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### **DIGITAL CONTROL DISCRETE MODELS OF ACTIVE VIBRATION ISOLATION**

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#### **ABSTRACT**

Application of vibration isolation systems with kinematic operation principle is a perspective trend in the development of protection systems against basement vibrations impact. Connections of the object with its basement represent stiff/rigid mechanisms which control relative position of the object and of its basement with help of information obtained from sensors of relative position of the object, its basement and accelerometers too, that are installed both at the object and on its basement. In case of basement motion absence, similar systems are insensitive to dynamic forces applied to the object, and only basement motion excitation sets the object in a relative motion with help of accelerometers. The vibration isolation problem could be achieved by the creation of digital control methods of drive actuation. Here two approaches could be used. The first one is based on invariant control principle. We can create such a mode when we compensate acceleration on the object in such a way that it comes to zero due to the processing of accelerometer signal placed on the basement. The second way consists of traditional methods to produce lower frequency filter. A third idea is to combine in a certain way these two approaches.

#### **INTRODUCTION**

Space technologies implementation goes in parallel to the necessity of devices and technological systems vibration isolation, under space condition. Vibration and shocks appear due to a number of reasons. For example, due to stabilization systems engines operation, various drives of life protection and power supply systems, movements of cosmonauts inside space objects. The vibration affects and in particular in low-frequency and infra-low-frequency spectrum range (0,01-0,1 Hz) violate both some crystal growth processes and microbiology processes. The required levels of vibroaccelerations in the same frequency region are equal to  $10^{-6}$ -  $10^{-5}$  g.

## DIGITAL CONTROL SYNTHESIS

Generalized scheme of the similar system of unidirectional operation consists of sensors of acceleration and of relative displacement, a regulator/controller, a drive mechanism with an engine (Fig.1) [1]. The sensors, the regulator/controller and the drive mechanism can be assembled basing on different physical principles. Depending on external impacts from the basement side it is necessary to choose characteristics of the regulator/controller, in order to provide the active vibration isolation system with the required dynamic properties. In principle, the similar system can exist without usual elastic elements that are typical for vibration isolation devices. Low own frequencies, inattainable in usual vibration isolation systems, are provided owing to regulator characteristics' selection: these frequencies are independent both of dynamic characteristics of protected object and of drive mechanisms with engines. This system also ensures absence of influence on dynamic characteristics of external support connections, such as cables pneumatic and hydraulic systems' manifolds/pipelines, which is of importance for space technology systems.

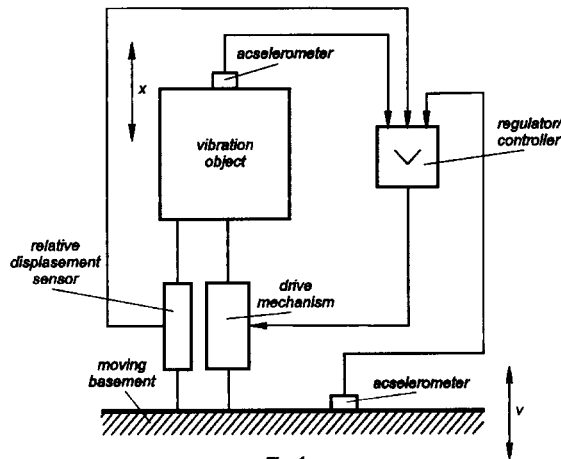


Fig. 1

A method of digital control of the active system of a vibration isolation, based on a combination the principle of invariancy and feedback control, is offered. Is shown, that a zero level of accelerations of protected object in the field of frequencies from zero was provided by the digital methods and formation of appropriate algorithm, and effect that above, than below excitation frequency. Here two approaches could be used. The first one is based on invariant control principle. We can create such a mode when we compensate acceleration on the object in such a way that it comes to zero due to the processing of accelerometer signal placed on the basement. In this case it is necessary to  $\ddot{\delta} = -\ddot{v}$ , but here the limitation of relative displacement requirement is not fulfilled. This approach is rather interesting, but can not be fully consistent. The second way consists of traditional methods to produce lower frequency filter. It is implemented by means of feedback control. It could be both non-complicated feedback control using amplification coefficients choosing for given sensors output data, and optimum feedback control. It can be realised by means of digital control.

We consider the first order filter with the continuous equations (Fig.1)

$$\begin{aligned}\ddot{\delta}(t) + \omega_1 \dot{\delta}(t) &= \omega_1 u(t) \\ \ddot{x}(t) &= \ddot{\delta}(t) + \ddot{v}(t)\end{aligned}$$

where  $\ddot{x}$ ,  $\ddot{v}$  - absolute acceleration of the object and basement,  $\delta$ ,  $\dot{\delta}$ ,  $\ddot{\delta}$  - relative displacement, velocity, acceleration,  $u(t)$  - signal of control,  $\omega_1$  - circular frequency of a cutoff of a filter of the first order. Designate on the time interval  $iT \leq t \leq (i+1)T$  the constant control signal  $u(t)=u(i)$ . Obtain the discrete equations

$$\begin{aligned}\begin{pmatrix} \delta(i+1) \\ \dot{\delta}(i+1) \\ \ddot{\delta}(i+1) \end{pmatrix} &= \begin{pmatrix} 1 & \frac{1}{\omega_1}(1 - e^{-\omega_1 T}) & 0 \\ 0 & e^{-\omega_1 T} & 0 \\ 0 & -\omega_1 e^{-\omega_1 T} & 0 \end{pmatrix} \begin{pmatrix} \delta(i) \\ \dot{\delta}(i) \\ \ddot{\delta}(i) \end{pmatrix} + \begin{pmatrix} T - \frac{1}{\omega_1}(1 - e^{-\omega_1 T}) \\ 1 - e^{-\omega_1 T} \\ \omega_1 e^{-\omega_1 T} \end{pmatrix} u(i)\end{aligned}$$

We added to this equations the output variable equation  $\ddot{x}(i) = \ddot{\delta}(i) + \ddot{v}(i)$   
The obtained system is unstable.

## FEEDBACK DIGITAL CONTROL

Introduce the feedback control  $u(i) = u_1(i) + u_2(i) + u_3(i)$ ,

where  $u_1(i)$  - control from a sensor of relative moving,  $u_2(i)$  - control from a sensor of accelerations of the object,  $u_3(i)$  - control from a sensor of accelerations of the basement.

**1 variant** (the relative displacement feedback)  $u_1(i) = k_1 \delta(i)$ ,  $u_2(i) = u_3(i) = 0$

Designate  $\theta(i) = T\dot{\delta}(i)$ ,  $\chi(i) = T^2\ddot{x}(i)$ ,  $\varphi(i) = T^2\ddot{v}(i)$ ,  $u'(i) = Tu(i)$

Obtain the dimensionless equations

$$\begin{aligned}\delta(i+1) &= (1 + b_1 k_1) \delta(i) + a_{12} \theta(i), \\ \theta(i+1) &= b_2 k_1 \delta(i) + a_{22} \theta(i), \\ \chi(i+1) &= b_3 k_1 \delta(i) + a_{32} \theta(i) + \varphi(i+1)\end{aligned}$$

The system stability is determined by the roots arrangement inside the unit circle on the complex plane. Obtain the two complex conjugate roots.

$$q_{1,2} = \pm j \sqrt{a_{22} + k_1 (b_1 a_{22} - b_2 a_{12})}, \quad k_1 = -\frac{1 + a_{22}}{b_1}$$

**2 variant** (the object acceleration feedback)  $u_2(i) = k_2 \ddot{x}(i)$

Obtain the dimensionless equations

$$\delta(i+1) = (1 + b_1 k'_1) \delta(i) + \left( a_{12} - \frac{b_1 \omega_1 k'_2}{T} \right) \theta(i),$$

$$\theta(i+1) = b_2 k'_1 \delta(i) + \left( a_{22} - \frac{b_2 \omega_1 k'_2}{T} \right) \theta(i),$$

$$\chi(i+1) = b_3 k'_1 \delta(i) + \left( a_{32} - \frac{b_3 \omega_1 k'_2}{T} \right) \theta(i) + \varphi(i+1)$$

The system is two zero roots  $q_{1,2} = 0$  (it is the aperiodic discrete system)

$$k'_1 = -\frac{1}{b_2}, \quad k'_2 = \frac{T(1 + a_{22} + b_1 k'_1)}{b_2 \omega_1}$$

**3 variant.** The invariant control in the combination with the relative displacement and object acceleration feedback control  $u(i) = u_1(i) + u_2(i) + u_3(i)$

We shall define a structure of invariant control by use of the laws of control on feedback. We shall put  $\chi(i) = 0$ , that corresponds to equality to zero of acceleration on the object in each moment of time. We shall enter acceleration on the basement  $\varphi(i)$ . Using z - transformation, we have

$$u_3(i) = b k'_1 \delta(i) + \frac{1}{b_3} \times \left\{ \left( a_{32} - \frac{b_3 \omega_1 k'_2}{T} \right) \theta(i) - \varphi(i+1) - \frac{b_3 k'_2}{T^2} \varphi(i) \right\}$$

## OPTIMAL REGULATOR SYNTHESIS

The optimal regulator is described by matrix equation

$$u(i) = -FX^*(i),$$

where  $F = [f_1 \ f_2 \ f_3]$  - is the feedback coefficients matrix

$X^*(i)^T = [\delta(i) \ \hat{\theta}(i) \ \varphi(i)]$ , - the state coordinates vector

The criterial function is minimized

$$J = \langle \delta^2(i) \rangle + q_2 \langle \dot{\chi}^2(i) \rangle + r \langle u^2(i) \rangle.$$

which is equivalent the standard matrix equation

$$J = \langle X^T(i)QX(i) \rangle + 2 \langle X^T(i)Mu(i) \rangle + \langle Ru^2(i) \rangle,$$

where

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & -\omega_1 T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & q_2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\omega_1 T & 1 \end{bmatrix} - \text{is the weight coefficients matrix;}$$

$$M = q_2 \begin{bmatrix} 0 \\ -\omega_1 T \\ 1 \end{bmatrix}, \quad R = [r + (\omega_1 T)q_2], \quad r \text{ and } q_2 - \text{the weight coefficients.}$$

Determine the feedback coefficients matrix

$$F = (R + B^T P B)^{-1} (B^T P A + M^T),$$

P-matrix is determined from the discrete matrix Ricaty equation

$$P(i+1) = Q + A^T P(i)A - (A^T P(i)B + M) \times (R + B^T P(i)B)^{-1} (B^T P(i)A + M^T)$$

The discrete matrix Ricaty - equation is consisted by the iteration procedure. Thus determine the feedback coefficients the control is

$$u(i) = -f_1(i)\delta(i) - f_2(i)\theta(i) - f_3(i)\varphi(i) + u_3(i),$$

The invariant control by the processing of  $\varphi(i)$  signal is

$$u_3(i) = \frac{1}{b_0} [b_2 u_3(i-2) + b_1 u_3(i-1) + a_2 \varphi(i-2) + a_1 \varphi(i-1) + a_0 \varphi(i)]$$

## RESULTS

The developed digital control method of the active vibration isolation ensures the stability and the vibration isolation effect at any low frequencies ( $\ddot{x} \approx 0$ ). The developed control algorithm can be applied for the wide class another drives for example the electromagnetic suspension

## REFERENCES

1. Rybak L.A, Siniov A.V. "Optimum regulator/controller synthesis of active vibration isolation system based on kinematic operation principle" J. Machine Engineering, No.6, pp.23-30, 1994