A new structure for nonlinear narrowband active noise control using Volterra filter

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ABSTRACT

Many structures and algorithms have been developed for linear narrowband active noise control (NANC) systems, which are effective in suppressing sinusoidal noise generated by rotating machines and devices with reciprocating motion. However, the conventional linear NANC systems may seriously suffer from performance degradation due to the nonlinear distortions which, large or small, usually exist in real-life sound fields. Development of nonlinear NANC systems to solve the above-mentioned problems has recently attracted a great deal of attention and several structures and algorithms have been proposed based on the use of adaptive Volterra filter. In those systems, the sinusoidal noise at the reference point is distorted by the nonlinear part of a primary path, eventually adding some high-order harmonic components to the primary noise being targeted. But the main body of the primary noise is still dominated by linearity of the primary path. This paper presents a new structure for a nonlinear NANC system, where the largest portion of the primary noise power can be reduced by the conventional adaptive linear filter and the remaining portion due to high-order harmonics may be effectively suppressed by an adaptive Volterra filter. Numerical simulation results reveal that the proposed system is very effective in suppressing sinusoidal noise distorted by nonlinearity of the primary path.

Keywords: Narrowband active noise control (NANC), Nonlinearity, Linear combiner, Volterra filter
I-INCE Classification of Subjects Number(s): 74.9

1. INTRODUCTION

Linear narrowband active noise control (NANC) system has been widely used that presents promising performance in cancelling sinusoidal noise generated by rotating machines and devices with reciprocating motion in a linear acoustic field (1, 2, 3, 4). In a linear NANC system, the filtered-X LMS (FXLMS) algorithm (5, 6) is most commonly used to update weights of the control filter. However, in some real-life applications, the linear NANC system may suffer from performance degradation when it is used to deal with noise distorted by nonlinearity (7, 8, 9, 10). To solve this problem, many network based structures and algorithms have been developed (7, 11, 12, 13). ANC systems using adaptive Volterra filters have also been proposed (14, 15, 16, 17).

In a recently developed nonlinear NANC system using Volterra filter (17), the reference signal is obtained via an acoustic sensor. In this system, an additive stochastic noise, which is usually regarded as an Gaussian white noise with zero mean, will slip into the reference signal. Obviously, this additive noise acts as a background noise and theoretically it is difficult or impossible to reduce it. In this paper, we consider a case that a non-acoustic sensor such as a tachometer is used as a reference sensor to obtain a piece of information on the noise signal frequency. Reference cosine and sine waves may be then generated by use of this piece of real-time information, which are fed to an NANC system with a linear combiner structure. The dominant portion of power of the primary noise will be mitigated by this conventional NANC system (3). Therefore, in this paper, we propose a new nonlinear NANC structure that consists of a linear combiner and a Volterra filter. The former is intended to remove the sinusoid in the primary noise that has the same frequency as the reference waves do, while the latter is designated to reduce a portion of power of the primary noise that is generated by the nonlinearity of the primary path. In the proposed NANC system, the linear combiner or the conventional NANC system and the adaptive Volterra filter are placed in parallel. Another linear combiner is designated to form a reference sinusoid such that the Volterra filter is able to implement ghe nonlinearity of the primary path.

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The remainder of this paper is organized as follows. The new nonlinear NANC system with two linear combiners and a Volterra filter is given in Section 2. In this Section, the FXLMS algorithm for updating weights of linear combiners and Volterra filter is also derived in detail. Representative simulation results will be provided in Section 3 to confirm the effectiveness of the proposed system. Section 4 concludes the paper.

2. NONLINEAR NANC SYSTEM USING LINEAR COMBINER AND VOLTERRA FILTER

2.1 Linear Combiner and Volterra filter

Linear combiner is a sufficient signal synthesizer when it is used in a linear NANC system (3). Many analyses have given us deep insight into performance of the combiner (18, 19, 20). General configuration of the linear combiner as used in a linear NANC system will be given in the following context when we introduce a new structure of nonlinear NANC system.

A causal Volterra filter with a finite memory of L_{V} and a finite order of P can be described by following relationships (21):

$$y(n) = \sum_{p=1}^{P} y_p(n) \tag{1}$$

$$y_p(n) = \sum_{m_1=0}^{L_{\mathrm{V}}-1} \sum_{m_2=m_1}^{L_{\mathrm{V}}-1} \cdots \sum_{m_p=m_{p-1}}^{L_{\mathrm{V}}-1} h_{p,m_1,m_2,\ldots,m_p}(n)x(n-m_1)x(n-m_2)\cdots x(n-m_p). \tag{2}$$

According to the above expressions, we clearly get that $x(n)$ and $y(n)$ are respectively input and output of the Volterra filter and $h_{p,m_1,m_2,\cdots,m_p}(n)$ is the pth-order weight corresponding to Volterra kernel with the same order, whereas n designates the time index. It is noted that, in fact, Volterra filter is derived from Volterra series (22). Therefore, considering symmetric properties of the series, we can remove redundant kernels without any loss of generality when Volterra filter is used (14, 15, 16, 21). Figure 1 shows a typical adaptive Volterra filter with both memory and order of 2.

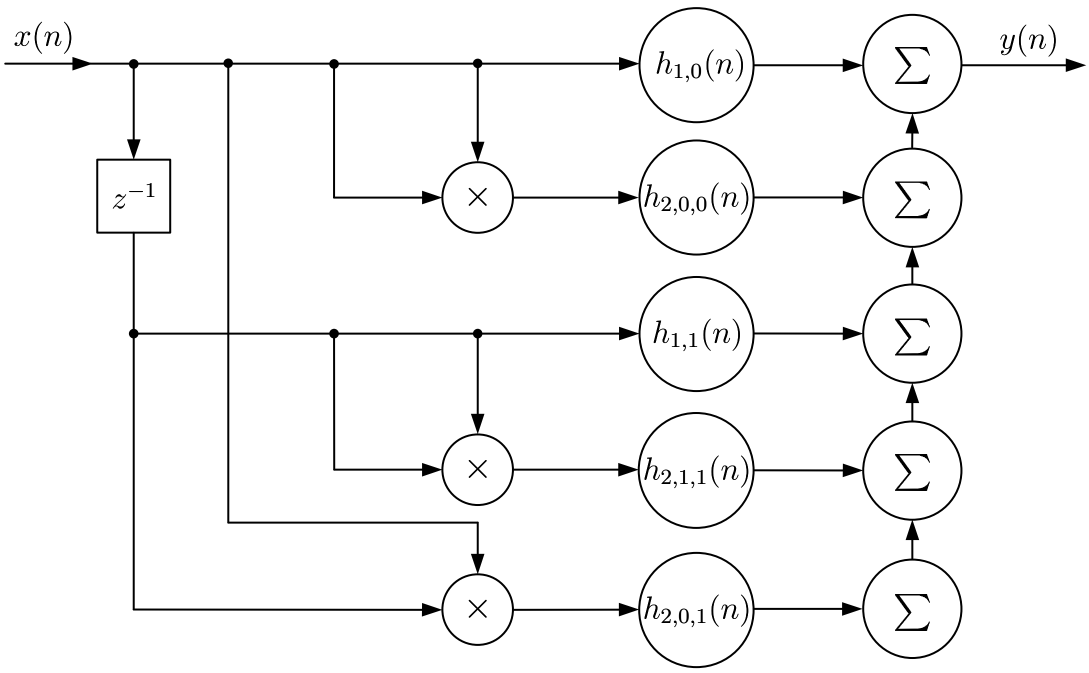


Figure 1 – A Volterra filter with both memory and order of 2.

From equations (1) – (2) and Figure 1, we have that, the first order weights $h_{1,0}(n)$ and $h_{1,1}(n)$ are linear coefficients since their corresponding kernels are $x(n)$ and $x(n-1)$, respectively. Memory and order of the filter can be set to be larger, which depends on exact conditions in real-life applications.

2.2 New Nonlinear NANC System

Figure 2 depicts the proposed nonlinear NANC system using linear combiner and Volterra filter for single tone. If nonlinear path exist in a NANC system, e.g., especially the primary path is nonlinear, the noise will be distorted and higher order harmonics with relatively smaller power will be excited at the cancelling point. Therefore, we can design a linear subsystem and a nonlinear subsystem to respectively reduce linear and nonlinear portions of the excited noise being targeted. In the new system considered and shown in Figure 2, the linear subsystem is constructed by a linear combiner with coefficients $\hat{a}(n)$ and $\hat{b}(n)$ while the nonlinear subsystem by a Volterra filter. In fact, as mentioned before, a Volterra filter with order 1 is a FIR filter which can be separated and replaced by a linear combiner, and finally to be used to suppress the main power linear portion. This is the essential idea in designing the new nonlinear system. Reference signal fed to the linear subsystem is generated from a synchronization signal, e.g., rotation speed, through which we can obtain frequency of the sinusoidal noise using simple linear regression estimate.

It should be noted that, another linear combiner with coefficients $\hat{c}(n)$ and $\hat{d}(n)$ is added into the system to synthesize coherent reference signal $x_r(n)$ to be fed to Volterra filter as shown in Figure 2. All control weights in the system are designed to be updated by conventional FXLMS algorithm in this paper.

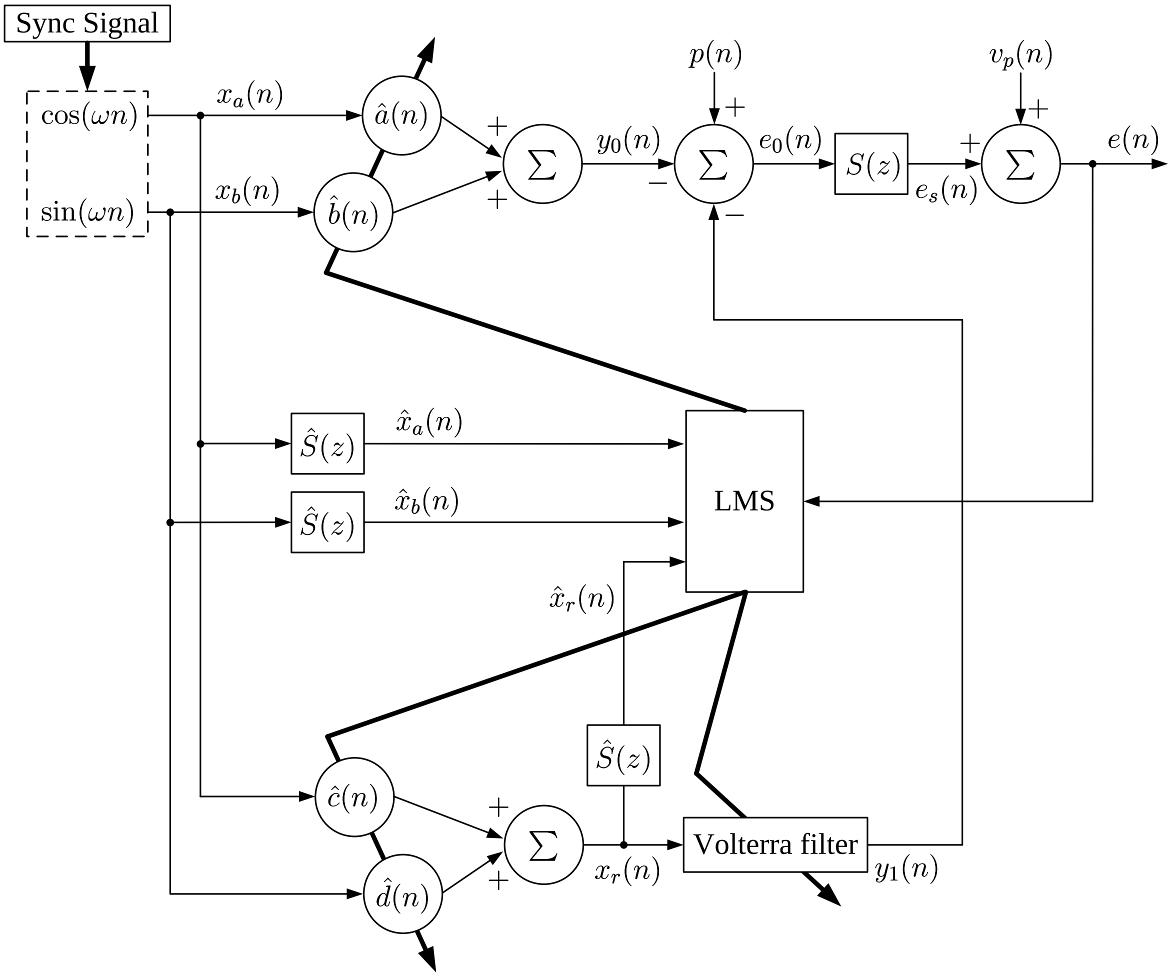


Figure 2 – Proposed nonlinear NANC system using linear combiner and Volterra filter (single tone).

Signal $p(n)$ is the primary noise being targeted. If the primary path is nonlinear, it can be expressed as a nonlinear function with respect to the original sinusoidal source $x(n)$ and in some scenarios it can be popularly written as a polynomial of $x(n)$ (15), such as

$$p(n) = a_1 x(n) + a_2 x^2(n) + a_3 x^3(n) \tag{3}$$

where, the source, reference signal generated by signal generater, and synthesized reference signal fed to Volterra filter are

$$x(n) = a x_a(n) + b x_b(n) \tag{4}$$

$$x_a(n) = \cos(\omega n),\ x_b(n) = \sin(\omega n) \tag{5}$$

$$x_r(n) = \hat{c}(n) x_a(n) + \hat{d}(n) x_b(n). \tag{6}$$

In above equations, a_1, a_2, a_3, a, and b are fixed coefficients which will be set in advance when conduct simulations. ω is the digital frequency of sinusoidal noise.

According to equations (1) – (2), the secondary sources produced by linear and nonlinear control filter, $y_0(n)$ and $y_1(n)$, can be respectively written as

$$y_0(n) = \hat{a}(n)x_a(n) + \hat{b}(n)x_b(n) \tag{7}$$

$$\begin{aligned} y_1(n) = & \sum_{i=0}^{L_V-1}\sum_{j=i}^{L_V-1} h_{2,i,j}(n)x_r(n-i)x_r(n-j) \\ & + \sum_{i=0}^{L_V-1}\sum_{j=i}^{L_V-1}\sum_{k=j}^{L_V-1} h_{3,i,j,k}(n)x_r(n-i)x_r(n-j)x_r(n-k) \\ & + \cdots . \end{aligned} \tag{8}$$

The residual noise signal, $e(n)$, can be derived as

$$\begin{aligned} e(n) &= e_s(n) + v_p(n) \\ &= \sum_{j=0}^{M-1} s_j e_0(n-j) + v_p(n) \end{aligned} \tag{9}$$

$$e_0(n) = p(n) - y_0(n) - y_1(n). \tag{10}$$

Where, $\{s\}_{j=0}^{M-1}$ are impulse response coefficients of secondary path $S(z)$ given in Figure 2. The secondary path is usually modeled as a FIR filter, i.e.,

$$S(z) = \sum_{j=0}^{M-1} s_j z^{-j} \tag{11}$$

and $v_p(n)$ is an additive noise which is usually regarded as a Gaussian white noise with zero-mean and variance σ_p^2.

2.3 FXLMS Algorithm for Coefficients Updating

So far, it is very easy to derive FXLMS algorithm for updating weights $\hat{a}(n)$, $\hat{b}(n)$, and $h_{p,m_1,m_2,\cdots,m_p}(n)$ as (only equations for 2 and 3 orders are given)

$$\hat{a}(n+1) = \hat{a}(n) + \mu_1 e(n)\hat{x}_a(n) \tag{12}$$

$$\hat{b}(n+1) = \hat{b}(n) + \mu_1 e(n)\hat{x}_b(n) \tag{13}$$

$$h_{2,i,j}(n+1) = h_{2,i,j}(n) + \mu_2 e(n)\hat{x}_r(n-i)\hat{x}_r(n-j), \quad i = 0,1,\ldots,L_V-1;\ j = i,i+1,\ldots,L_V-1 \tag{14}$$

$$h_{3,i,j,k}(n+1) = h_{3,i,j,k}(n) + \mu_3 e(n)\hat{x}_r(n-i)\hat{x}_r(n-j)\hat{x}_r(n-k), \quad i = 0,1,\ldots,L_V-1;\ j = i,i+1,\ldots,L_V-1;\ k = j,j+1,\ldots,L_V-1. \tag{15}$$

Where, μ_1, μ_2, and μ_3 are step sizes. Signals $\hat{x}_a(n)$, $\hat{x}_b(n)$, and $\hat{x}_r(n)$ are filtered-X reference signal obtained through reference signals $x_a(n)$, $x_b(n)$, and $x_r(n)$ after being filtered by secondary path estimate, $\hat{S}(z)$ with impulse response coefficients $\{\hat{s}\}_{m=0}^{\hat{M}-1}$, and they can be respectively derived as

$$\hat{x}_a(n) = \sum_{m=0}^{\hat{M}-1} \hat{s}_m x_a(n-m),\ \hat{x}_b(n) = \sum_{m=0}^{\hat{M}-1} \hat{s}_m x_b(n-m) \tag{16}$$

$$\hat{x}_r(n) = \sum_{m=0}^{\hat{M}-1} \hat{s}_m x_r(n-m). \tag{17}$$

Here, if we use off-line modeling method to estimate FIR-type secondary path $S(z)$ given above, its estimate can also be written as a similar FIR function

$$\hat{S}(z) = \sum_{m=0}^{\hat{M}-1} \hat{s}_m z^{-m}. \tag{18}$$

Derivation of difference equations for updating coefficients $\hat{c}(n)$ and $\hat{d}(n)$ is relatively more complicated. We apply the steepest descent method and choose the cost function as

$$J(n) = \frac{1}{2}e^2(n). \tag{19}$$

The gradient for updating $\hat{c}(n)$ is derived as

$$\begin{aligned}\frac{\partial J(n)}{\partial \hat{c}(n)} &= e(n)\frac{\partial e(n)}{\partial \hat{c}(n)} \\ &= s_0 e(n)\frac{\partial e_0(n)}{\partial \hat{c}(n)} \\ &= -s_0 e(n)\frac{\partial y_1(n)}{\partial \hat{c}(n)} \\ &= -s_0 e(n) I[x_r(n)] x_a(n)\end{aligned} \tag{20}$$

where

$$\begin{aligned}I[x_r(n)] = 2h_{2,0,0}(n)x_r(n) &+ \sum_{j=1}^{L_V-1} h_{2,0,j}(n)x_r(n-j) \\ &+ 3h_{3,0,0,0}(n)x_r^2(n) + 2\sum_{k=1}^{L_V-1} h_{3,0,0,k}(n)x_r(n)x_r(n-k) \\ &+ \sum_{j=1}^{L_V-1}\sum_{k=j}^{L_V-1} h_{3,0,j,k}(n)x_r(n-j)x_r(n-k).\end{aligned}$$

Following the same way, we easily have gradient for updating $\hat{d}(n)$

$$\frac{\partial J(n)}{\partial \hat{d}(n)} = -s_0 e(n) I[x_r(n)] x_b(n). \tag{21}$$

With gradients derived above, the FXLMS algorithm for updating $\hat{c}(n)$ and $\hat{d}(n)$ can be finally given as

$$\hat{c}(n+1) = \hat{c}(n) + \mu_4 e(n) I[\hat{x}_r(n)]\hat{x}_a(n) \tag{22}$$

$$\hat{d}(n+1) = \hat{d}(n) + \mu_4 e(n) I[\hat{x}_r(n)]\hat{x}_b(n) \tag{23}$$

where μ_4 is the step size.

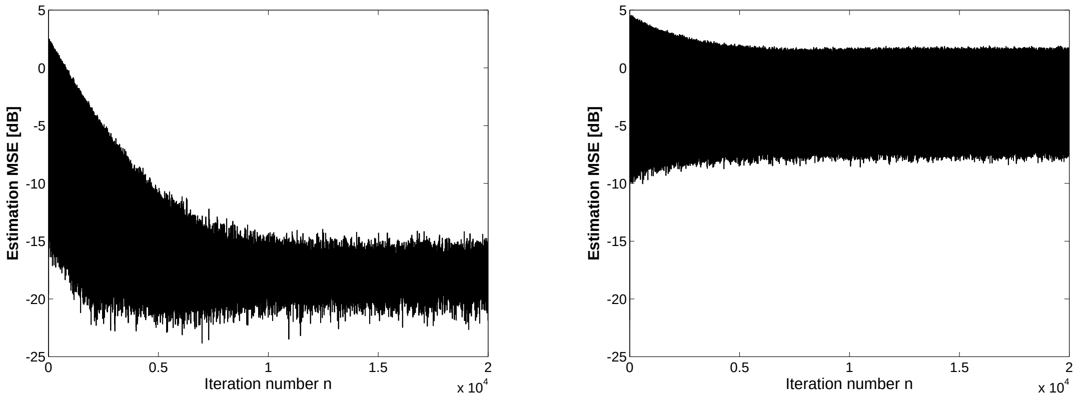
3. SIMULATIONS

Extensive simulations have been conducted for many scenarios to illustrate the effectiveness of the proposed nonlinear NANC system in reducing sinusoidal noise distorted by nonlinear primary path. In our simulations, the nonlinear behavior exhibited in the primary path is described by polynomial equation (3). Some common conditions for conducting simulations are given in Table 1. In addition, the FIR-type secondary path cutoff frequency is set to be 0.4π and it is identified by offline adaptive LMS algorithm. We performed 40 independent runs in all simulations to do the ensemble average.

Table 1 – Common conditions for simulations.

Parameters	Value
ω	0.2π
a_1	1.0
a, b	1.0
σ_p^2	0.01 (-20 dB)
$M, \hat{M}$	11

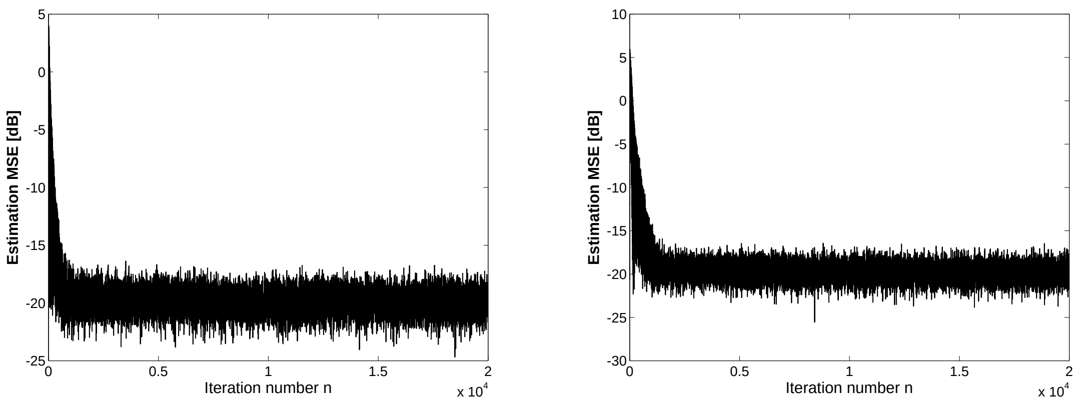
Firstly, we confirmed performance degeneration of the linear NANC system when used to reduce nonlinearly distorted sinusoidal noise. Representative residual noise powers (MSEs) are shown in Figure 3. It is clear that when the power of nonlinear portion in $p(n)$ is much smaller than that of linear portion, the steady-state residual noise power can be reduced in certain degree. However, if the power of nonlinear portion becomes larger than or similar to that of linear portion, the linear system will suffer from obvious performance degradation.



(a) (b)

Figure 3 – Comparison of residual noise powers with linear NANC system for distorted sinusoidal noise ($\mu_1 = 0.001$). (a) $a_2 = 0.08$, $a_3 = -0.04$. (b) $a_2 = 0.8$, $a_3 = -0.4$.

Figure 4 provides comparison of residual noise powers produced by the conventional NANC system and our proposed nonlinear NANC system. Here, the memory and order of Volterra filter used are respectively set to be 2 and 3. The steady-state residual noise powers depicted in Figure 4 show the noise level have been sufficiently reduced, which proves the proposed system is of good robustness since the residual noise powers in steady state are reduced to the same level without changing the memory and order of Volterra filter even power of the nonlinear portion in $p(n)$ becomes more larger.



(a) (b)

Figure 4 – Comparison of residual noise powers with proposed nonlinear NANC system for distorted sinusoidal noise ($L_V = 2$, $P = 3$, $\mu_1 = \mu_4 = 0.005$, $\mu_2 = \mu_3 = 0.002$). (a) $a_2 = 0.08$, $a_3 = -0.04$. (b) $a_2 = 0.8$, $a_3 = -0.4$.

Figures 3 and 4 give us proofs that nonlinear kernels contained in Volterra filter can significantly and effectively compensate limitations of linear filter. We have confidence that, with larger memory and higher order, the proposed nonlinear system may response more complicated nonlinearity, which however needs more supports through our future works.

Finally, to preliminary pay insight into convergence of the proposed nonlinear system, mean estimation errors of several weights are picked up and given in Figure 5. Since the iteration number in simulations is not relatively enough, we can not have good idea of complete trends of all weights. However, it shows some of them fluctuate in small range and the fluctuations will not affect convergence of the whole system. Certainly, it is very important to analyze performance of the system in deepth through investigating convergence of control filter weights in mean and mean-square sense and this work is one of our interesting future topics.

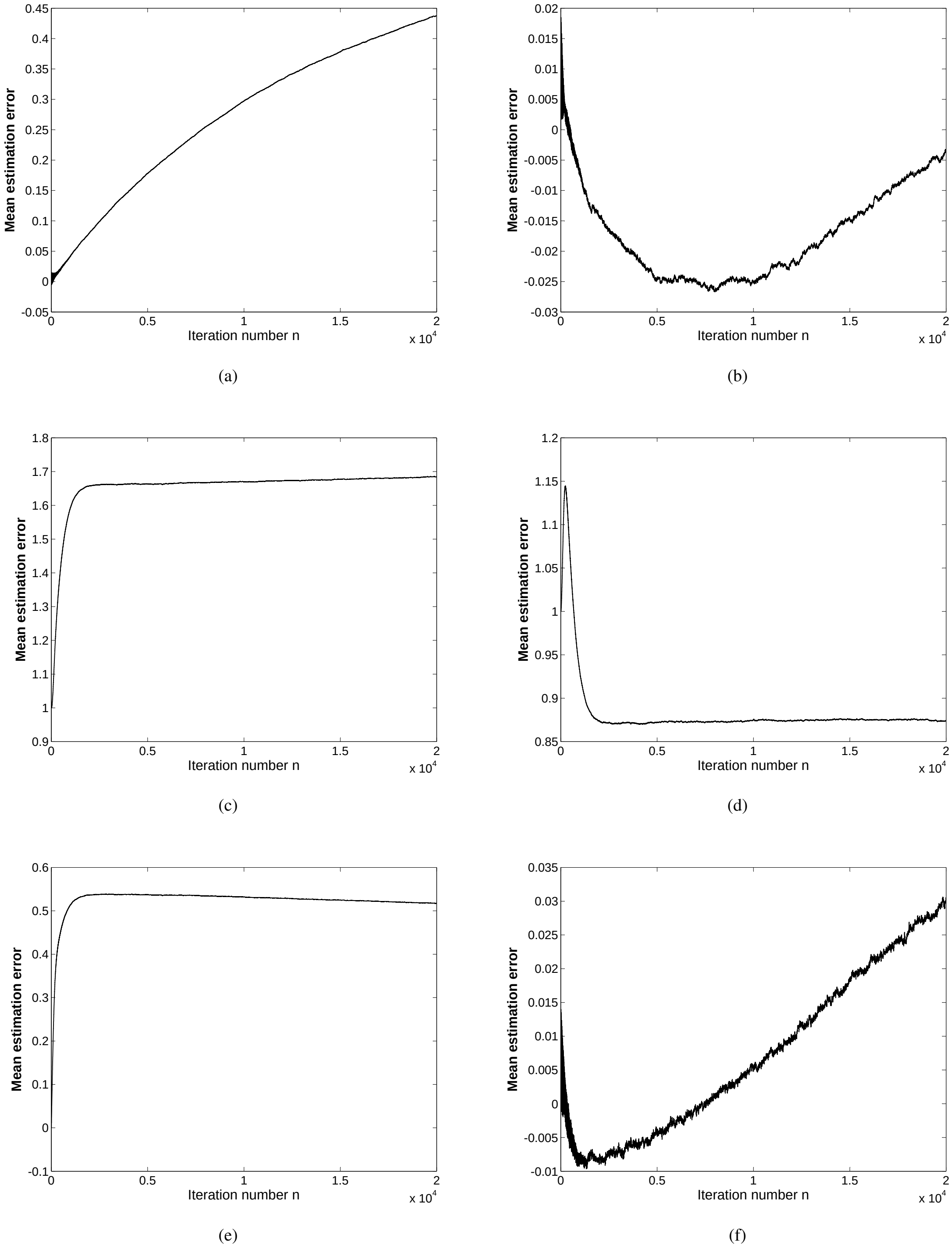


Figure 5 – Mean estimation errors of noise control filter weights (simulation conditions are the same as in Figure 4(b)). (a) $E[\hat{a}(n)]$. (b) $E[\hat{b}(n)]$. (c) $E[\hat{c}(n)]$. (d) $E[\hat{d}(n)]$. (e) $E[\hat{h}_{2,0,0}(n)]$. (f) $E[\hat{h}_{3,0,0,0}(n)]$.

4. CONCLUSIONS

In this paper, a nonlinear NANC system using Volterra filter is proposed. Effectiveness of the new system in suppressing sinusoidal noise distorted by nonlinear primary path has been verified through extensive simulations. The proposed system can significantly compensate for limitations of the conventional NANC system. Topics for further research include a) extension of the proposed structure to multi-frequency case; b) analysis of statistical properties of the proposed system, etc.

ACKNOWLEDGMENT

This work was supported in part by the National Natural Science Foundations of China (61201364) and the Fundamental Research Funds for the Central Universities (NS2012088).

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