allow shock-free flow at a given free stream Mach number, but the flow is not shock-free if the Mach number is changed or if the shape of the airfoil is perturbed. In the shock reflection problem studied here, the shape of the Mach stem varies with the Mach number of the incident shock. Thus, the Mach stem only has to allow a shock-free transonic flow behind it at a single Mach number and not for a range of Mach numbers. As a result, the possibility of shock-free flow cannot be ruled out theoretically.

A conceivable alternative to shock-free flow behind the Mach stem is that the supersonic bubble is terminated by a shock, as is typically the case for steady transonic flows over an airfoil. However, there would then be two triple points, the front supersonic triple point and the rear subsonic triple point. Since non-singular, subsonic triple points do not exist, the flow velocity would presumably be have to be singular at the rear sonic point, and the whole triple point puzzle would return. These considerations suggest that the flow behind the Mach stem is continuous, but a rigorous demonstration of the existence of shock-free flows behind the Mach stem is likely to be very difficult.

ACKNOWLEDGMENTS. The work of JKH was partially supported by the NSF. We wish to thank Y. Zheng for helpful comments on this work.

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