Parameters design of a nonlinear membrane absorber applied to an acoustic cavity

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ABSTRACT

The targeted energy transfer (TET) phenomenon has been demonstrated and analyzed between an acoustic medium inside a parallelepiped cavity and a thin viscoelastic membrane that is mounted on one wall of the cavity and that is working as a nonlinear absorber or a nonlinear energy sink (NES). Based on the desired working zone of the NES and the two thresholds of the zone which have been obtained, this paper investigates the parameters analysis of a nonlinear membrane absorber to design the NES. The physical parameters of the membrane and the place of the membrane on the wall of the cavity are studied. It can finally provide us to determine where is the better place for the membrane and which parameter affects mainly the desired working zone for the NES.

Keywords: Acoustic cavity, Membrane absorber   I-INCE Classification of Subjects Number(s): 51

1. INTRODUCTION

The concept of targeted energy transfer (TET) was proposed by Vakakis and Gendelman (1, 2), in 2001, which is a new passive technique for reducing noise and vibration. A purely nonlinear absorber is often spoken of the nonlinear energy sink (NES). Firstly, the NES was used in view of applications in the field of mechanical vibrations (3, 4, 5, 6). In acoustic, the TET phenomenon has been firstly observed and analyzed inside one tube (1D acoustic system) by a membrane NES or a loudspeaker nonlinear absorber (7, 8, 9, 10). In (11),the TET phenomenon can be also observed inside an acoustic cavity (3D acoustic system) by a membrane NES. By considering one acoustic mode of 3D acoustic cavity and one membrane NES, the desired working zone for the membrane NES inside 3D acoustic cavity have been defined and the two thresholds of the zone have also been determined analytically and semi-numerically, respectively.

In the paper, a parametric analysis of the membrane absorber is performed by using the system with one acoustic mode of the acoustic cavity and one membrane NES, described in (11). The parameters are the physical parameters of the membrane and the place of the membrane on the wall of the cavity. The results show the influence of each parameter firstly for the nonlinear normal modes (NNMs) and the periodic forced responses of the system, then for the value of the plateau, the desired working zone of the NES and its two thresholds. Finally, we can conclude on where is the better place for the membrane and which parameter affects mainly the desired working zone for the NES.

2. THE TET PHENOMENON OF THE SYSTEM

2.1 Description of the system

The system described in (11) is composed of a primary linear system coupled by a nonlinear system NES as shown in Figure 1. The primary linear system is an acoustic medium inside a parallelepiped cavity with dimensions L_x, L_y and L_z. We assume that all the walls are rigid. The NES is a thin viscoelastic membrane that is mounted on one wall of the cavity. The eigenfrequencies of the acoustic cavity and the acoustic pressure of

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a mode marked by the integers l, m and n are described by the following form:

$$\begin{aligned} f_{lmn} &= \frac{c_0}{2}\sqrt{(\frac{l}{L_x})^2+(\frac{m}{L_y})^2+(\frac{n}{L_z})^2}, \\ p_r(x,y,z,t) &= P_{lmn}(x,y,z)\ p(t) = \cos(\frac{l\pi x}{L_x})\cos(\frac{m\pi y}{L_y})\cos(\frac{n\pi z}{L_z})\ p(t), \end{aligned} \tag{1}$$

where P_{lmn} is the mode shape and $p(t)$ is the acoustic pressure amplitude. The position of the membrane center is defined as (x_m, y_m, z_m), ($x_m = L_x$ in Figure 1).

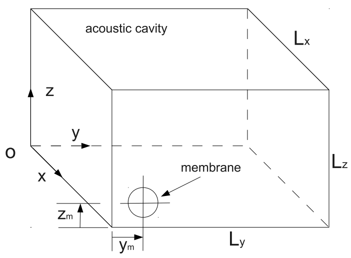


Figure 1 – Schema of the acoustic cavity with a membrane.

We assume that the first few modes of the cavity are separated in frequency, and we focus on the interaction between one mode of the acoustic cavity and the membrane. To analyze the TET phenomenon, we consider a simplified model with two DOFs system: one for the acoustic cavity and another one for the NES. The final system with two DOFs is in the following form:

$$\begin{aligned} m_a\ddot{p}+\lambda\dot{p}+k_a p+\frac{\rho_a^2 c_0^2 s_m}{2}P_{lmn}(x_m,y_m,z_m)\ddot{q} &= f(t), \\ m_m\ddot{q}+k_1\eta\dot{q}+k_3(q^3+2\eta q^2\dot{q})-\frac{s_m}{2}P_{lmn}(x_m,y_m,z_m)p &= 0, \end{aligned} \tag{2}$$

where,

$$\begin{aligned} s_m = \pi R^2, \quad m_m = \frac{\rho_m h s_m}{3}, \quad k_1 = \frac{1.015^4\pi^5}{36}\frac{Eh^3}{(1-\nu^2)R^2}, \\ k_3 = \frac{8\pi Eh}{3(1-\nu^2)R^2}, \quad P_{lmn}(x_m,y_m,z_m) = \cos(\frac{l\pi x_m}{L_x})\cos(\frac{m\pi y_m}{L_y})\cos(\frac{n\pi z_m}{L_z}), \\ V = L_xL_yL_z, \quad m_a = \frac{\rho_a V}{(2-\delta_{l0})(2-\delta_{m0})(2-\delta_{n0})}, \quad k_a = \frac{\rho_a c_0^2 V\pi^2((\frac{l}{L_x})^2+(\frac{m}{L_y})^2+(\frac{n}{L_z})^2)}{(2-\delta_{l0})(2-\delta_{m0})(2-\delta_{n0})}. \end{aligned} \tag{3}$$

and for $\delta_{l0}, \delta_{m0}, \delta_{n0}$,

$$\begin{aligned} l = 0, \delta_{l0} = 1; l \neq 0, \delta_{l0} = 0; \\ m = 0, \delta_{m0} = 1; m \neq 0, \delta_{m0} = 0; \\ n = 0, \delta_{n0} = 1; n \neq 0, \delta_{n0} = 0. \end{aligned} \tag{4}$$

$q(t)$ is the transversal displacement of the membrane center (direction Ox in Figure 1) and $p(t)$ is the acoustic pressure amplitude. The coefficients k_1 and k_3 stand for the linear and nonlinear stiffness, respectively. R and h are the radius and the thickness of the membrane, respectively. E, ν, η and ρ_m are the Young's modulus, the Poisson's ratio, the viscous parameter and the density of the membrane, respectively. λ is a coefficient of a viscous term for the acoustic damping and $f(t)$ is a forcing term on the acoustic cavity.

Notice that the terms of coupling are represented principally by $P_{lmn}(x_m, y_m, z_m)$, where the important factor is the placement of the membrane. i.e., if $P_{lmn}(x_m, y_m, z_m) = 0$, the center of the membrane is on a node of the acoustic mode, then the coupling between the two equations is null.

For convenience, the two equations of the system are divided by the masses m_a and m_m respectively, and the acoustic pressure amplitude $p(t)$ is replaced by the acoustic displacement amplitude $u(t) = \frac{p(t)}{\rho_a c_0 \omega_{lmn}}$, so that the two equation share the same unity, (accelarations, m/s^2). Finally, the system reads

$$\begin{aligned} \ddot{u}+\mu\dot{u}+\omega_{lmn}^2 u+\phi\ddot{q} &= F(t), \\ \ddot{q}+\mu_{m1}\dot{q}+\mu_{m2}q^2\dot{q}+\beta q^3-\gamma u &= 0, \end{aligned} \tag{5}$$

where

$$\begin{gathered}\mu=\frac{\lambda}{m_a},\quad \omega_{lmn}=\pi c_0\sqrt{(\frac{l}{L_x})^2+(\frac{m}{L_y})^2+(\frac{n}{L_z})^2},\\ \phi=\frac{s_m\rho_a c_0}{2m_a\omega_{lmn}}\cos(\frac{l\pi x_m}{L_x})\cos(\frac{m\pi y_m}{L_y})\cos(\frac{n\pi z_m}{L_z}),\quad F(t)=\frac{f(t)}{m_a\rho_a c_0\omega_{lmn}},\\ \mu_{m1}=\frac{k_1\eta}{m_m},\quad \mu_{m2}=\frac{2k_3\eta}{m_m},\quad \beta=\frac{k_3}{m_m},\\ \gamma=\frac{s_m\rho_a c_0\omega_{lmn}}{2m_m}\cos(\frac{l\pi x_m}{L_x})\cos(\frac{m\pi y_m}{L_y})\cos(\frac{n\pi z_m}{L_z}).\end{gathered}\tag{6}$$

For all the parameters, we choose $l=0, m=1, n=0$, the dimensions of the cavity are $L_x=1$ m, $L_y=2.2$ m, $L_z=1.6$ m and the position of the membrane is $x_m=L_x, y_m=\frac{L_y}{6}, z_m=\frac{L_z}{6}$. The values of the membrane and air parameters are: $R=0.04$ m, $h=0.00039$ m, $\eta=0.00062$ s, $E=1.48$ MPa, $\nu=0.49$, $\rho_m=980$ kg m^{-3}, $\rho_a=1.3$ kg m^{-3},$c_0=350$ m s^{-1}.

2.2 NNMs and response to periodic forcing

It's important to look for the NNMs and the periodic forced solutions of the system, because the TET phenomenon is caused by a 1:1 resonance capture (2). A harmonic balance method (HBM) with a single term is applied to analyze the NNMs and the periodic forced responses of the system. For the NNMs, we remove the force $F(t)$ and all the dampings in the system and take the solutions as follows $u(t)=u_{1c}\cos(\omega t)$ and $q(t)=q_{1c}\cos(\omega t)$. The displacements u_{1c} and q_{1c} are obtained as the function of the angular frequency ω:

$$q_{1c}=\sqrt{\frac{4}{3\beta}(\omega^2-\frac{\gamma\phi\omega^2}{\omega^2-\omega_{010}^2})},\quad u_{1c}=-\frac{\phi\omega^2}{\omega^2-\omega_{010}^2}q_{1c}.\tag{7}$$

For the periodic forced responses, since the nonlinear terms of [5] are of the forms q^3 and $q^2\dot{q}$, it is worth to choose a phase condition on $q(t)$ instead of $F(t)$ to get easy calculation. We set $\dot{q}(0)=0$, and take the force $F(t)$ and the displacements $u(t)$ and $q(t)$ in the following form:

$$\begin{aligned}&F(t)=F_{1c}\cos(\omega t)+F_{1s}\sin(\omega t),\\ &u(t)=u_{1c}\cos(\omega t)+u_{1s}\sin(\omega t),\\ &q(t)=q_{1c}\cos(\omega t)\end{aligned}\tag{8}$$

The following four algebraic equations are obtained for the eight unknowns F_{1c}, F_{1s}, F_1, u_{1c}, u_{1s}, u_1, q_{1c} and ω:

$$\begin{aligned}&(\omega_{010}^2-\omega^2)u_{1c}+\mu\omega u_{1s}-\phi\omega^2 q_{1c}=F_{1c},\\ &(\omega_{010}^2-\omega^2)u_{1s}-\mu\omega u_{1c}=F_{1s},\\ &\frac{3}{4}\beta q_{1c}^3-\omega^2 q_{1c}-\gamma u_{1c}=0,\\ &-\frac{1}{4}\mu_{m2}\omega q_{1c}^3-\mu_{m1}\omega q_{1c}-\gamma u_{1s}=0,\\ &F_1=\sqrt{F_{1c}^2+F_{1s}^2},\quad u_1=\sqrt{u_{1c}^2+u_{1s}^2}.\end{aligned}\tag{9}$$

Taking the amplitude q_{1c} and the angular frequency ω as the master parameters, closed form expressions can be easily obtained for the response surfaces.

2.3 The desired working zone of the NES and the two thresholds

Through analyzing the extrema of the curves of periodic forced responses $u(t)$ versus ω for a constant levels of forcing F_1, the desired working zone is determined, where there are exactly three extrema. About the threshold F_b for the beginning of TET, the analytical expression is obtained as following:

$$F_b=\bar{F_1}=\sqrt{(\mu\bar{\omega}\bar{u_{1s}})^2+((\omega_{010}^2-\bar{\omega}^2)\bar{u_{1s}}-\mu\bar{\omega}\bar{u_{1c}})^2}.\tag{10}$$

And the expression of the value of the limited amplitude, namely the value of the plateau, is also obtained:

$$\bar{u_1}=\frac{1}{\gamma}\sqrt{(\frac{3}{4}\beta\bar{q_{1c}}^3-\bar{\omega}^2\bar{q_{1c}})^2+(\frac{1}{4}\mu_{m2}\bar{\omega}\bar{q_{1c}}^3+\mu_{m1}\bar{\omega}\bar{q_{1c}})^2}.\tag{11}$$

where, $\bar{\omega}, \bar{u_{1c}}, \bar{u_{1s}}, \bar{q_{1c}}$ are the values of the limit point solution in (11).

About the threshold F_e for the appearance of undesired periodic regimes is obtained by using a formula of the level of forcing F_1, which is long but which depends only on ω ($\omega<\omega_{010}$):

$$\begin{aligned}&q_{1c}=\sqrt{\frac{4}{3\beta}(\omega^2-\frac{\gamma\phi\omega^2}{\omega^2-\omega_{010}^2})},\quad u_{1c}=\frac{1}{\gamma}(\frac{3}{4}\beta q_{1c}^3-\omega^2 q_{1c}),\quad u_{1s}=-\frac{1}{\gamma}(\frac{1}{4}\mu_{m2}\omega q_{1c}^3+\mu_{m1}\omega q_{1c}),\\ &F_1=\sqrt{((\omega_{010}^2-\omega^2)u_{1c}+\mu\omega u_{1s}-\phi\omega^2 q_{1c})^2+((\omega_{010}^2-\omega^2)u_{1s}-\mu\omega u_{1c})^2}.\end{aligned}\tag{12}$$

3. PARAMETRIC ANALYSIS

Based on these coefficients of the system [5], we choose the analyzed parameters by the physical parameters of the membrane: the radius R, the thickness h and the damping coefficient η and the position of the membrane on the wall of the cavity y_m (here, x_m and z_m can not work because of $l = 0, n = 0$). Then we consider their values like those in the following table:

Table 1 – The values of the parameters R, h, y_m, μ and η.

R(m)	0.04	0.08	0.12
h(m)	0.00018	0.00039	0.00062
y_m	$L_y/6$	$L_y/4$	$L_y/3$
η (s^{-1})	0.0001–0.002		

3.1 Radius of the membrane

For analyzing the influence of the radius of the membrane R, we choose the R values according to Table 1 and fix the other parameters as follows: $h = 0.00039$m, $y_m = L_y/6$, $\mu = 0.014$, $\eta = 0.00062$ s^{-1}. Based on the solutions [7], the NNMs of the system for the three different R are shown in Figure 2, where we can see that in this time the radius of the membrane affect both the shapes of $q(t)$ and $u(t)$. When the value of the parameter R is higher, the maximum point (the limit point) of the second NNM of $u(t)$ and the oblique characteristic value of the NES in (12) will become higher.

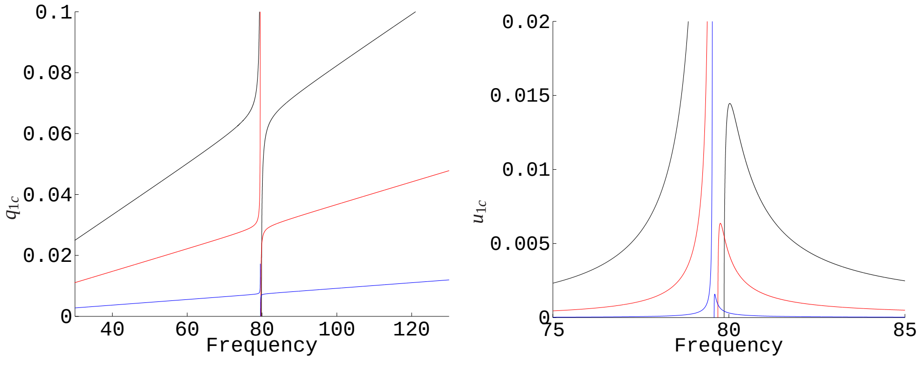


Figure 2 – The NNMs S11+ and S11- of the system for three different R. Blue curve: $R = 0.04$m, red curve: $R = 0.06$m and black curve: $R = 0.08$m.

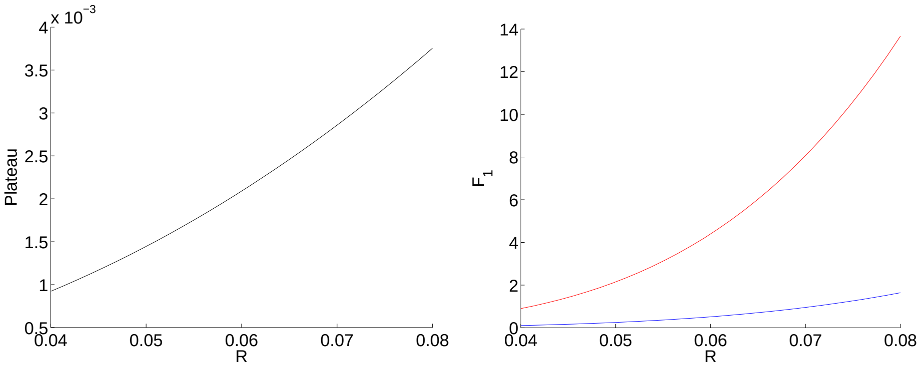


Figure 3 – (a): the amplitude of the plateau. (b): the two thresholds F_b and F_e. Blue curve: F_b, red curve: F_e.

Based on the solutions [9] and the solutions of the extrema in (11), the periodic forced responses of the system with the same levels of forcing F_1 and the location of the extrema of $u(t)$ in the plane (Frequency, F_1)

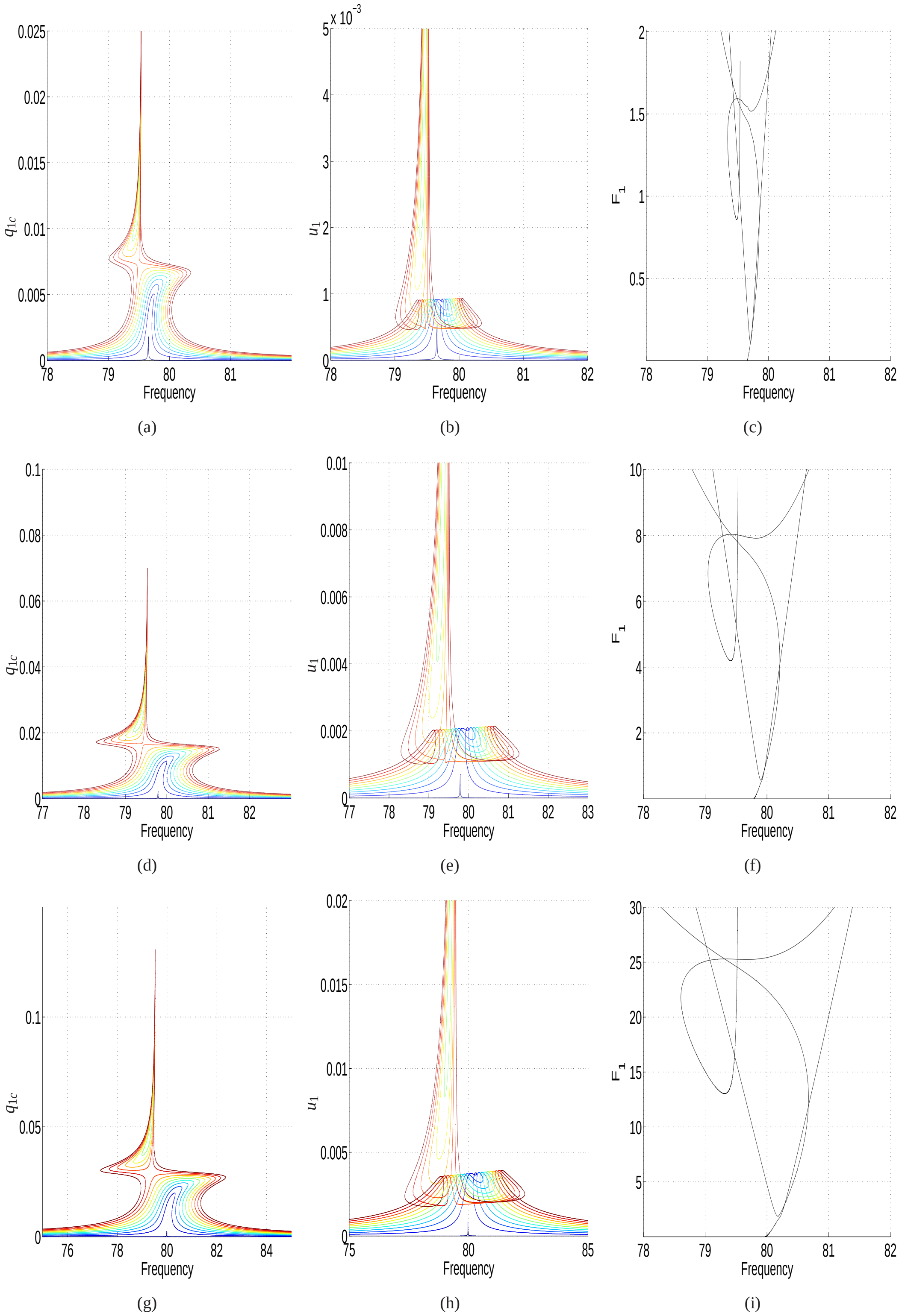


Figure 4 – The periodic forced responses of the system and the location of the extrema of $u(t)$ in the plane (Frequency, F_1). (a)(b)(c): $R = 0.04$m, $F_1 = [0.01 : 0.2 : 2.01]$, (d)(e)(f): $R = 0.06$m, $F_1 = [0.01 : 1 : 10.01]$ and (g)(h)(i): $R = 0.08$m, $F_1 = [0.01 : 3 : 30.01]$.

are shown in Figure 4 for three different R, where we can see that all three membranes for three different R can work. Based on the analytical formulas [10], [11] and [12], the value of the plateau and the two thresholds F_b and F_e according to h are also shown in Figure 3. When the value of the parameter R is larger, the value and the width of the plateau become larger. At the same time, the desired working zone grows much larger, where the two thresholds F_b and F_e become simultaneously larger.

Here, the radius of the membrane can much affect the characteristic of the membrane. Depending on the requirements and the considered applications, different choices can be made, such as, to adjust the value of the plateau to the desired value and also to set the highest level of the source as the threshold F_e.

3.2 Thickness of the membrane

We choose the h values consigned in Table 1 and fix the other parameters as follows: $R = 0.04$m, $y_m = L_y/6$, $\mu = 0.014$, $\eta = 0.00062$ s^{-1}. The NNMs of the system for the three different h are shown in Figure 5. We can see that for $q(t)$, there are not many differences for three different h, while for $u(t)$, the larger the value of the parameter h has, the higher the maximum point of the second NNM becomes.

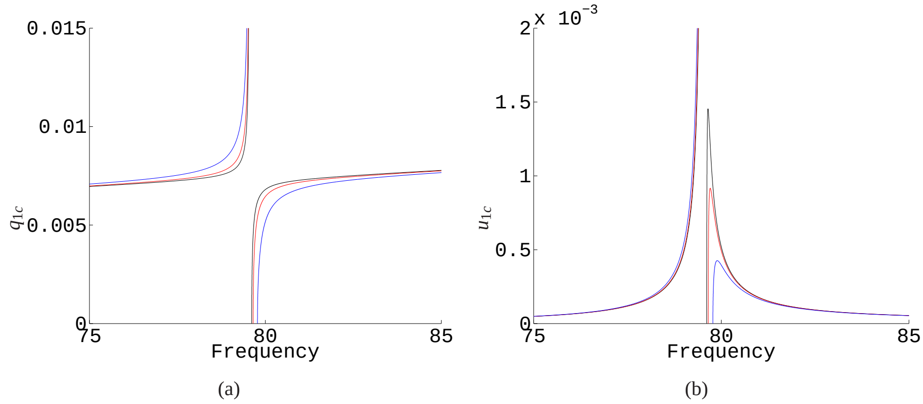


Figure 5 – The NNMs of the system for three different h. Blue curve: $h = 0.00018$m, red curve: $h = 0.00039$m and black curve: $h = 0.00062$m.

The periodic forced responses of the system with the same levels of forcing F_1 and the location of the extrema of $u(t)$ in the plane (Frequency, F_1) for three different h are also performed. And all three membranes can work for three different h. The value of the plateau and the two thresholds F_b and F_e according to h are also obtained, as shown in Figure 6. We can observe that when the thickness of the membrane h is larger, the value of the plateau becomes higher and the width of the plateau for the same level of forcing F_1 becomes more narrow, but the desired working zone between the two thresholds F_b and F_e becomes a little larger, where the threshold F_b does not nearly change and the threshold F_e becomes a little larger.

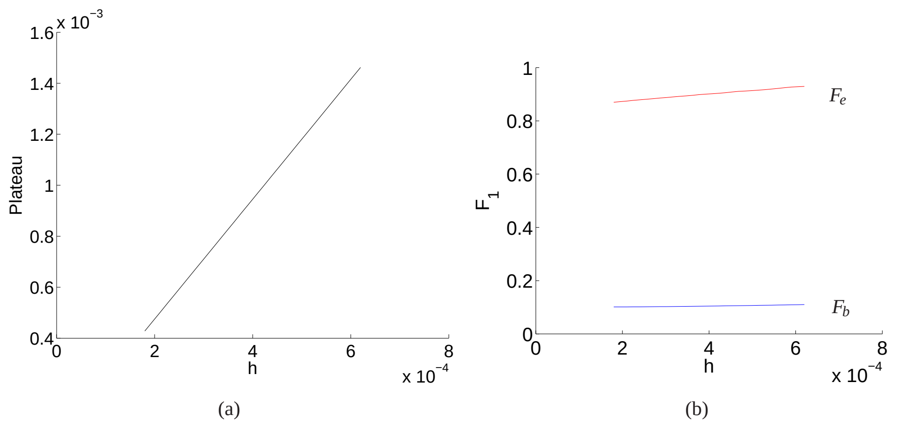
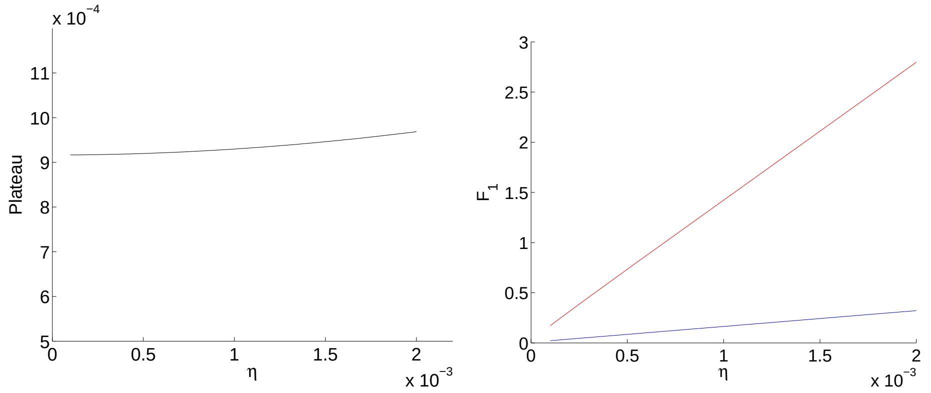


Figure 6 – (a): the amplitude of the plateau. (b): the two thresholds F_b and F_e. Blue curve: F_b, red curve: F_e.

Finally, it is difficult to choose the proper thickness of the membrane h for the TET phenomenon: a small thickness seems best suited to the applications where noise levels remain relatively low, while a large thickness will have one relatively larger desired working zone.

3.3 Damping of the membrane

To analyze the influence of the damping coefficient of the membrane η, we choose the values η in Table 1 and fix the other parameters as follows: $R = 0.04$m, $h = 0.00039$m, $y_m = L_y/6$, $\mu = 0.014$. At first, we assume that when η varies, the membrane can work well, namely that there is always the TET phenomenon. Then, we can apply our method to analyze the damping coefficient of the membrane η. The value of the plateau and the two thresholds F_b and F_e are shown in Figure 7. We can see that the value of the plateau does not change a lot and when the parameter η is larger, the desired working zone for the NES becomes larger where the threshold F_b does not change a lot.



(a) (b)

Figure 7 – (a): the amplitude of the plateau. (b): the two thresholds F_b and F_e. Blue curve: F_b, red curve: F_e.

Then we analyze the periodic forced responses of the system to verify whether the membrane works with three values of η, which is taken by the minimum, middle and maximum η values in the range of $[0.0001, 0.002]$, as shown in Figure 8. We can see that for $\eta = 0.0001$ s^{-1} and $\eta = 0.001$ s^{-1}, there is TET, but TET is more effective for the latter η value. For $\eta = 0.002$ s^{-1}, there is no TET. According to (13), there is a critical value for the nonlinear damping and above the value, no bifurcation of the invariant manifold exists and the quasi-periodic solutions can not be found, namely no TET. Here, we also find that above a certain η value, the TET phenomenon occur no longer.

3.4 Position of the membrane

For the position of the membrane on the wall of cavity in the system [5], only the position y_m in the direction Oy in Figure 1 can affect the TET phenomenon. To analyze the influence of the parameter y_m, we fix the other parameters as follows: $R = 0.04$m, $h = 0.00039$m, $\mu = 0.014$, $\eta = 0.00062$ s^{-1}. We have known that the coupling of the system [5] is determined by the coefficients ϕ and γ, which are according to the cosine function $cos(\frac{\pi y_m}{L_y})$. So, for $y_m = L_y/2$, there is no coupling in the system [5] and the membrane will not work. At this time, the center of the membrane is on the nodal plane ($y_m = L_y/2$, $\forall x_m$, $\forall z_m$) of the mode P_{010}. Therefore, we choose the values y_m according to Table 1. Here, we cite the letter N to present the position of the membrane ($y_m = L_y/N$). For small N, the membrane is close to the nodal plane; for large N, the membrane is far from the nodal plane.

The NNMs of the system for three different N are shown in Figure 9. We can see that for $q(t)$, there are not many differences, while for $u(t)$, the larger N is, the higher the maximum point of the second NNM becomes. The periodic forced responses of the system and the location of the extrema of $u(t)$ in the plane (Frequency, F_1) for three different N are also studied. And the membrane can work for all three positions.

The value of the plateau and the two thresholds F_b and F_e according to N are shown in Figure 10. We can observe that when N is larger, the value of the plateau becomes smaller and the desired working zone for the NES becomes larger (the threshold F_b does not change a lot, while the threshold F_e changes more). Therefore, the larger N is better for the membrane in this example. That is to say, it's better to mount the membrane close to one side of the cavity in direction Oy.

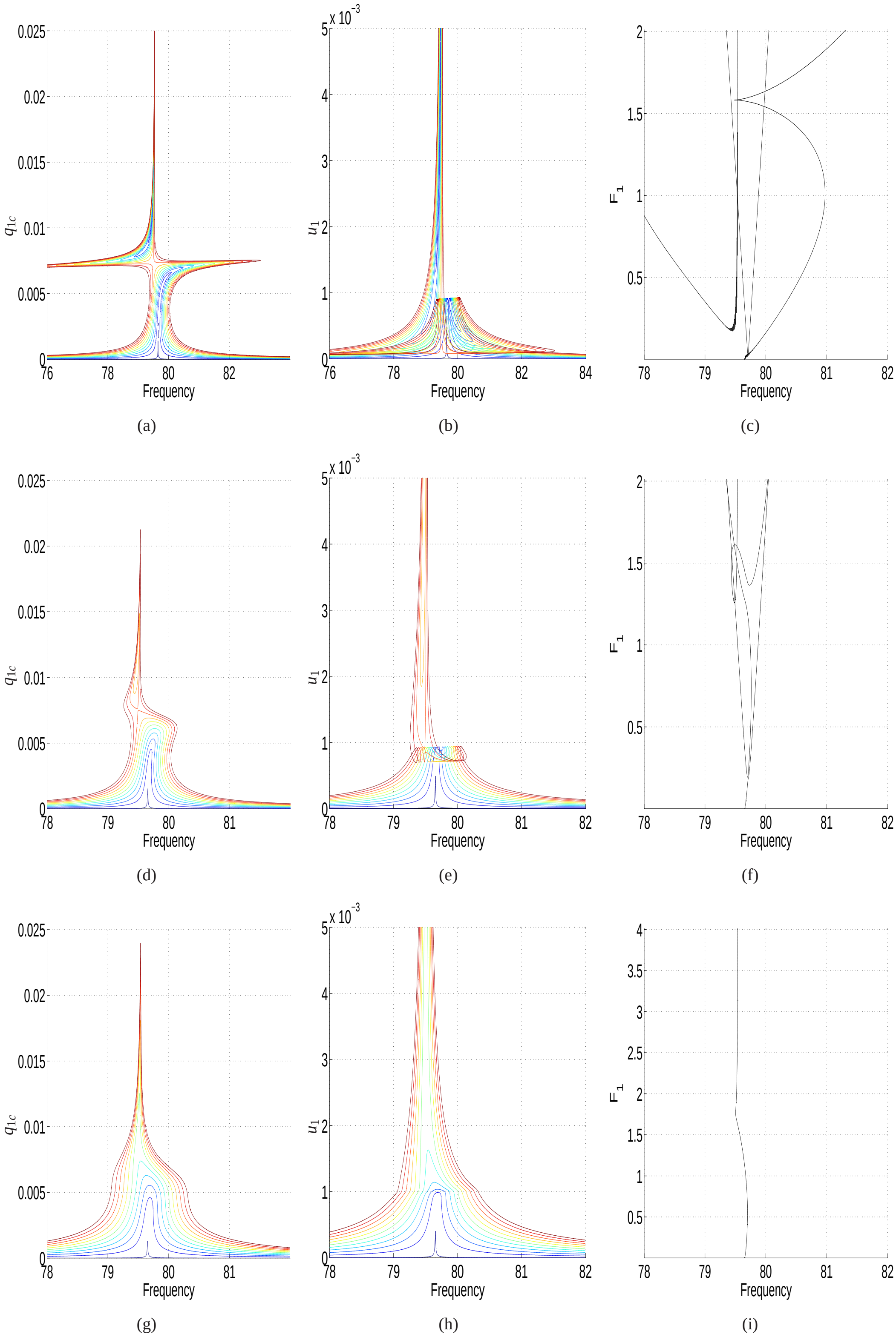
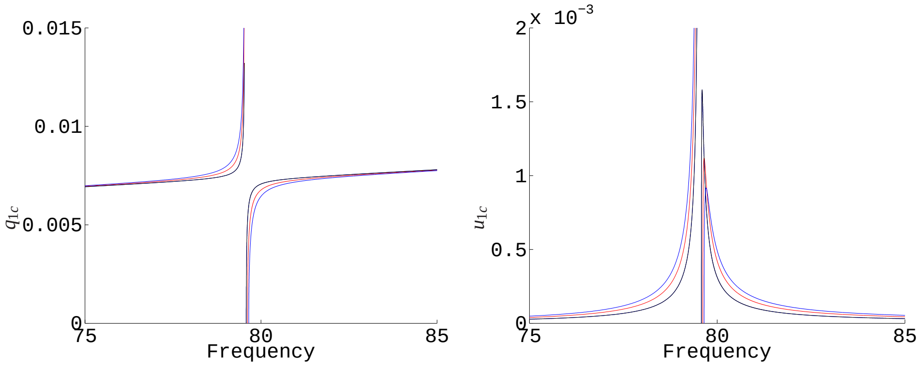
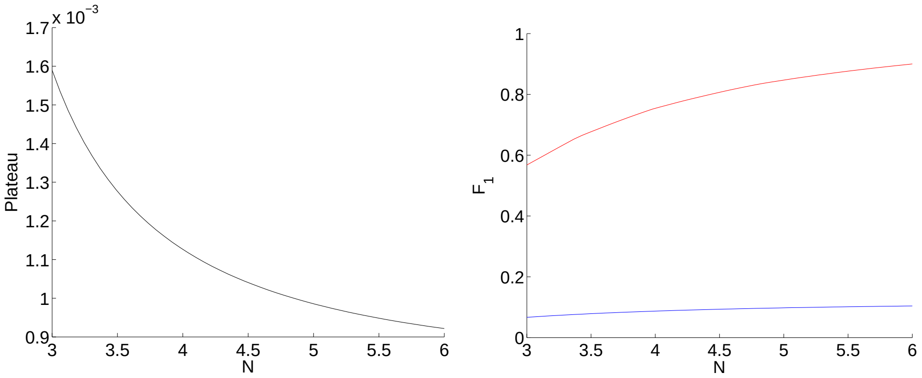


Figure 8 – The periodic forced responses of the system and the location of the extrema of $u(t)$ in the plane (Frequency, F_1). (a)(b)(c): $\eta = 0.0001$, $F_1 = [0.01 : 0.2 : 2.01]$, (d)(e)(f): $\eta = 0.001$, $F_1 = [0.01 : 0.4 : 4.01]$ and (g)(h)(i): $\eta = 0.002$, $F_1 = [0.01 : 0.4 : 4.01]$.



(a) (b)

Figure 9 – The NNMs S11+ and S11- of the system for three different y_m. Blue curve: $y_m = L_y/6$, red curve: $y_m = L_y/4$ and black curve: $y_m = L_y/3$.



(a) (b)

Figure 10 – (a): the amplitude of the plateau. (b): the two thresholds F_b and F_e. Blue curve: F_b, red curve: F_e. N stands for $y_m = L_y/N$.

4. CONCLUSIONS

In the paper, we have performed a parametric analysis of the membrane absorber, which includes the physical parameters of the membrane, the position of the membrane on the wall of the cavity. Among the physical parameters of the membrane, it is the radius of the membrane that has mainly affected the TET phenomenon. For the larger value radius of the membrane, we have got the much larger desired working zone, where the two thresholds F_b and F_e have become simultaneously larger, but at this time we have got the larger value of the plateau. Concerning the thickness of the membrane, it has mainly affected the value of the plateau, the smaller value of the thickness, the smaller value of the plateau. For the damping of the membrane, it has mainly affected the desired working zone: a large value of damping trends to enlarge the desired working zone. But there is a limited value of damping for the TET phenomenon. Indeed according to the particular considered application and its needs, different choices can be made and we can fit the parameters of the membrane in one direction or another to give the TET phenomenon and get the desired effect. Moreover, we have found the better positions for the membrane on the wall of the cavity, where it is better to mount the membrane far from the nodes, nodal lines and nodal planes of the mode.

REFERENCES

1. Gendelman OV, Manevitch LI, Vakakis AF, M'Closkey R. Energy pumping in nonlinear mechanical oscillators: Part I - Dynamics of the underlying hamiltonian systems. Journal of Applied Mechanics. 2001;68:34–41.

2. Vakakis AF, Gendelman OV. Energy pumping in nonlinear mechanical oscillators: Part II - Resonance capture. Journal of Applied Mechanics. 2001;68:42–48.

3. Gendelman OV, Gourdon E, Lamarque CH. Quasiperiodic energy pumping in coupled oscillators under periodic forcing. Journal of Sound and Vibration. 2006;294:651–662.

4. Gendelman OV. Targeted energy transfer in systems with non-polynomial nonlinearity. Journal of Sound and Vibration. 2008;315:732–745.

5. Starosvetsky Y, Gendelman OV. Dynamics of a strongly nonlinear vibration absorber coupled to a harmonically excited two-degree-of-freedom system. Journal of Sound and Vibration. 2008;312:234–256.

6. Starosvetsky Y, Gendelman OV. Vibration absorption in systems with a nonlinear energy sink : Nonlinear pumping. Journal of Sound and Vibration. 2009;324:916–939.

7. Cochelin B, Herzog P, Mattei PO. Experimental evidence of energy pumping in acoustics. C R Mecanique. 2006;334(11):639–644.

8. Bellet R, Cochelin B, Herzog P, Mattei PO. Experimental study of targeted energy transfer from an acoustic system to a nonlinear membrane absorber. Journal of Sound and Vibration. 2010;329:2768–2791.

9. Mariani R, Bellizzi S, Cochelin B, Herzog P, Mattei PO. Toward an adjustable nonlinear low frequency acoustic absorber. Journal of Sound and Vibration. 2011;330:5245–5258.

10. Bellet R, Cochelin B, Côte R, Mattei PO. Enhancing the dynamic range of targeted energy transfer in acoustics using several nonlinear membrane absorbers. Journal of Sound and Vibration. 2012;331:5657–5668.

11. Shao J, Cochelin B. Theoretical and numerical study of targeted energy transfer inside an acoustic cavity by a non-linear membrane absorber. International Journal of Non-Linear Mechanics. 2014;http://dx.doi.org/10.1016/j.ijnonlinmec.2014.04.008i.

12. Shao J, Wu X, Cochelin B. Targeted energy transfer in two degrees-of-freedom linear system coupled by one nonlinear absorber. The 21st International Congress on Sound and Vibration. July, 2014;Beijing, China.

13. Gourdon E, Alexander NA, Taylor CA, Lamarque CH, Pernot S. Nonlinear energy pumping under transient forcing with strongly nonlinear coupling: Theoretical and experimental results. Journal of Sound and Vibration. 2007;300:522–551.