An explicit time-domain finite-element method for room acoustics simulation

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ABSTRACT

The time-domain finite-element method (TD–FEM) is a reliable numerical method for room acoustics simulation. With the recent progress in computer technology, the use of FEM at high frequencies has become a realistic option. However, efficient simulation at high frequencies is still cited as a challenging task. One of the key to realize such simulation is to reduce the discretization error called dispersion error. In this paper, the applicability of an explicit TD–FEM with a dispersion reduction technique called modified integration rule on room acoustics simulation at high frequencies is tested. First, an explicit scheme with frequency-independent impedance boundary condition is derived for addressing boundaries with finite impedance. Then, the accuracy and efficiency of the explicit TD–FEM is demonstrated in comparison with an implicit TD–FEM for sound field analysis inside a cubic cavity with rigid boundaries at the frequencies up to 8 kHz. Finally, the accuracy of the explicit TD–FEM is examined for sound field analyses with finite impedance boundaries. Results showed that the explicit method offered the same accurate results as the implicit method with less computational time, whereas the implicit method was more efficient in term of memory requirement.

Keywords: Room acoustics simulation, Time-domain finite-element method, Dispersion error
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(See http://www.inceusa.org/links/Subj%20Class%20-%20Formatted.pdf.)

1. INTRODUCTION

The time-domain finite-element method (TD-FEM) is a powerful wave-based numerical method for room acoustics simulation with complex boundary conditions. Thanks to a drastic advancements in computer technology, the application of TD-FEM to room acoustics simulation at several kilohertz frequencies is becoming a realistic option [1, 2] although the cost-efficient simulation at high frequencies is still considered to be a difficult task due to the high computational cost. Inherent discretization error called dispersion error coming from spatial and time discretizations of computational domain is one of the reasons for the high computational cost.

The dispersion error is defined as the difference between the exact wave velocity and numerical wave velocity, and the specific features are different numerical wave velocity at each frequency and the numerical anisotropy of error magnitude in multi-dimensional analysis. Because of this error, a spatial discretization requirement known as a rule of thumb is imposed in the mesh generation process to yield reliable results. Also, time discretization error must be considered in time-domain analysis. These requirements engender the solution of large-scale problems with many degrees of freedom (DOF) and many time steps for the simulation at high frequencies.

Many dispersion reduction methods have been developed to increase the computational efficiency [2, 3, 4, 5, 6, 7]. The authors have also proposed some TD-FEM [2, 7] to solve the large-scale problems efficiently. Also, an explicit TD-FEM with a simple dispersion reduction technique called modified integration rules (MIR) [5] exists, in which fourth-order accuracy with respect to the dispersion error can be obtained for idealized case using square or cubic FEs. Although the explicit method is very attractive because of its simplicity, the dissipation term for treating the absorption at boundaries, which is important for room acoustics simulation, was not considered in the formulation presented in the literature. When a sound field inside room with finite

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impedance boundaries is analyzed using this explicit method, a time derivative of sound pressure related to the dissipation term has to be approximated by less-accurate backward difference even though time derivatives of other physical quantities can be approximated by the second-order accurate central difference. Therefore, the accuracy and efficiency of the explicit TD-FEM on room acoustics simulation still remain unclear, especially for high frequency region.

The purpose of this paper is to present the accuracy and efficiency of the explicit TD-FEM on room acoustics simulation at high frequencies through the numerical experiments in three-dimensions. First, we describe the theory of the explicit TD-FEM with dissipation term as well as an implicit TD-FEM. Then, the accuracy and efficiency of the explicit method is demonstrated in the comparison with the implicit method for sound field analysis inside a cubic cavity with rigid boundaries. Here, the accuracies of both methods are estimated using the analytical solution. Finally, the effect of using the backward difference approximation in the explicit method with dissipation term on resulting accuracy is tested for sound field analysis inside a cubic cavity with finite impedance boundaries.

2. TIME-DOMAIN FINITE-ELEMENT METHOD

2.1 Implicit method

We consider a closed sound field Ω with rigid boundary, vibration boundary and impedance boundary governed by the wave equation. By introducing the FE approximations to sound pressure and weight function in the weak form derived from the wave equation, the semi-discretized matrix equation for the sound field can be obtained as

$$\boldsymbol{M}\ddot{\boldsymbol{p}} + c_0^2\boldsymbol{K}\boldsymbol{p} + c_0\boldsymbol{C}\dot{\boldsymbol{p}} = \boldsymbol{f}, \tag{1}$$

where $\boldsymbol{M}$, $\boldsymbol{K}$ and $\boldsymbol{C}$, respectively, denote the global mass matrix, the global stiffness matrix, and the global dissipation matrix constructed by respective element matrices $\boldsymbol{m}_\mathrm{e}$, $\boldsymbol{k}_\mathrm{e}$ and $\boldsymbol{c}_\mathrm{e}$. Further, $\boldsymbol{p}$, $\boldsymbol{f}$, c_0, respectively, denote the sound pressure vector, the external force vector, and the speed of sound. $\cdot$ and $\cdot\cdot$ respectively signify first-order and second-order derivatives with respect to time. The respective element matrices for $\boldsymbol{M}$, $\boldsymbol{K}$ and $\boldsymbol{C}$ are defined as

$$\boldsymbol{m}_\mathrm{e} = \int_{\Omega_\mathrm{e}} \boldsymbol{N}^T\boldsymbol{N}d\Omega, \tag{2}$$

$$\boldsymbol{k}_\mathrm{e} = \int_{\Omega_\mathrm{e}} \nabla\boldsymbol{N}^T\nabla\boldsymbol{N}d\Omega, \tag{3}$$

$$\boldsymbol{c}_\mathrm{e} = \frac{1}{z_n}\int_{\Gamma_\mathrm{e}} \boldsymbol{N}^T\boldsymbol{N}d\Gamma. \tag{4}$$

Here, $\boldsymbol{N}$ and z_n, respectively, represent the shape function and the normalized acoustic impedance ratio. Ω_e and Γ_e respectively represent the region and surface areas to be integrated. The sound pressure $\boldsymbol{p}$ in time domain is calculable applying a direct time integration method such as Newmark β method [8] to Eq. (1).

An efficient formulation [7] is used to solve the Eq. (1) with many DOF, in which 8-node hexahedral elements and Fox-Goodwin method, which is a family of Newmark method, are respectively used for spatial and time discretizations, with MIR [5]. A krylov subspace iterative method is also used to solve the large-scale linear system of equations at each time step efficiently.

MIR is a simple method to reduce the dispersion error by modifying numerical integration points of $\boldsymbol{m}_\mathrm{e}$ and $\boldsymbol{k}_\mathrm{e}$ from conventional points, and it provides fourth-order accuracy with respect to dispersion error for an idealized case. The modified integration points for FG method are $\alpha_m = \alpha_k = \pm\sqrt{2/3}$ with unit weight coefficient, where α_m, α_k are numerical integration points in Gauss-Legendre rule for calculating $\boldsymbol{m}_\mathrm{e}$ and $\boldsymbol{k}_\mathrm{e}$. Further advantages can be obtained by using the MIR, i.e., relaxation effect of stability condition and improvement in convergence property of an iterative method.

For the analysis using only rectangular FEs in three dimensions, the relaxed stability condition is given as [7]

$$\Delta t \leq \frac{1}{c_0\sqrt{\frac{1}{d_x^2} + \frac{1}{d_y^2} + \frac{1}{d_z^2}}}, \tag{5}$$

where d_x, d_y and d_z respectively represent element length in x, y and z directions. The three-dimensional dispersion relation and the efficiency of this formulation can also be found in a reference [7].

2.2 Explicit method

In contrast to the implicit method, the explicit method solves first-order ordinary differential equation. By introducing a diagonal mass matrix $\boldsymbol{D}$ and a vector $\dot{\boldsymbol{p}} = \boldsymbol{v}$, the second-order ordinary differential equation of

Eq. (1) can be transformed into [4]

$$D\dot{p} = Mv, \quad (6)$$

$$D\dot{v} = f - c_0^2 Kp - cC\dot{p}. \quad (7)$$

Discretizations of $\dot{p}$ in Eq. (6) and $\dot{v}$ in Eq. (7) using second-order accurate central difference and $\dot{p}$ in Eq. (7) using first-order accurate backward difference lead to the following explicit scheme as

$$p^n = p^{n-1} + \Delta t D^{-1} M v^{n-\frac{1}{2}}, \quad (8)$$

$$v^{n+\frac{1}{2}} = v^{n-\frac{1}{2}} + \Delta t D^{-1}\left[f^n - c_0^2 K p^n - \frac{c_0}{\Delta t} C(p^n - p^{n-1})\right], \quad (9)$$

where n represents the time step. For further numerical efficiency, M and K are stored by sparse matrix storage format and lumped C is used. Here, these techniques are naturally used in the above-mentioned implicit method. Therefore, two sparse matrix-vector products $Mv^{n-\frac{1}{2}}$ and Kp^n are the main numerical operations of this scheme. According to a reference [5], an element matrix of D is lumped using standard Gauss-Legendre rule. Meanwhile, element matrices m_e and k_e are constructed using MIR where the respective modified integration points are given as [5]

$$\alpha_k = \sqrt{\frac{2}{3}},\ \alpha_m = \sqrt{\frac{1}{3}\left[4 - \left(\frac{c_0 \Delta t}{d}\right)^2\right]}. \quad (10)$$

Here, d is the element length of cubic elements. This scheme provides fourth-order accuracy with respect to dispersion error for idealized case using cubic elements in free space [5].

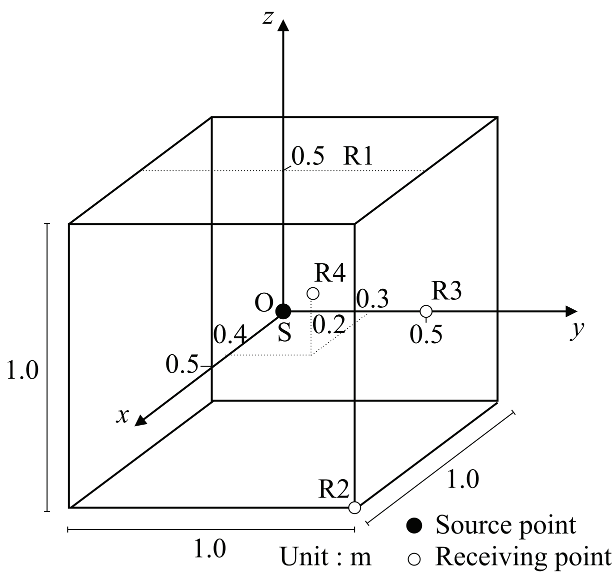

Figure 1 – A benchmark problem, B0-1T.

3. NUMERICAL EXPERIMENT

3.1 Cubic cavity with rigid boundaries

The sound field analysis inside a cubic cavity with rigid boundaries, which is a problem, “B0-1T, Task A”, in the benchmark platform on computational methods for architectural/environmental acoustics [9], was selected to compare the accuracy and efficiency between the implicit method and the explicit method because the analytical method [10] is available. It is a problem of computing transient response at three receiving points R2, R3 and R4 inside a cubic cavity (Figure 1), in which c_0 and air density ρ_0 were respectively assumed to be 343.7 m/s and 1.205 kg/m^3. For quantitative estimation of accuracy of either formulation, the relative error in the sound pressure between the analytical method and the numerical method was defined as

$$||e||_2 = \frac{1}{N_{\mathrm{step}}} \cdot \frac{||p_{\mathrm{ana.}} - p_{\mathrm{FEM}}||_2}{||p_{\mathrm{ana.}}||_2}, \quad (11)$$

where $p_{\mathrm{ana.}}$ and p_{FEM} are the sound pressure obtained from the analytical method and FE analysis, respectively. N_{step} and $||\cdot||$ represent the number of time step and 2-norm, respectively.

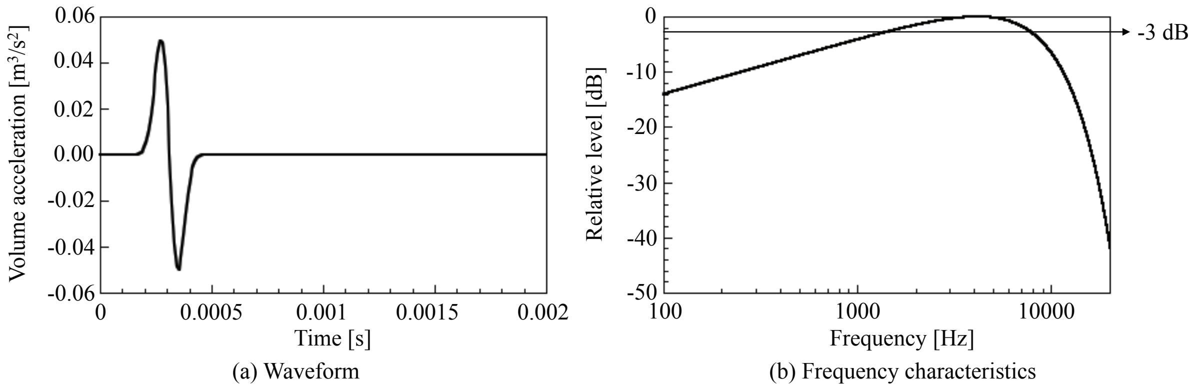

Figure 2 – Waveform of sound source and its frequency characteristics.

3.1.1 FE setup

The sound source used here was a modulated Gaussian pulse. Its waveform and frequency characteristics are depicted in Figure 2. Here, upper limit frequency was assumed as 8 kHz where the gain is -3 dB. An FE mesh with spatial resolution λ/d=6.8 was constructed using only cubic elements, where λ represents the wavelength at upper limit frequency. The DOF of the FE mesh is 4,173,281. The sound pressure was calculated up to 0.01 s with Δt=1/96,000 s, which was determined by the stability condition of Eq. (5). It is noted that Eq. (5) is not exact stability condition for the explicit method, but it is considered as the safe side because the stability condition of the explicit method in two-dimensional analysis is slightly looser than that of the implicit method [5)].

A diagonal scaled conjugate gradient iterative solver was used to solve the linear system of equations in the implicit method, in which the convergence tolerance used in a stopping criterion was set to 10^{-6}.

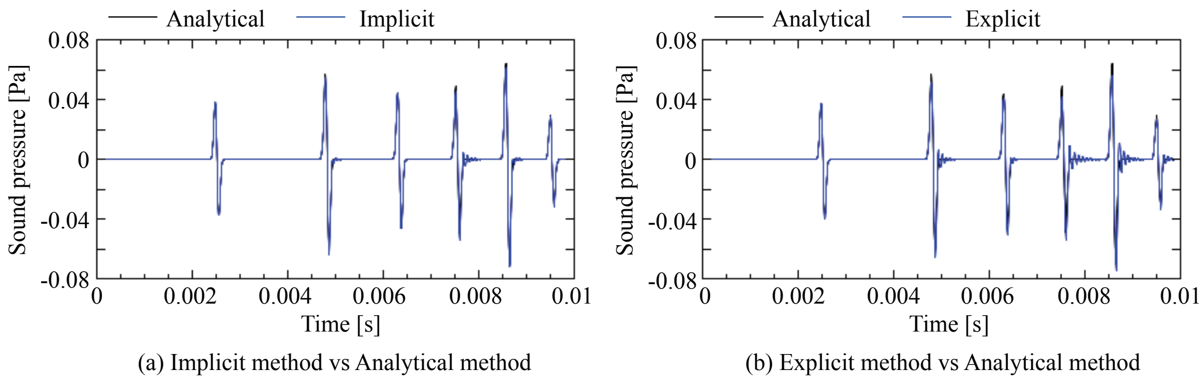

Figure 3 – Comparisons of sound pressures at R2 obtained from analytical method and TD-FEM with FE mesh of λ/d=6.8: (a) Analytical vs Implicit and (b) Analytical vs Explicit.

Table 1 – The relative error $||e||_2$ at each receiving point for implicit and explicit methods with FE mesh of λ/d=6.8

	$\|\|e\|\|_2$, %		
Method/Receiving point	R2	R3	R4
Implicit	0.013	0.012	0.019
Explicit	0.025	0.024	0.038

3.1.2 Results and discussion

Figure 3 (a) and (b) respectively present the comparisons of the sound pressures at R2 obtained from the analytical method and the implicit/explicit methods. Overall, the sound pressures obtained by both implicit and explicit methods agree well with the analytical solution, but larger numerical oscillation can be observed in the sound pressure obtained from the explicit method. Table 1 lists the relative error at each receiving points for both schemes where the relative errors of the explicit method are approximately two times larger than those of the implicit method at all receiving points. This is because the magnitude of the spatial discretization error in the explicit method is larger than that in the implicit method, which can be confirmed using the two-dimensional dispersion relation in both methods. The dispersion relation of the implicit method is given as [5)]

$$\frac{|c_0 - c^h|}{c_0} \approx \frac{(kd)^4}{480}|(1-\tau^4) - 3\cos^2\theta\sin^2\theta|, \tag{12}$$

with

$$\tau = c_0 \Delta t / d. \tag{13}$$

Here c^h, k and θ represent numerical wave velocity, wavenumber and a direction of wave propagation in two dimensional polar coordinate system. For the explicit method, the relation is [5)]

$$\frac{|c_0 - c^h|}{c_0} \approx \frac{(kd)^4}{1440}|(8-10\tau^2+2\tau^4) - (19-20\tau^2+5\tau^4)\cos^2\theta\sin^2\theta|. \tag{14}$$

The relationships between maximum dispersion errors and spatial resolutions calculated by the dispersion relations are presented in Figure 4.

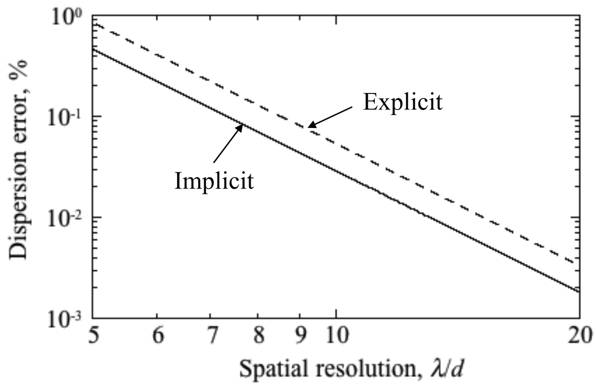

Figure 4 – A comparison of dispersion errors of the implicit and the explicit methods as a function of spatial resolution.

The maximum dispersion errors at the upper limit frequency are 0.13% in the implicit method and 0.24% in the explicit method, respectively. For the explicit method, the use of the FE mesh with approximately λ/d=8.0 is necessary to obtain the same accurate results as the implicit method. Considering this, a comparison of sound pressures obtained from the analytical method and the explicit method with FE mesh of λ/d=8.0 is shown in Figure 5. Here, the DOF of the FE mesh was 6,539,203 and Δt was set to 1/120,000 s. The agreement of sound pressures between them was improved with the relative error of 0.013%, which is the same magnitude as the implicit method.

In comparison of computational cost in both methods to obtain the same accurate results, computational time of the explicit method was only 1/2.4 of the implicit method, whereas the explicit method required 1.4 times larger memory than the implicit method.

3.2 Cubic cavity with impedance boundaries

Sound fields inside the cubic cavity (Figure 1) with finite impedance boundaries were computed to reveal the effect of the use of the backward difference in dissipation term of the explicit method on resulting accuracy, and the computed sound pressures were compared with reference solution obtained by the implicit method. In the computations, the different degrees of absorption was given on the floor. The accuracy was also estimated using the relative error defined by Eq. (11), but the reference solution was used instead of the analytical solution.

3.2.1 FE setup

Regarding the absorption conditions, frequency independent normalized acoustic impedance ratios z_n's corresponding to statistical absorption coefficient α_s of 0.2, 0.4, 0.6 and 0.8 were given for the floor, where the z_n's were calculated using Paris' formula [11)] with an assumption that z_n is a real number. z_n corresponding to α_s=0.01 was given for remaining boundaries.

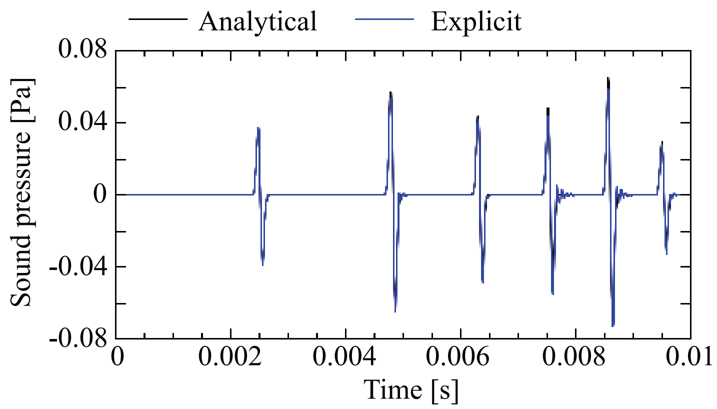

Figure 5 – A comparison of sound pressures at R2 obtained from analytical method and explicit method with FE mesh of λ/d=8.0.

The FE meshes with λ/d=6.8 and 8.0 were respectively used for the implicit method and the explicit method. The sound pressure was calculated up to 0.01 s with Δt of 1/120,000 s. Other settings are the same as previous section.

3.2.2 Results and discussion

As an example, Figure 6 (a) and (b) respectively present a comparison of sound pressure at R3 obtained from the implicit method and the explicit method for the cases with α_s = 0.2 and 0.8, in which good agreement can be found in sound pressures calculated by both methods, irrespective of degree of absorption. Table 2 lists the mean relative error over receiving points for all cases. Here, reference value of the mean relative error is also presented as "rigid", which was calculated from a condition that all boundaries are rigid. From the table, the magnitude of mean relative errors for all absorbing conditions are almost the same as the reference value from which it can be concluded that the use of first order backward difference in dissipation term has no effect on resulting accuracy. Further, this indicates that the use of Δt determined by the stability condition of Eq. (5) provides sufficiently small value for approximating $\dot{\boldsymbol{p}}$ in Eq. (7).

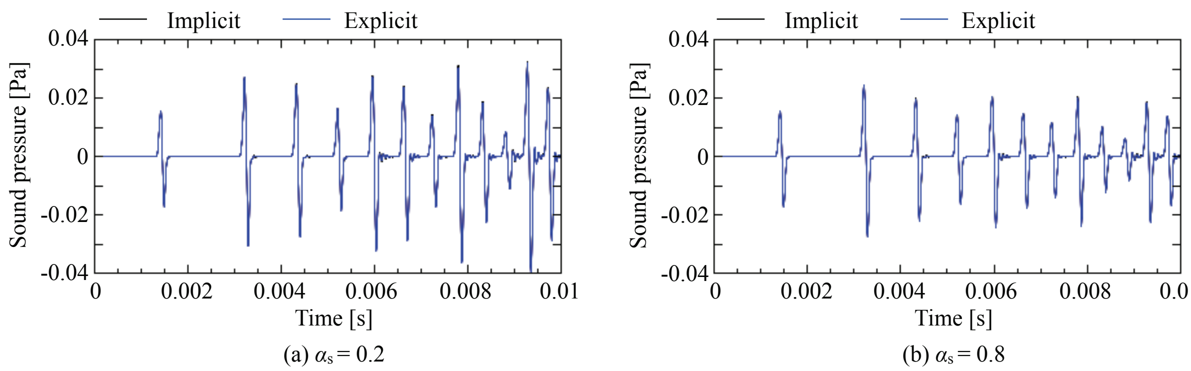

Figure 6 – Comparisons of sound pressures at R3 obtained from implicit method and explicit method: (a) α_s=0.2 and (b) α_s=0.8.

Table 2 – The mean relative error $\overline{||e||}_2$ for various absorption conditions

		$\overline{||e||}_2$, %		
Rigid	α_s=0.2	α_s=0.4	α_s=0.6	α_s=0.8
2.57×10^{-3}	2.12×10^{-3}	2.14×10^{-3}	2.69×10^{-3}	3.90×10^{-3}

4. CONCLUSIONS

In this paper, the accuracy and efficiency of an explicit TD-FEM with MIR on room acoustics simulation at high frequencies was tested through the numerical experiments in three-dimensions, in which the computational efficiency was estimated with the implicit TD-FEM for idealized case using only cubic FEs.

The numerical results showed that the explicit method is computationally more efficient than the implicit method from the perspective of computational time. It was also revealed that the backward difference approximation of first order time derivative of sound pressure in the dissipation term has no effect on resulting accuracy with the time interval determined by the stability condition of Eq. (5). Future works are necessary to reveal the applicability of the explicit method for more generalized cases using the rectangular and the distorted FEs. Derivations of dispersion relation and stability condition of the explicit method in three dimensional sound field are also subject of future research.

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