A stability analysis of cluster active control system of sinusoidal sound in free space

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ABSTRACT

Generally, the implementation of fully coupled multichannel active noise control system which has advantages in improving noise reduction and extending quiet zone requires considerable processing power. A practical method to decrease processing power is to decentralize the control; that is, implement many single input, single output independent controllers operating simultaneously instead of a large multiple input, multiple output system. However, the drawback of decentralized control is the risk of global instability. A new method was proposed for constructing an active noise control system to balance the complexity in multichannel systems and instability in the decentralized system. The proposed method consists of several independent multichannel sub-systems called cluster active control system, which is a modified version of the decentralized system and can improve stability at a reasonable computational cost. The purpose of this paper is to derive conditions under which globally stable control system behavior can be obtained in the case of cluster control for a sinusoidal disturbance. Hence, theoretical analysis and simulations are carried out to achieve the main objective which is to give practical conditions derived from the zero-pole map of the control system. These conditions only take into account the geometrical arrangement of the secondary sources and error sensors.

Keywords: active noise control, cluster active control; free space

1. INTRODUCTION

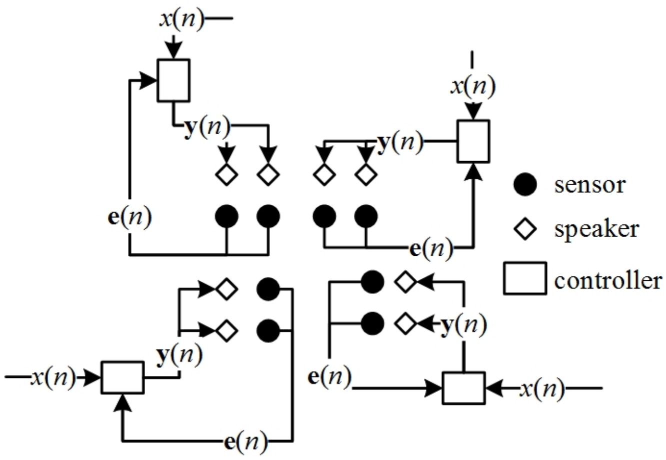

Figure 1 – Diagram of cluster active control system

Generally, multichannel active noise control system uses signals from all error sensors and the secondary path transfer functions between all secondary sources and all error sensors are required to adjust the output of each secondary source, which is called fully coupled control system or centralized control system (1). Such multichannel system has been extensively used to improve

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noise reduction and extend quiet zone (2,3). However, when the number of secondary sources and error sensors becomes large, the complexity of centralized controllers makes the implementation of such a control strategy difficult or expensive. To reduce the complexity, the decentralized strategy has been used recently, where each independent single channel controller is designed based on its own secondary path and the error sensor (4) but not take into account the acoustic coupling from other secondary sources, so it has benefits of simple design and flexible hardware. However, there might be a performance loss and instability due to neglect the interactions for the acoustic coupling.

Recently, in order to alleviate the contradiction between the complexity and instability, a new strategy called cluster system whose structure is in between centralized and decentralized is proposed, shown as Figure 1. A cluster system is also centralized system which contains a subset of secondary sources and error sensors. In this study, stability of the cluster system aimed at tonal noise in free space is investigated by analyzing the equivalent transfer function to get the principle of geometrical arrangement of the secondary sources and error sensors.

2. THEORY

2.1 Algorithm Implement

For the simplification of the problem, consider a cluster active control system which has I sub-systems. The number of error sensors and secondary loudspeakers of each individual subsystem is N. The system is implemented by FxLMS algorithm. Obviously, such a cluster system can be degenerated into the decentralized system when N is equal to one. In order to keep the analysis relatively straightforward, we will assume that the primary disturbances of each error sensors are sinusoidal which is related to a single reference signal by a primary plant model and that all subsystems use the same reference signal, shown as Equation (1), where ω is digital frequency, n is sampling index.

$$x = \cos \omega n \tag{1}$$

The number of sensors and loudspeakers of entire system is NI, the secondary path response from the jth loudspeaker to the kth sensor shown as Equation (2).

$$S_{j,k}(e^{i\omega}) = A_{j,k} e^{i\varphi_{j,k}} \tag{2}$$

So that the relevant filtered-x signal shown as Equation (3) is given by

$$r_{j,k}(n) = A_{j,k} \cos(\omega n + \varphi_{j,k}) \tag{3}$$

Assuming each subsystem implements simultaneously with same adaptive step which is small enough and the length of adaptive filter is L. The error equation and update equation can be written as follow

$$\mathbf{e}(n) = \mathbf{d}(n) + \mathbf{R}(n)^T \mathbf{w}(n) \tag{4}$$

$$\mathbf{w}(n+1) = \mathbf{w}(n) - 2\mu \hat{\mathbf{R}}(n)\mathbf{e}(n) \tag{5}$$

where $\mathbf{e}(n)$ is the error vector, $\mathbf{d}(n)$ is the primary sound vector and μ is an adaptive step. $\mathbf{R}(n)$ and $\hat{\mathbf{R}}(n)$ are the filtered-x signal matrixes shown as follow, they are both block matrixes and $\hat{\mathbf{R}}(n)$ is a diagonal block matrix.

$$\mathbf{R}(n) = \begin{pmatrix} \mathbf{R}_{1,1}(n) & \cdots & \mathbf{R}_{1,I}(n) \\ \vdots & \ddots & \vdots \\ \mathbf{R}_{I,1}(n) & \cdots & \mathbf{R}_{I,I}(n) \end{pmatrix}_{LIN \times IN} \tag{6}$$

$$\hat{\mathbf{R}}(n) = \begin{pmatrix} \mathbf{R}_{1,1}(n) & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{2,2}(n) & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{R}_{I,I}(n) \end{pmatrix}_{LIN \times IN} \tag{7}$$

$\mathbf{R}_{p,q}(n)$ in Equation (6) and Equation (7) represents that the filtered-x signals in the matrix are relevant about the path from the loudspeakers of the pth subsystem to the sensors of the qth subsystem, show as Equation (8).

$$\mathbf{R}_{p,q}(n)=\begin{pmatrix}\mathbf{r}_{(p-1)N+1,(q-1)N+1}(n) & \cdots & \mathbf{r}_{(p-1)N+1,qN}(n)\\ \vdots & \ddots & \vdots \\ \mathbf{r}_{pN,(q-1)N+1}(n) & \cdots & \mathbf{r}_{pN,qN}(n)\end{pmatrix}_{LN\times N} \tag{8}$$

$$\mathbf{r}_{j,k}(n)=\left(r_{j,k}(n) \quad \cdots \quad r_{j,k}(n-L+1)\right)^{T} \tag{9}$$

$\mathbf{w}(n)$ is the vector of adaptive filter coefficients given by

$$\mathbf{w}(n)=\left(\mathbf{w}_1(n)^T \quad \cdots \quad \mathbf{w}_I(n)^T\right)^T \tag{10}$$

$$\mathbf{w}_p(n)=\left(\mathbf{w}'_{(p-1)N+1}(n)^T \quad \cdots \quad \mathbf{w}'_{pN}(n)^T\right)^T \tag{11}$$

$$\mathbf{w}'_j(n)=\left(w_{j,0}(n) \quad \cdots \quad w_{j,L-1}(n)\right)^T \tag{12}$$

where $w_{j,l}(n)$ in Equation (12) is the lth coefficient of the adaptive filter connect to the jth loudspeakers. $w_{(p-1)N+j,l}(n)$ represent the lth coefficient of the jth filter in the pth subsystem. Thus the update equation of $w_{(p-1)N+j,l}(n)$ can be written as Equation (13) according to Equation (5).

$$w_{(p-1)N+j,l}(n+1)=w_{(p-1)N+j,l}(n+1)-2\mu\sum_{k=1}^{N} r_{(p-1)N+j,(p-1)N+k}(n-l)e_{(p-1)N+k}(n) \tag{13}$$

2.2 Equivalent Transfer Function

The Z-transform of Equation (4) is shown as

$$\mathbf{E}(z)=\mathbf{D}(z)+\frac{1}{2}\mathbf{S}^T\sum_{l=0}^{L-1}e^{-i\omega l}\mathbf{W}_l(e^{-i\omega}z)+\frac{1}{2}\mathbf{S}^H\sum_{l=0}^{L-1}e^{i\omega l}\mathbf{W}_l(e^{i\omega}z) \tag{14}$$

In this equation, the superscript H represenst Hermitian conjugate and $\mathbf{S}$ is the secondary path response matrix given by

$$\mathbf{S}=\begin{pmatrix}\mathbf{S}(1,1) & \cdots & \mathbf{S}(1,I)\\ \vdots & \ddots & \vdots\\ \mathbf{S}(I,1) & \cdots & \mathbf{S}(I,I)\end{pmatrix}_{IN\times IN} \tag{15}$$

$$\mathbf{S}(p,q)=\begin{pmatrix}S_{(p-1)N+1,(q-1)N+1} & \cdots & S_{(p-1)N+1,qN}\\ \vdots & \ddots & \vdots\\ S_{pN,(q-1)N+1} & \cdots & S_{pN,qN}\end{pmatrix}_{N\times N} \tag{16}$$

where $\mathbf{S}(p,q)$ represents the response matrix from the loudspeakers in the pth subsystem to the sensors in the qth subsystem.

In Equation (14), $\mathbf{W}_l(z)$ is shown as Equation (17) where $W_{j,l}(z)$ is the Z-transform of $w_{j,l}(n)$, $\mathbf{W}_l(z)$ can be derived from the Z-transform of Equation (13), shown as Equation (18).

$$\mathbf{W}_l(z)=\left(W_{1,l}(z) \quad \cdots \quad W_{IN,l}(z)\right)^T \tag{17}$$

$$\mathbf{W}_l(z)=-\mu U(z)[e^{-i\omega l}\hat{\mathbf{S}}\mathbf{E}(e^{-i\omega}z)+e^{i\omega l}\overline{\hat{\mathbf{S}}}\mathbf{E}(e^{i\omega}z)] \tag{18}$$

$$U(z)=\frac{1}{z-1} \tag{19}$$

In Equation (15), $\hat{\mathbf{S}}$ is another version response matrix of the secondary path which only contains the path of each subsystem itself but ignores the influence of the other subsystem, i.e.

$$\hat{\mathbf{S}}=\begin{pmatrix} \mathbf{S}(1,1) & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{S}(2,2) & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{S}(I,I) \end{pmatrix}_{IN\times IN} \tag{20}$$

Combining Equation (14) with Equation (18), we get the Z domain formula of entire systemas follows

$$\begin{aligned}\mathbf{E}(z)=\mathbf{D}(z)-\frac{1}{2}\mu U(e^{-i\omega}z)\mathbf{S}^{T}[L\bar{\hat{\mathbf{S}}}\mathbf{E}(z)+\hat{\mathbf{S}}\mathbf{E}(e^{-i2\omega}z)\sum_{l=0}^{L-1}e^{-i2\omega l}] \\ -\frac{1}{2}\mu U(e^{i\omega}z)\mathbf{S}^{H}[L\hat{\mathbf{S}}\mathbf{E}(z)+\bar{\hat{\mathbf{S}}}\mathbf{E}(e^{i2\omega}z)\sum_{l=0}^{L-1}e^{i2\omega l}]\end{aligned} \tag{21}$$

The summation parts of Equation (21), shown as Equation (22), contains the factor $\mathbf{E}(e^{\pm i2\omega}z)$ make the system transfer function unobtainable directly. So we further assume that the reference signal is synchronously sampled and L sample points just samples several integral periods of the reference signal (5). Under this premise, $L\omega$ equals to the integral multiples of π, thus Equation (22) equal to zero.

$$\sum_{l=0}^{L-1}e^{\pm i2\omega l}=\frac{1-e^{\pm i2L\omega}}{1-e^{i2\omega}} \tag{22}$$

We eliminate $\mathbf{E}(e^{\pm i2\omega}z)$ from Equation (21), the system can be rewritten as follows

$$\mathbf{E}(z)=\mathbf{D}(z)-\beta[U(e^{-i\omega}z)\mathbf{S}^{T}\bar{\hat{\mathbf{S}}}+U(e^{i\omega}z)\mathbf{S}^{H}\hat{\mathbf{S}}]\mathbf{E}(z) \tag{23}$$

$$\beta=\frac{1}{2}\mu L \tag{24}$$

where β is named normalized adaptive step. Substituting Equation (19) into Equation (23), the equivalent transfer function can be obtained as follows

$$\mathbf{H}(z)=\frac{\mathbf{E}(z)}{\mathbf{D}(z)}=[\mathbf{I}+\beta\mathbf{G}(z)]^{-1} \tag{25}$$

$$\mathbf{G}(z)=\frac{\mathbf{S}^{T}\bar{\hat{\mathbf{S}}}(e^{i\omega}z-1)+\mathbf{S}^{H}\hat{\mathbf{S}}(e^{-i\omega}z-1)}{z^{2}-2\cos\omega z+1} \tag{26}$$

where the symbol $\bar{\hat{\mathbf{S}}}$ represents the conjugation of $\hat{\mathbf{S}}$, superscript T represent transposition, $\mathbf{I}$ is identity matrix. $\mathbf{H}(z)$ is the transfer function matrix with size of $NI\times NI$, shows that the each individual error signal is relevant to all primary signals because of the interactions of other loudspeakers.

In addition, notice that Equation (25) represents a multi-channel feedback system, the open-loop transfer function matrix is $\mathbf{G}$(z) and the feedback gain is β.

3. Simulation of Simple Physical Condition

3.1 Single Small Subsystem

In this section, we consider a simple physical condition of a cluster active control system, operating in free space, to obtain the principle of system geometrical layout. Firstly, we begin with one small subsystem with two loudspeakers and two sensors which is a centralized system exactly. The positions of the loudspeakers and sensors are shown as Figure 2. The loudspeakers and sensors are placed in the corner of a square with its size of 1. Notice that the size of 1 is normalized length.

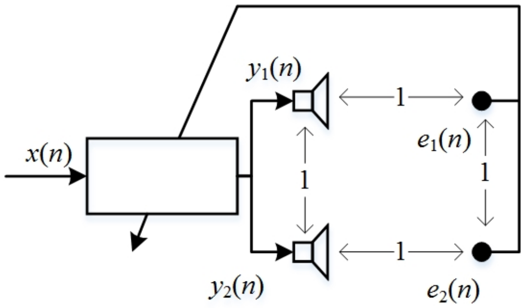


Figure 2 – Two-channel small subsystem of cluster active control system

If we further assume that the loudspeakers and sensors both have the electronic response of 1 normalized unit. Thus we obtained the secondary path response matrix in free space

$$\mathbf{S} = \hat{\mathbf{S}} = \mathbf{S}(1,1) = \begin{pmatrix} e^{-ik} & e^{-i\sqrt{2}k}/\sqrt{2} \\ e^{-i\sqrt{2}k}/\sqrt{2} & e^{-ik} \end{pmatrix} \tag{26}$$

where k is the acoustic wavenumber, equal to the angular frequency divided by the sound speed, and should not be confused with the iteration number.

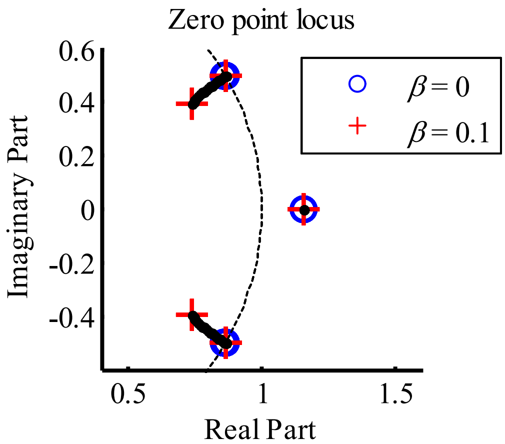


Figure 3 – Zero point locus of single subsystem

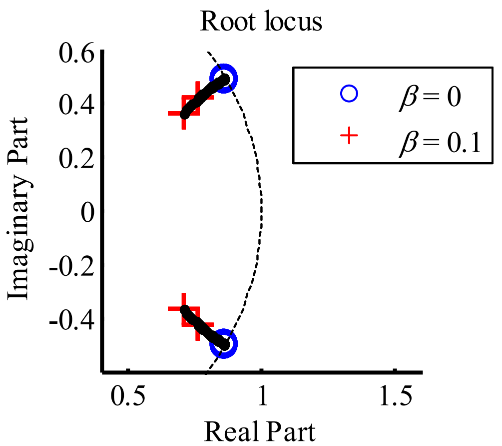


Figure 4 – Root locus of single subsystem

In this case, $\mathbf{H}(z)$ is 2×2 and contains four transfer functions. The zero point locus of $\mathbf{H}(z)$ with β

changing from 0 to 0.1 is shown as Figure 3. In this figure, the blue circles are the zero points with β=0, red crosses are the zero points with β=0.1. Notice that there are two conjugate zero points locate on the unit circle invariably although β is changed, and the angle of the invariable locations is just the digital frequency ω. It is shown that the steady-state error of the system is zero if system is stable.

The root locus of $\mathbf{H}(z)$ with β changing from 0 to 0.1 is shown as Figure 4. We can see that when β>0, the pole points are located within the unit circle, so that the β>0 which can makes the system stable is existent. The root locus can verify whether the system is stable intuitively.

3.2 The Cluster System with Two Subsystems

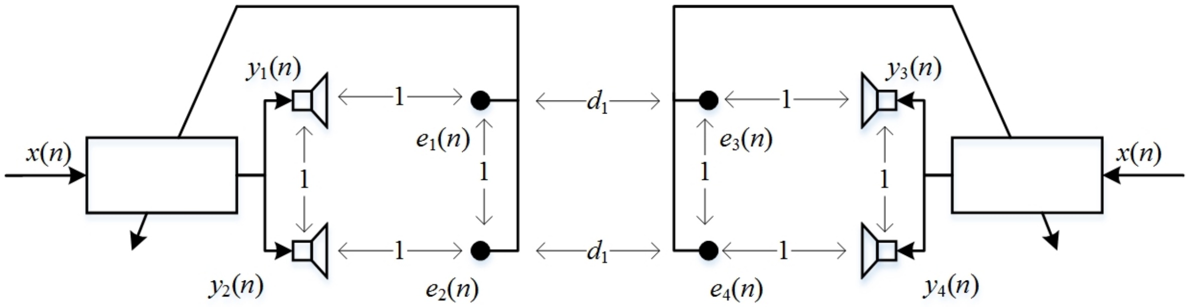


Figure 5 – Symmetrical placement of the cluster system

We can assume that the cluster system is constructed by two subsystems both of which are the same with the mentioned one above. The system placement is shown as Figure 5. Two same subsystems are placed "face to face" symmetrically, and the distance between two subsystems is d_1.

Table 1 – Stability of the system with different d_1

d_1	<0	0	>0
Stability	unstable	critical stable	stable, error converge to 0

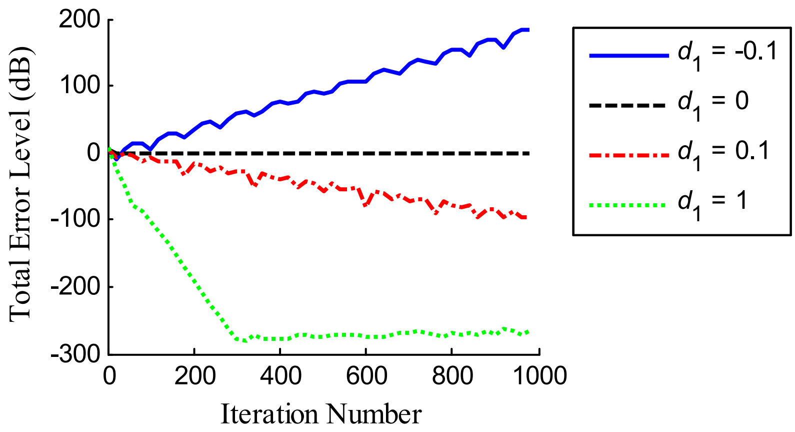


Figure 6 – The learning curves with different d_1

The stability of the system with different d_1 is shown as Table 1. Figure 6 shows the learning curves of the system when d_1 is -0.1, 0, 0.1 and 1, respectively. We can see that when the two subsystems overlap with each other, taking d_1=-0.1 as example, the cluster system becomes unstable. The root locus of $\mathbf{H}(z)$ under this situation is shown as Figure 7. The figure shows that there are one locus locating outside of the unit circle, which leading to the unstability.

When d_1=0, the system reaches critical stable state, that is the relative distance of the error sensors of the subsystems can be ignored considering the wavelength at low frequency. Under this situation, the magnitude response at ω of $\mathbf{H}(z)$ is shown as Equation (27). There are two transfer functions obtaining -6dB response at each row, which means that the magnitude of each error is one, so the system is critical stable. The same result can be represented by the root locus, shown as

Figure 8.

$$20\log_{10}\left|\mathbf{H}(e^{-i\omega})\right| = \begin{pmatrix} -6 & -259 & -6 & -259 \\ -272 & -6 & -272 & -6 \\ -6 & -259 & -6 & -259 \\ -272 & -6 & -272 & -6 \end{pmatrix} (\mathrm{dB}) \tag{27}$$

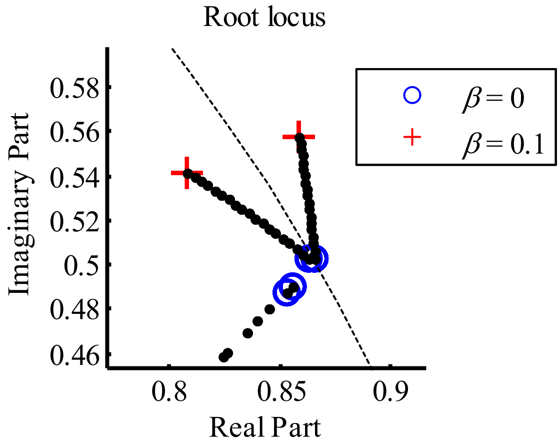


Figure 7 The root locus when d_1=-0.1

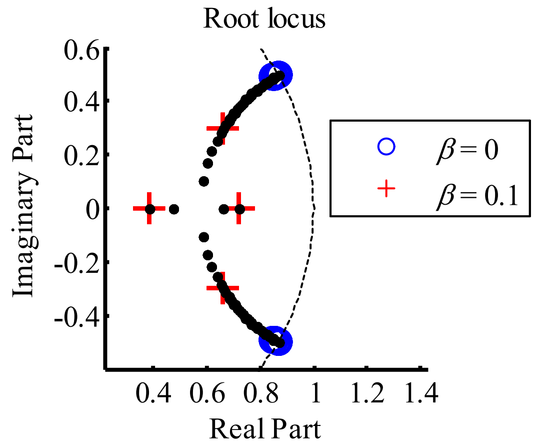
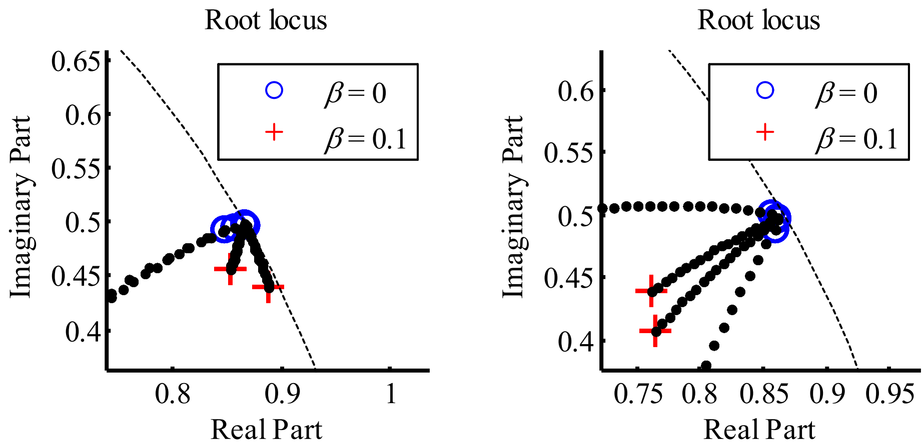


Figure 8 –The root locus when d_1=0



(a) d_1=0.1 (b) d_1=1

Figure 9 – The root locus when d_1>0

When d_1>0, the root locus is within the unit circle when β is changed from 0 to 0.1, whether d_1 is 0.1 or 1. It reveals that the system reaches stable state, shown as Figure 9. The locus which is shown as Figure 9(a), is much closer to unit circle than the locus shown as Figure 9(b). This explains why the convergence rate with d_1=1 is much faster than d_1=1.

The influence of d_1 on convergence rate is studied by comparing with the learning curves in different d_1, shown as Figure 10. Combining Figure 6, we can see that the convergence rate is increased when d_1 is increased from 0. The increment of convergence rate is getting smaller and smaller when d_1 is increased unceasingly. Particularly, after d_1 is larger than 0.5, the increment of convergence rate can be ignored. When d_1 is very large (e.g. 10000), the zero point locus and root locus of the whole system are exactly the same with the single subsystem which mentioned in Section 3.1. Namely, the interaction between two subsystems is so weak that can be completely ignored. That is, the two subsystems run independently.

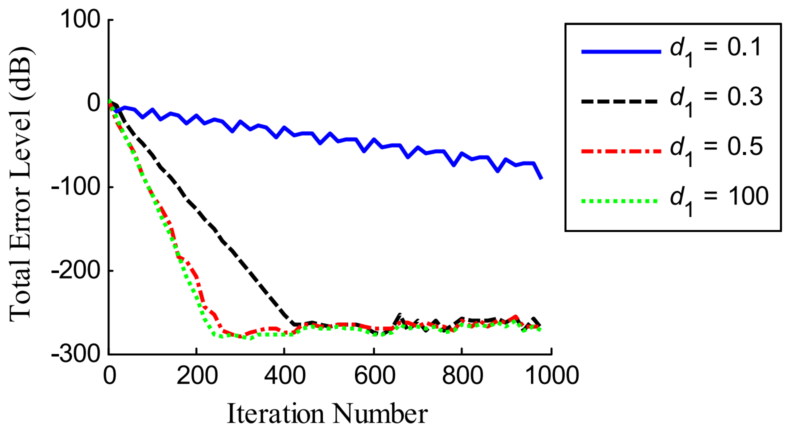


Figure 10 – The learning curves with increasing d_1

4. Conclusions

In this study, the equivalent transfer function of the cluster active noise control system for sinusoidal signals is deduced from the Z-transform of the active control algorithm. The stability and steady-state error of the system are obtained by analyzing the zero-pole map of the equivalent transfer function.

Considering the simplest situation, we assume that the cluster system is made up of two small subsystems. The subsystems are placed "face to face" with distance d_1 in a free space. The simulation results show that when two subsystems overlap with each other, i.e. d_1<0, the system become unstable because of the root locus is located outside the unit circle. If the difference of the error signal between two subsystems can be ignored at low frequency, i.e. d_1<0, the system reaches critical stable state. The magnitude response of the equivalent transfer function explains the reason. In this condition, the system is ineffective.

The effectively arrangement requires that the two subsystems are departed with distance which is larger than 0.5. If the distance is smaller than 0.5, the system is stable and the interactions between subsystems will lead to a slow convergence rate. However, if the distance is larger than 0.5, the system can achieve the fastest convergence rate and must be stable.

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