inter.noise 2014
MELBOURNE AUSTRALIA
16-19 NOVEMBER

# Approximate formulae for the average one sided specific radiation wave impedance of a finite rectangular panel

John Laurence DAVY[1]; David James LARNER[2]; Robin R. WAREING[3], John R. PEARSE[4]

[1,2] RMIT University, Australia

[3,4] University of Canterbury, New Zealand

## ABSTRACT

The authors have previously published approximate formulae for the average one sided specific radiation wave impedance of a finite rectangular panel mounted in a rigid infinite baffle. The panel's transverse vibration was due to a (possibly forced) two dimensional bending plane wave propagating in the panel without reflection at the edges of the panel. The average was over all the surface area of the panel and over all possible azimuthal angles of propagation direction. The radiation from waves propagating in different directions was assumed to be uncorrelated. These approximate formulae were derived from the 1982 research of Thomasson whose approximate formulae only covered the high and low frequency regions and not the mid frequency region. This paper presents more accurate versions of some of the approximate formulae. When the bending wave number is larger than the wave number of sound, the real part of the impedance is smaller than that for the case studied by Maidanik and Leppington. This is because correlated reflections are not included the case analyzed in this paper. When the bending wave number is smaller than or equals the wave number of sound, the real part of the impedance is the same for both cases.

Keywords: Radiation, Impedance, Rectangle    I-INCE Classification of Subjects Number(s): 23.1

## 1. INTRODUCTION

The average one sided specific radiation wave impedance of a finite rectangular panel mounted in an infinite rigid baffle is of importance for the prediction of sound insulation, sound absorption and sound scattering. If the wave number of the (possibly forced) bending waves in the panel is less than or equal to the wave number of sound in the fluid medium into which the panel is radiating sound energy, the real part of the average specific forced radiation impedance is independent of the vibration pattern of the panel (1). If the wave number of the bending waves is greater than the wave number of sound, the real part of the average specific forced radiation impedance depends on the pattern of vibration (1). This paper assumes that the panel's transverse vibration is due to a (possibly forced) two dimensional bending plane wave propagating in the panel without reflection at the edges of the panel. The average is over all the surface area of the panel and over all possible azimuthal angles of propagation direction. The radiation from waves propagating in different directions is assumed to be uncorrelated.

When the wave number of the (possibly forced) bending waves is less than or equal to the wave number of sound, the authors (2) have derived approximations for the average specific forced radiation impedance. These approximations were an extension of Thomasson's approximations (3). Thomasson (3) published numerical calculations of the average specific forced radiation wave impedance of a square of side length $2e$ for a forcing sound wave number $k$ in half octave steps of $ke$ from 0.25 to 64 and in 15° steps of the incident angle of the forcing sound wave from 0° to 90°. Thomasson (3) also published approximate formulae for values of $ke$ above and below his published numerical results. The authors (2) were able to combine Thomasson's high and low frequency approximations (3) to cover the whole frequency range. The authors (2) showed that the real part of these approximations were not quite as good as those published by Davy (4), but Davy (4) did not give approximations for the

[1] john.davy@rmit.edu.au
[2] david.larner@rmit.edu.au
[3] robin.wareing@pg.canterbury.ac.nz
[4] john.pearse@canterbury.ac.nz

imaginary part or cover the case when the bending wave number is greater than the wave number of sound.

Since writing (2), the authors (5) have extended their approximations to cover the case when the bending wave number is greater than the wave number of the sound in air. Examination of the form of the high frequency approximation for the real part of the impedance when the bending wave number is greater than the wave number of sound has led to a high frequency approximation for the imaginary part of the impedance when the bending wave number is less than the wave length of sound. Combined with Davy's (4) approximation for real part, this new approximation is better than Thomasson's (3) high frequency approximation which is a complex number formula giving both the real and imaginary parts of the impedance.

When the bending wave number is equal to the wave number of sound, Davy's (4) high frequency approximation for the real part and a slight variant of Thomasson's (3) high frequency approximation for the imaginary part are used. These give better approximations than the use of Thomasson's combined complex approximate formula for both real and imaginary parts. The values of the impedance when the bending wave number is equal to the wave number of sound are important because they are used to interpolate values when the bending wave number is immediately above and below the wave number of sound.

This paper presents new approximations for the imaginary part of the average specific radiation impedance. Together with Davy's (4) approximations for the real part, these approximations are better than those previously published by the authors (2, 5).

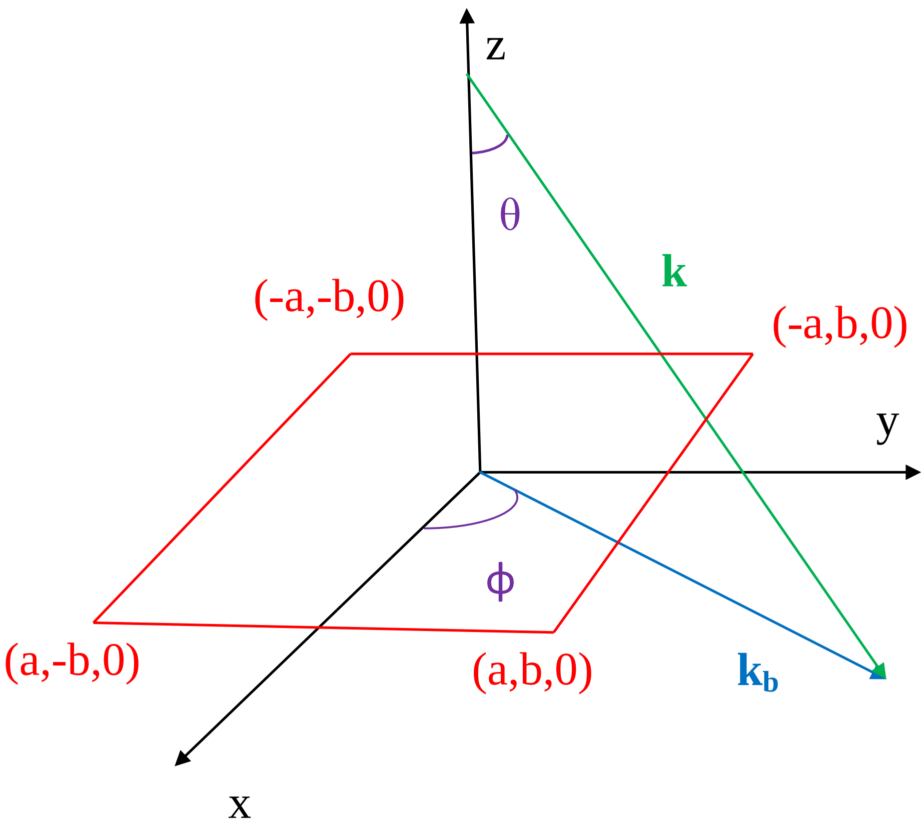


Figure 1. The geometry of the problem considered in this paper. Note that if $|\mathbf{k}_b|$ is greater than $|k|$, $\theta$ does not exist as a real angle.

## 2. NUMERICAL CALCULATIONS

The geometry of the problem considered in this paper is shown in Figure 1. A rectangle, with sides of length $2a$ and $2b$ parallel to the $x$ and $y$ axes respectively, is mounted in an infinite rigid baffle lying in the *x*-*y* plane with the centre of the rectangle at the co-ordinate system origin. A transverse velocity two dimensional plane wave with wave number vector $\mathbf{k}_b$ is propagating in the rectangle at an azimuthal angle $\phi$ to the *x*-axis. If $|\mathbf{k}_b| \le k$, where *k* is the wave number of sound in the surrounding compressible fluid medium on one side of the rectangle, then the wave in the panel may be forced by an incident three dimension plane wave in the surrounding medium of wave number $k = |\mathbf{k}|$ which is incident at an angle of $\theta$ to the normal to the rectangle.

The specific radiation wave impedance is the ratio of the radiated complex number sound pressure at a point on the surface of a radiating panel to the complex number transverse velocity of the panel at the same point. Because the specific radiation wave impedance will vary with position on the finite rectangular panel, the average is taken over the radiating surface of the panel. The specific radiation wave impedance may also vary with the azimuthal angle of propagation of the transverse velocity wave in the finite rectangular panel and the average will also be taken over azimuthal angle. The impedances in this paper are normalized by dividing by the characteristic impedance of the fluid medium.

Figure 2 and Figure 3 show the numerically calculated real part and imaginary part respectively of the normalized surface averaged and azimuthally averaged specific radiation wave impedance as a function of the ratio $\mu$ of the transverse wave number $k_b$ of a square panel of side length 2*e*, mounted in an infinite rigid baffle, to the wave number *k* of sound in the fluid medium into which the panel is radiating. The legend shows the value of *ke*.

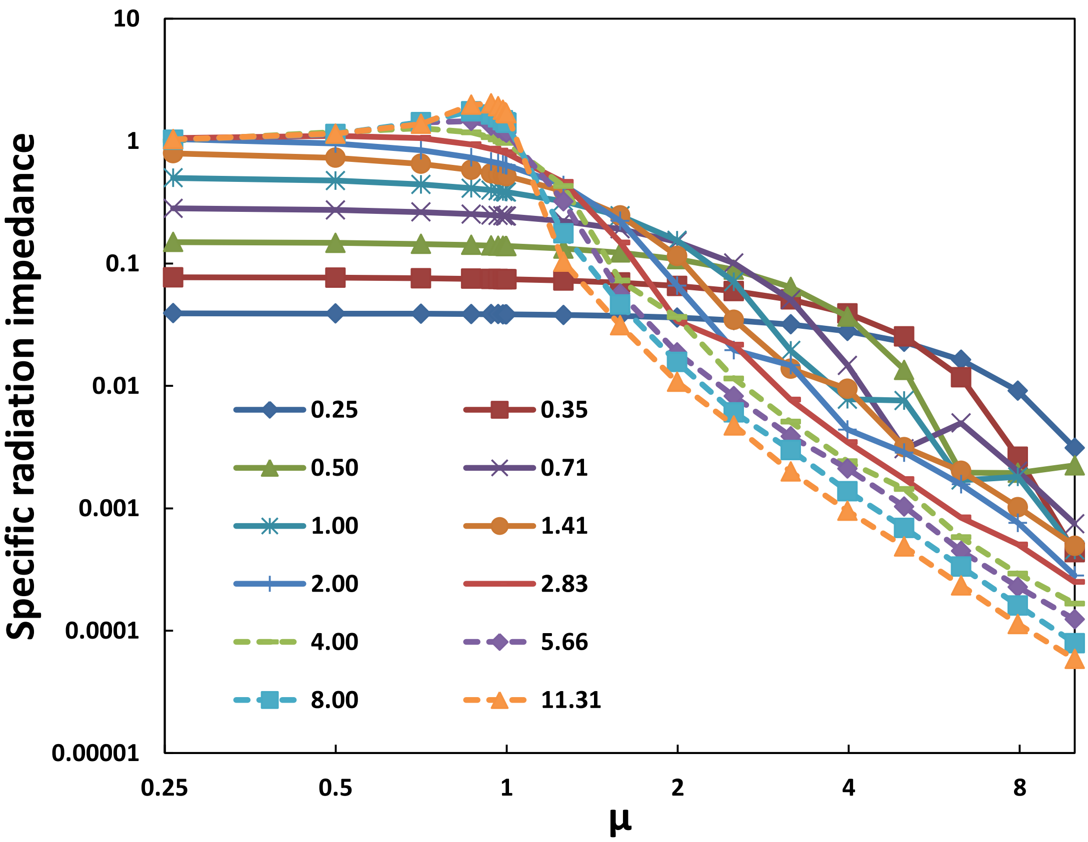


Figure 2. The numerically calculated real part of the normalized surface averaged and azimuthally averaged specific radiation impedance as a function of the ratio $\mu$ of the transverse wave number $k_b$ of a square panel, of side length 2*e* mounted in an infinite rigid baffle, to the wave number *k* of sound in the medium into which the panel is radiating. The legend shows the value of *ke*.

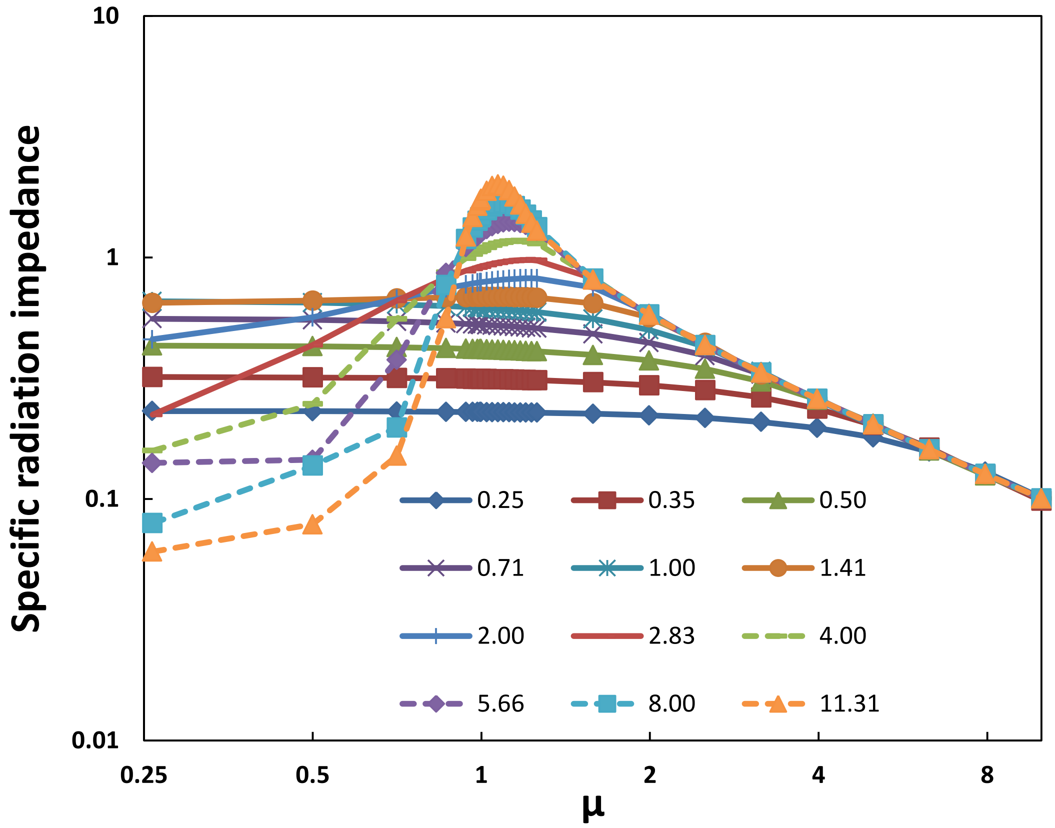


Figure 3. The numerically calculated imaginary part of the normalized surface averaged and azimuthally averaged specific radiation impedance as a function of the ratio $\mu$ of the transverse wave number $k_b$ of a square panel, of side length 2e mounted in an infinite rigid baffle, to the wave number $k$ of sound in the medium into which the panel is radiating. The legend shows the value of $ke$.

## 3. APPROXIMATE FORMULAE

Calculate

$$\mu = \frac{k_b}{k} = \sin(\theta), \tag{1}$$

where the second equality only applies if $k_b \leq k$. $\mu$ is the ratio of $k_b$ and $k$, which are respectively the transverse wave number in the rectangular panel and sound wave number in the surrounding compressible fluid medium into which the panel is radiating. $\theta$ is the angle of incidence of an incoming three dimensional plane wave in the surrounding fluid medium which could generate the transverse wave in the rectangular panel.

Calculate

$$ke = \frac{2kab}{a+b}, \tag{2}$$

where 2$a$ and 2$b$ are the lengths of the sides of the rectangle and 2$e$ is the length of the sides of a equivalent square.

If $\mu \leq 1$, calculate

$$g = \sqrt{1-\mu^2} = \cos(\theta), \tag{3}$$

and

$$p = \sqrt{\frac{\pi}{2ke}}\,. \tag{4}$$

Set $w_r = 1.3$ and $w_i = 0.88$ and calculate $g_x$ where *x* equals *r* and *i*.

$$g_x = \min\left(w_x p, 1\right) \text{ where } x \text{ equals } r \text{ or } i\,. \tag{5}$$

Calculate

$$z_{lr} = \frac{2k^2 ab}{\pi}\,. \tag{6}$$

If $g_r \le g \le 1$, calculate

$$z_{hr} = \frac{1}{g}\,. \tag{7}$$

If $g = 0$ calculate

$$z_{hr} = z_{0hr} = \frac{2}{3p} - 0.124\,. \tag{8}$$

If $0 < g < g_r$ calculate $z_{0hr}$ using Eq. (8) and $z_{mhr}$ using Eq. (7) with $g = g_r$. Interpolate

$$z_{hr} = \frac{\left(g_r - g\right) z_{0hr} + \left(g - 0\right) z_{mhr}}{g_r - 0}\,. \tag{9}$$

Calculate

$$z_r = \frac{1}{\sqrt[3]{\frac{1}{z_{lr}^3} + \frac{1}{z_{hr}^3}}}\,, \tag{10}$$

where $z_{lr}$ is calculated using Eq. (6) and $z_{hr}$ is calculated using Eq. (7), Eq. (8) or Eq. (9).

Calculate

$$z_{li} = \frac{2k}{\pi}\left[bH\left(\frac{a}{b}\right) + aH\left(\frac{b}{a}\right)\right], \tag{11}$$

where

$$H(q) = \ln\left(\sqrt{1+q^2} + q\right) - \frac{\sqrt{1+q^2} - 1}{3q}\,. \tag{12}$$

If $g_i \le g \le 1$, calculate

$$z_{hi} = \frac{2}{\pi k e g^3}\,. \tag{13}$$

Calculate

$$z_i = \frac{1}{\sqrt[4]{\frac{1}{z_{li}^4} + \frac{1}{z_{hi}^4}}}\,. \tag{14}$$

If $g = 0$, calculate

$$z_{0hi} = 0.95\frac{2}{3p} + 0.07\,. \tag{15}$$

Calculate

$$z_i = z_{0i} = \min\left(z_{li}, z_{0hi}\right). \tag{16}$$

If $0 < g < g_i$, calculate $z_{0i}$ using Eq. (15) and Eq. (16), and $z_{mi}$ using Eq. (13) and Eq. (14) with $g = g_i$. Interpolate

$$z_{is} = \frac{\left(g_i - g\right)z_{0i} + \left(g - 0\right)z_{mi}}{g_i - 0}\,. \tag{17}$$

If $ke \geq 8$, calculate

$$z_{1hi} = \frac{0.68}{ke}\,. \tag{18}$$

Calculate

$$z_i = \begin{cases} z_{is} & \text{if } ke < 8 \\ \max\left(z_{is}, z_{1hi}\right) & \text{if } ke \geq 8 \end{cases}. \tag{19}$$

If $\mu \leq 1$, calculate

$$z = z_r + jz_i\,, \tag{20}$$

where $z_r$ is given by Eq. (10) and $z_i$ is given by Eq. (14), Eq. (16) or Eq. (19).

Else if $\mu > 1$, set $h_r = 1.7$ and $h_i = 1.3$ and calculate $\mu_x$ where *x* equals *r* and *i*.

$$\mu_x = \sqrt{1 + \frac{\pi h_x^2}{2ke}} \text{ where } x \text{ equals } r \text{ or } i\,. \tag{21}$$

If $\mu \geq \mu_r$, calculate

$$z_r = \frac{2}{\pi ke\left(\mu^2 - 1\right)^{3/2}}\,. \tag{22}$$

If $1 < \mu < \mu_r$ calculate the real part $z_{mr}$ using Eq. (22) with $\mu = \mu_r$. Calculate the real part $z_{1r}$ as described for the $\mu \leq 1$ case with $\mu = 1$.

Interpolate

$$z_r = \frac{\left(\mu_r - \mu\right)z_{1r} + \left(\mu - 1\right)z_{mr}}{\mu_r - 1}\,. \tag{23}$$

If $ke \leq \sqrt{2}$, calculate the imaginary part $z_{1i}$ as described for the $\mu \leq 1$ case with $\mu = 1$. Calculate

$$z_{mi} = \frac{1}{\sqrt{\mu^2 - 1}} \tag{24}$$

Calculate

$$z_i = \frac{1}{\sqrt[4]{\frac{1}{z_{1i}^4} + \frac{1}{z_{mi}^4}}} \tag{25}$$

Else if $ke > \sqrt{2}$ proceed as follows
If $\mu \geq \mu_i$, calculate

$$z_i = \frac{1}{\sqrt{\mu^2 - 1}}. \tag{26}$$

If $1 < \mu < \mu_i$ calculate the imaginary part $z_{mi}$ using Eq. (26) with $\mu = \mu_i$.
Calculate the imaginary part $z_{1i}$ as described for the $\mu \leq 1$ case with $\mu = 1$.
Interpolate

$$z_i = \frac{(\mu_i - \mu) z_{1i} + (\mu - 1) z_{mi}}{\mu_i - 1}. \tag{27}$$

If $\mu > 1$, calculate

$$z = z_r + jz_i. \tag{28}$$

where $z_r$ is given by Eq. (22) or Eq. (23) and $z_i$ is given by Eq. (25), Eq. (26) or Eq. (27).

The area averaged and azimuthally averaged specific radiation wave impedance of a finite rectangular panel is given by Eq. (20) or Eq. (28).

## 4. ACCURACY

Table 1 to Table 4 show the amount in decibels by which the approximate formulae presented in this paper exceed the numerically calculated values. For the case of the real part when $\mu \leq 1$ shown in Table 1, the average, standard deviation, maximum and minimum of the differences are -0.03, 0.16, 0.32 and -0.61 dB. These values are better than those for the authors' previous approximate formulae of 0.09, 0.31, 0.77 and -0.99 dB. In Table 2 for the imaginary part when $\mu \leq 1$, the values are -0.01, 0.45, 2.11 and -1.56 dB. Again these are better than the values for the authors' previous approximate equations of -0.33, 0.84, 2.33 and -2.64 dB. Table 3 shows the differences for the real part when $\mu \geq 1$ and gives values of -0.02, 1.09, 6.23 and -2.44 dB. The average is better than the authors' previous value of 0.12 and the other three values are the same. The differences for the imaginary part when $\mu \geq 1$ are shown in Table 4. They produce values of 0.02, 0.13, 0.42 and -0.37 dB which are less than or equal to the authors' previous values of 0.02, 0.32, 1.02 and -1.45 dB.

## 5. DISCUSSION

The biggest difficulties with the approximate method occur in the cross over from the low frequency formulae to the high frequency formulae which occurs about $ke$ equals 2. Better agreement is obtained for the real part of the impedance when $\mu$ is less than 1 and for the imaginary part when $\mu$ is greater than 1. For the imaginary part when $\mu$ is less than 1, the main problem is ripple in the impedance as a function of $ke$ when $\mu$ is close to one. For the real part of the impedance when $\mu$ is greater than 1, problems are caused by the ripple that occurs for larger values of $\mu$ and smaller values of $ke$. However as noted in the introduction when $\mu$ is greater than 1, there is a large variation of the real part of the impedance with the vibration pattern of the rectangular panel which depends on the exact edge conditions (1). Thus the calculated values of the impedance when $\mu$ is greater than 1 are only an indication unless the exact boundary conditions are known and used in the calculations.

Table 1. The difference in decibels between the approximate formulae presented in this paper and numerical calculations for the real part of the average specific radiation wave impedance of a square panel of side length $2e$ when $\mu \le 1$.

| $\mu$ / $ke$ | 0.000 0° | 0.259 15° | 0.500 30° | 0.707 45° | 0.866 60° | 0.940 70° | 0.966 75° | 0.985 80° | 0.996 85° | 1.000 90° |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.25 | 0.06 | 0.06 | 0.07 | 0.08 | 0.07 | 0.06 | 0.04 | 0.03 | 0.01 | -0.01 |
| 0.35 | 0.11 | 0.11 | 0.13 | 0.14 | 0.13 | 0.09 | 0.07 | 0.03 | 0.00 | -0.04 |
| 0.50 | 0.19 | 0.20 | 0.22 | 0.23 | 0.19 | 0.12 | 0.07 | 0.01 | -0.06 | -0.14 |
| 0.71 | 0.27 | 0.29 | 0.32 | 0.31 | 0.22 | 0.09 | 0.01 | -0.09 | -0.20 | -0.32 |
| 1.00 | 0.22 | 0.24 | 0.28 | 0.27 | 0.17 | 0.02 | -0.08 | -0.20 | -0.33 | -0.48 |
| 1.41 | -0.18 | -0.14 | -0.03 | 0.08 | 0.10 | 0.03 | -0.03 | -0.13 | -0.25 | -0.40 |
| 2.00 | -0.61 | -0.56 | -0.40 | -0.18 | 0.02 | 0.07 | 0.06 | 0.01 | -0.06 | -0.18 |
| 2.83 | -0.17 | -0.22 | -0.54 | -0.57 | -0.33 | -0.16 | -0.11 | -0.09 | -0.11 | -0.19 |
| 4.00 | 0.26 | 0.09 | -0.15 | -0.37 | -0.34 | -0.16 | -0.08 | -0.05 | -0.06 | -0.14 |
| 5.66 | -0.09 | 0.04 | 0.05 | -0.04 | -0.32 | -0.19 | -0.10 | -0.04 | -0.04 | -0.12 |
| 8.00 | -0.06 | 0.04 | 0.04 | -0.05 | -0.19 | -0.22 | -0.12 | -0.03 | -0.01 | -0.09 |
| 11.31 | -0.05 | -0.01 | 0.01 | 0.10 | 0.04 | -0.22 | -0.15 | -0.04 | 0.02 | -0.06 |
| 16.00 | 0.04 | 0.00 | -0.01 | -0.02 | -0.08 | -0.14 | -0.18 | -0.06 | 0.03 | -0.03 |
| 22.63 | 0.02 | 0.00 | -0.01 | 0.03 | 0.08 | 0.16 | -0.17 | -0.09 | 0.05 | -0.01 |
| 32.00 | 0.01 | 0.00 | 0.01 | 0.01 | 0.01 | -0.09 | -0.03 | -0.13 | 0.04 | 0.02 |
| 45.25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.03 | 0.02 | -0.14 | 0.03 | 0.04 |
| 64.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | -0.08 | -0.06 | 0.00 | 0.06 |

Table 2. The difference in decibels between the approximate formulae presented in this paper and numerical calculations for the imaginary part of the average specific radiation wave impedance of a square panel of side length $2e$ when $\mu \le 1$ .

| $\mu$ / $ke$ | 0.000 0° | 0.259 15° | 0.500 30° | 0.707 45° | 0.866 60° | 0.940 70° | 0.966 75° | 0.985 80° | 0.996 85° | 1.000 90° |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.25 | 0.10 | 0.10 | 0.11 | 0.12 | 0.13 | 0.14 | 0.14 | 0.14 | 0.14 | 0.14 |
| 0.35 | 0.19 | 0.20 | 0.21 | 0.23 | 0.26 | 0.27 | 0.27 | 0.28 | 0.28 | 0.28 |
| 0.50 | 0.36 | 0.36 | 0.35 | 0.32 | 0.28 | 0.23 | 0.20 | 0.17 | 0.14 | 0.10 |
| 0.71 | 0.48 | 0.47 | 0.42 | 0.34 | 0.20 | 0.07 | -0.01 | -0.09 | -0.18 | -0.28 |
| 1.00 | -0.37 | -0.36 | -0.33 | -0.29 | -0.27 | -0.27 | -0.27 | -0.28 | -0.30 | -0.32 |
| 1.41 | -1.56 | -1.16 | -0.67 | -0.62 | -0.49 | -0.37 | -0.30 | -0.24 | -0.17 | -0.10 |
| 2.00 | -1.13 | -1.13 | -0.63 | 0.04 | -0.22 | -0.23 | -0.20 | -0.16 | -0.10 | -0.03 |
| 2.83 | 2.11 | 0.48 | -0.98 | -0.18 | 0.01 | -0.13 | -0.14 | -0.12 | -0.07 | -0.01 |
| 4.00 | -0.73 | 0.47 | -0.04 | -0.92 | 0.44 | 0.02 | -0.06 | -0.08 | -0.05 | 0.02 |
| 5.66 | 1.17 | -0.52 | 0.76 | -0.73 | 0.18 | 0.24 | 0.05 | -0.03 | -0.03 | 0.04 |
| 8.00 | -0.43 | 0.47 | -0.50 | 0.56 | -0.82 | 0.58 | 0.21 | 0.03 | -0.01 | 0.05 |
| 11.31 | -0.05 | 0.13 | 0.42 | 0.23 | -0.95 | 0.61 | 0.45 | 0.11 | 0.00 | 0.04 |
| 16.00 | 0.59 | 0.18 | -0.02 | -0.26 | 0.20 | -0.59 | 0.84 | 0.23 | 0.01 | 0.04 |
| 22.63 | 0.02 | -0.11 | 0.09 | -0.13 | 0.65 | -1.05 | 0.13 | 0.43 | 0.03 | 0.03 |
| 32.00 | 0.11 | 0.03 | -0.06 | 0.10 | -0.53 | -0.27 | -0.87 | 0.75 | 0.08 | 0.02 |
| 45.25 | -0.22 | 0.09 | -0.11 | -0.18 | 0.31 | 0.90 | -0.94 | 0.59 | 0.15 | 0.02 |
| 64.00 | -0.15 | -0.04 | -0.08 | 0.05 | 0.28 | -0.46 | 0.28 | -0.62 | 0.27 | 0.01 |

When the bending wave number is larger than the wave number of sound, the real part of the impedance for the anechoic boundary condition case studied in this paper is smaller than that for the simply supported panel case studied by Maidanik (6, 7) and Leppington *et al*. (8). This is because correlated reflections are not included the case analyzed in this paper. The formulae of Maidanik (6, 7) and Leppington *et al.* (8) for the case when $\mu > 1$ are only valid if $ke(\mu-1) >> 1$. Under this condition, the formula of Leppington *et al.* (8) tends to two times (3 dB greater than) the value given by the formulae presented in this paper when $\mu$ tends to infinity. This is because the value of the term in the formula of Leppington *et al.* (8) which is not used in this paper tends from below to the value of the term in the equation of Leppington *et al.* (8) which is used in this paper when $\mu$ tends to infinity. When the bending wave number is less than or equal to the wave number of sound, the real part of the impedance is the same for both cases.

One advantage with the approximate formulae presented in this paper over the authors' (2, 5) previous approximate formulae is that no complex number operations are needed.

Table 3. The difference in decibels between the approximate formulae presented in this paper and numerical calculations for the real part of the average specific radiation wave impedance of a square panel of side length $2e$ when $\mu \geq 1$.

| $\mu$ <br> $ke$ | 1.00 | 1.26 | 1.58 | 2.00 | 2.51 | 3.16 | 3.98 | 5.01 | 6.31 | 7.94 | 10.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.25 | -0.01 | -0.01 | 0.01 | 0.07 | 0.18 | 0.40 | 0.77 | -0.27 | -1.90 | -2.44 | -0.80 |
| 0.35 | -0.04 | -0.14 | -0.23 | -0.32 | -0.39 | -0.41 | -0.95 | -2.22 | -1.99 | 1.46 | 6.23 |
| 0.50 | -0.14 | -0.28 | -0.44 | -0.62 | -0.84 | -1.35 | -2.19 | -0.99 | 4.29 | 1.26 | -2.38 |
| 0.71 | -0.32 | -0.44 | -0.56 | -0.71 | -1.20 | -1.82 | 0.29 | 3.96 | -1.25 | -0.37 | 0.87 |
| 1.00 | -0.48 | -0.47 | -0.40 | -0.45 | -1.35 | 0.84 | 1.56 | -1.50 | 1.92 | -1.43 | 1.51 |
| 1.41 | -0.40 | -0.18 | 0.10 | -0.64 | 0.26 | 0.83 | -0.80 | 0.81 | -0.35 | -0.45 | -0.33 |
| 2.00 | -0.18 | 0.07 | 0.26 | -0.27 | 1.24 | -0.96 | 1.02 | -0.25 | -0.76 | -0.68 | 0.59 |
| 2.83 | -0.19 | 0.23 | -0.21 | 1.11 | -0.72 | 0.34 | 0.54 | 0.40 | 0.45 | -0.41 | -0.39 |
| 4.00 | -0.14 | 0.52 | 0.73 | -0.73 | 0.54 | 0.64 | 0.61 | -0.29 | 0.57 | 0.47 | -0.12 |
| 5.66 | -0.12 | 0.90 | 0.28 | 0.68 | 0.48 | 0.32 | -0.29 | -0.36 | 0.17 | 0.05 | -0.34 |
| 8.00 | -0.09 | 0.04 | -0.33 | -0.06 | 0.30 | -0.06 | 0.04 | -0.12 | -0.04 | 0.06 | 0.09 |
| 11.31 | -0.06 | 0.86 | -0.13 | 0.07 | -0.14 | 0.19 | 0.15 | -0.10 | -0.03 | 0.07 | -0.11 |

Table 4. The difference in decibels between the approximate formulae presented in this paper and numerical calculations for the imaginary part of the average specific radiation wave impedance of a square panel of side length $2e$ when $\mu \geq 1$.

| $\mu$ <br> $ke$ | 1.00 | 1.26 | 1.58 | 2.00 | 2.51 | 3.16 | 3.98 | 5.01 | 6.31 | 7.94 | 10.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.25 | 0.14 | 0.17 | 0.20 | 0.25 | 0.29 | 0.30 | 0.23 | 0.07 | -0.11 | -0.18 | -0.07 |
| 0.35 | 0.28 | 0.33 | 0.38 | 0.42 | 0.41 | 0.28 | 0.07 | -0.11 | -0.12 | 0.01 | 0.09 |
| 0.50 | 0.10 | 0.18 | 0.26 | 0.29 | 0.20 | 0.02 | -0.11 | -0.08 | 0.04 | 0.05 | -0.01 |
| 0.71 | -0.28 | -0.15 | -0.03 | 0.01 | -0.07 | -0.14 | -0.10 | 0.01 | 0.02 | -0.01 | 0.01 |
| 1.00 | -0.32 | -0.18 | -0.10 | -0.14 | -0.22 | -0.17 | 0.00 | 0.02 | -0.01 | 0.01 | 0.00 |
| 1.41 | -0.10 | -0.14 | -0.25 | -0.37 | -0.26 | 0.01 | -0.01 | -0.01 | 0.00 | 0.01 | 0.01 |
| 2.00 | -0.03 | 0.03 | 0.34 | -0.07 | 0.07 | 0.00 | 0.02 | -0.01 | 0.00 | 0.00 | 0.00 |
| 2.83 | -0.01 | 0.10 | -0.03 | 0.04 | 0.01 | 0.02 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| 4.00 | 0.02 | 0.25 | -0.05 | 0.03 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 5.66 | 0.04 | 0.07 | 0.08 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 8.00 | 0.05 | -0.08 | -0.02 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 11.31 | 0.04 | 0.06 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

## 6. CONCLUSIONS

An approximate method for calculating both the real and the imaginary parts of the single sided normalized specific radiation wave impedance of a finite rectangular panel has been presented. The results of the approximate method have been compared with numerical calculations. For the real part, the approximate method is between 0.32 dB higher and -0.61 dB lower than numerical calculations, when the ratio of the transverse wave number in the panel to the wave number in the medium surrounding the panel $\mu$ is less than or equal to one. For the imaginary part, the approximate method is between 2.11 dB higher and -1.56 dB lower than numerical calculations when $\mu$ is less than or equal to one. For the real part, the approximate method is between 6.23 dB higher and -2.44 dB lower than numerical calculations when $\mu$ is greater than or equal to one. For the imaginary part, the approximate method is between 0.42 dB higher and -0.37 dB lower than the numerical calculations, when $\mu$ is greater than or equal to one. These maxima and minima are less than or equal in magnitude to the maxima and minima obtained by the authors' previous approximate method (2, 5).

## REFERENCES

1. Squicciarini C, Thompson DJ, Corradi R. The effect of different combinations of boundary conditions on the average radiation efficiency of rectangular plates. J Sound Vib. 2014;333(17):3931-48.

2. Davy JL, Larner DJ, Wareing RR, Pearse JR. The specific forced radiation wave impedance of a finite rectangular panel excited by a plane sound wave. The 21st International Congress on Sound and Vibration; 13-17 July 2014; Beijing, China 2014. p. 1-8.

3. Thomasson S-I. Theory and experiments on the sound absorption as function of the area, Report TRITA-TAK-8201. Stockholm: Department of Technical Acoustics, Royal Institute of Technology; 1982. 1-68 p.

4. Davy JL. The forced radiation efficiency of finite size flat panels that are excited by incident sound. J Acoust Soc Am. 2009;126(2):694-702.

5. Davy JL, Larner DJ, Wareing RR, Pearse JR. The average specific forced radiation wave impedance of a finite rectangular panel. J Acoust Soc Am. 2014;136(2):525-36.

6. Maidanik G. Response of ribbed panels to reverberant acoustic fields. J Acoust Soc Am. 1962;34:809-26.

7. Maidanik G. Erratum: Response of ribbed panels to reverberant acoustic fields. J Acoust Soc Am. 1975;57:1552.

8. Leppington FG, Broadbent EG, Heron KH. The acoustic radiation efficiency of rectangular panels. Proc R Soc London, Ser A. 1982;382:245-71.