Optimal design of unconstrained damping material on a thin panel by using topology optimization

Takashi YAMAMOTO[1]; Takayuki YAMADA[2]; Kazuhiro IZUI[2]; Shinji NISHIWAKI[2]

[1] Kogakuin University, Japan

[2] Kyoto University, Japan

ABSTRACT

Damping material is usually applied on the steel panel of a vehicle to reduce vibration level. On the other hand, the weight reduction is also required to improve fuel consumption. Therefore, modal loss factors induced by damping material on the steel panel of a vehicle body structure need to be maximized with a given volume. In this paper we propose a practical design method to maximize modal loss factors by optimizing the material distribution of damping material under a prescribed volume constraint. The modal loss factor of an eigensmode can be written as the ratio of the strain energy stored in the damping material over the total strain energy in the system under consideration. In the proposed method, we assume the eigenvectors are almost the same as the eigenvectors when damping material is removed. The modal loss factor can then be expressed by using a corresponding eigenvalue where the mass density of the damping material is ignored whereas the stiffness is taken into account. Several numerical examples are provided to show the optimal distributions of the damping material by using a flat panel. Damping material is distributed in the domain where the strain energy is stored, which agrees well with our experiences. Moreover, by applying a sensitivity filter that utilizes a weighted average of design sensitivities over local area, damping material can be distributed collectively in a single domain to meet practical requirements for manufacturing,

Keywords: Damping material, Optimal design, Simulation, Topology optimization
I-INCE Classification of Subjects Number(s): 47.3

1. INTRODUCTION

Annoying noise in a passenger compartment of a vehicle must be reduced to make passengers comfortable during driving a vehicle. Noise in the passenger compartment is mainly generated by the vibration of thin panels surrounding acoustic cavity in the compartment. Vibration damping material is applied on thin panels of automotive body structure, which is one of the common design methodologies to suppress vibration of thin panels. At the same time, the weight of a vehicle is also must be reduced to improve fuel economy that is one of the most essential environmental performance. To meet both of the requirements for performance and weight discussed above, the vibration reduction by a given amount of damping material should be maximized.

Several design methods to optimize the layout of damping sheets have been proposed to reduce the forced response level of thin panels. Yuge et al. (1) proposed an algorithm to find an optimal layout of damping material on a vibrating surface to minimize dynamic mean compliance or square of displacement by using topology optimization based on a homogenization method Kang et al. (2) investigated the optimal distribution of damping material in vibrating structures subject to harmonic excitations by using topology optimization method to minimize the structural vibration level at specified positions with a given amount of damping material. Zheng et al. (3) presented an optimization study to minimize the vibrational energy of beams with passive constrained layer damping treatment. Lassila et al. (4) found the optimal damping set of a two-dimensional membrane such that the energy of the membrane is minimized at some fixed end by using shape optimization scheme based on level set methods.

Damping material should suppress the spectrum level at resonance frequency by increasing damping characteristics of a system of interest. The objective function is basically based on vibration displacement in the prescribed frequency range in the design methods mentioned above. Vibration displacement can be reduced by increasing the stiffness or the weight of the system of interest. If damping material works as a stiffener or

[1]takashi_yamamoto@cc.kogakuin.ac.jp

an additional mass, the vibration displacement level can not be considerably reduced and it does not be applied efficiently in terms of weight. Moreover, frequency resolution must be fine enough to calculate/predict response level at resonance frequencies otherwise, iterative calculation in optimization process can not be stable. Fine frequency resolution leads to the increasing number of frequency response analysis and takes much calculation time.

Another property is modal loss factors to evaluate damping characteristics of a system of interest. Modal loss factors can be takes as objective function to be maximized in the optimization of layout of damping material. Modal loss factors can be calculated exactly by complex eigenmode analysis. However, it usually takes a long time as degrees of freedoms of a finite elemental model increase. Therefore, the application of complex eigenmode analysis is limited to a FE model with small degrees of freedoms. Consequently, modal strain energy (MSE) method is widely applied to estimate modal loss factors approximately by assuming that complex eigenvectors can be approximately identical with real eigenmode vectors when damping property of a system of interest is ignored (5, 6). In MSE method, modal loss factors are obtained by the loss factor of a damping material multiplied by a ratio of modal strain energy stored in damping material to total modal strain energy. Ling et. al (7) proposed a design method of damping material by maximizing modal loss factors calculated by MSE method. However, in the method modal strain energy in each element has to be summed up over the domain of damping material. Moreover, the summation of modal strain energy is not calculated in general eigenvalue analysis code, and some user subroutine for the summation has to be added.

Therefore, in the study presented here, we propose an approximate but a practical formulation to predict modal loss factors by using only real eigenvalues assuming that eigenvectors does not change much when damping material is applied on a steel panel. We also propose a optimal design method for the distribution of damping material to maximize modal loss factors obtained by the approximate method proposed here. The remainder of this paper is organized as follows: Section 2 develops a new formulation to evaluate modal loss factors by using only real eigenvalues. In Section 3, we propose a topology optimization method for the layout of damping material on thin steel panels to maximize modal loss factors. Several design examples are presented in Section 4 to demonstrate that the proposed method can provide optimal distribution for damping material. The final section summarizes the results and provides conclusions.

2. MODAL LOSS FACTORS OF A STRCUTURE WITH DAMPING MATERIAL

In this study, damping material is supposed to be applied on thin steel panels such as body structure of a vehicle and eigenvalues of the system of interest are calculated by finite element analysis.

2.1 Modal strain energy method

We briefly explain about modal strain energy method here. Governing equation for complex eigenvalue analysis is written as

$$\lambda_i \boldsymbol{M}\boldsymbol{\phi}_i = \boldsymbol{K}\boldsymbol{\phi}_i, \tag{1}$$

where $\boldsymbol{K}$ and $\boldsymbol{M}$ are stiffness matrix and mass matrix, respectively, and λ_i and $\boldsymbol{\phi}_i$ are i-th order complex eigenvalue and complex right eigenvector. This equation can be written as

$$\lambda_i^R(1+j\eta_i)\boldsymbol{M}\boldsymbol{\phi}_i = \boldsymbol{K}\boldsymbol{\phi}_i, \tag{2}$$

where λ_i^R is real part of λ_i, η_i is i-th modal loss factor, and j is imaginary unit. When $\boldsymbol{\phi}_i$ is assumed to be approximately represented by real eigenmode $\boldsymbol{\phi}_i^R$, and $\boldsymbol{\phi}_i^R$ is multiplied from left side, the equation is rewritten as

$$\lambda_i^R(1+j\eta_i)(\boldsymbol{\phi}_i^R)^T\boldsymbol{M}\boldsymbol{\phi}_i^R \approx (\boldsymbol{\phi}_i^R)^T(\boldsymbol{K}^R + j\boldsymbol{K}^I)\boldsymbol{\phi}_i^R, \tag{3}$$

where $\boldsymbol{K}^R$ and $\boldsymbol{K}^I$ are real part and imaginary part of complex stiffness matrix $\boldsymbol{K}$. By comparing real and imaginary part of both side of the equation, modal loss factor η_i is given as

$$\eta_i = \frac{(\boldsymbol{\phi}_i^R)^T\boldsymbol{K}^I\boldsymbol{\phi}_i^R}{(\boldsymbol{\phi}_i^R)^T\boldsymbol{K}^R\boldsymbol{\phi}_i^R}. \tag{4}$$

When n_d is the number of damping material applied and η_s $(s = 1, 2, \cdots, n_d)$ is loss factor of damping material s, imaginary part of stiffness matrix $\boldsymbol{K}^I$ is expressed as $\boldsymbol{K}^I = \sum_{s=1}^{n_d} \eta_s \boldsymbol{K}_s^R$. Substituting this expression in Eq. (3), modal loss factor η_i can be written as

$$\eta_i = \sum_{s=1}^{n_d} \eta_s \xi_{si}, \tag{5}$$

where ξ_{si} is the ratio of modal strain energy stored in damping material to total modal strain energy for damping material s, and defined as $\xi_{si} = (\boldsymbol{\phi}_i^R)^T \boldsymbol{K}_s^R \boldsymbol{\phi}_i^R / (\boldsymbol{\phi}_i^R)^T \boldsymbol{K}^R \boldsymbol{\phi}_i^R$.

2.2 Modal loss factors by real eigenvalues

ξ_{si} is not calculated by general eigenvalue solvers. Accordingly, a user-subroutine has to be coded. Therefore ,in the study presented here, we have developed a new approximate formulation to estimate modal loss factors by using only real eigenvalues.

Mass and stiffness matrix of a system including damping material are expressed respectively as

$$\boldsymbol{M} = \boldsymbol{M}^p + \sum_{s=1}^{n_d} \boldsymbol{M}_s^d, \tag{6}$$

$$\boldsymbol{K} = \boldsymbol{K}^p + \sum_{s=1}^{n_d} \boldsymbol{K}_s^d + j \sum_{s=1}^{n_d} \eta_s \boldsymbol{K}_s^d, \tag{7}$$

where $\boldsymbol{M}^p$ and $\boldsymbol{K}^p$ are mass and stiffness matrix of thin steel panel, respectively. $\boldsymbol{M}_s^d$ and $\boldsymbol{K}_s^d$ are mass matrix and real part of stiffness matrix of damping material s, respectively. Damping characteristics of thin steel panel is supposed to be ignored since it is usually much smaller than that of damping material.

Now we assume $\boldsymbol{\phi}_i$ is approximately represented by real eigenmode $\boldsymbol{\phi}_i^R$ as discussed in the previous section. By multiplying $\boldsymbol{\phi}_i^R$ from left side of Eq. (3), the equation is then written as

$$\lambda_i^R (1 + j\eta_i)(\boldsymbol{\phi}_i^R)^T (\boldsymbol{M}^p + \boldsymbol{M}^d)\boldsymbol{\phi}_i^R = (\boldsymbol{\phi}_i^R)^T (\boldsymbol{K}^p + \boldsymbol{K}^d + j \sum_{s=1}^{n_d} \eta_s \boldsymbol{K}_s^d)\boldsymbol{\phi}_i^R. \tag{8}$$

Here $\boldsymbol{M}^d$ and $\boldsymbol{K}^d$ are defined as $\boldsymbol{M}^d = \sum_{s=1}^{n_d} \boldsymbol{M}_s^d$ and $\boldsymbol{K}^d = \sum_{s=1}^{n_d} \boldsymbol{K}_s^d$, respectively. By comparing real part and imaginary part for both side of the equation, i-th modal loss factor η_i is given by

$$\eta_i = \frac{(\boldsymbol{\phi}_i^R)^T \sum_{s=1}^{n_d} \eta_s \boldsymbol{K}_s^d \boldsymbol{\phi}_i^R}{(\boldsymbol{\phi}_i^R)^T (\boldsymbol{K}^p + \boldsymbol{K}^d)\boldsymbol{\phi}_i^R}. \tag{9}$$

This equation can be rewritten as

$$\eta_i = \sum_{s=1}^{n_d} \eta_s \left(1 - \frac{(\boldsymbol{\phi}_i^R)^T (\boldsymbol{K}^p + \sum_{t=1,t\neq s}^{n_d} \boldsymbol{K}_t^d)\boldsymbol{\phi}_i^R}{(\boldsymbol{\phi}_i^R)^T (\boldsymbol{K}^p + \boldsymbol{K}_s^d + \sum_{t=1,t\neq s}^{n_d} \boldsymbol{K}_t^d)\boldsymbol{\phi}_i^R} \right) = \sum_{s=1}^{n_d} \eta_s \left(1 - \frac{1+\varepsilon_t^d}{1+\varepsilon_s^d+\varepsilon_t^d} \right), \tag{10}$$

where ε_s^d and ε_t^d are defined as $\varepsilon_s^d = (\boldsymbol{\phi}_i^R)^T \boldsymbol{K}_s^d \boldsymbol{\phi}_i^R / (\boldsymbol{\phi}_i^R)^T \boldsymbol{K}^p \boldsymbol{\phi}_i^R$, $\varepsilon_t^d = \sum_{t=1,t\neq s}^{n_d} (\boldsymbol{\phi}_i^R)^T \boldsymbol{K}_t^d \boldsymbol{\phi}_i^R / (\boldsymbol{\phi}_i^R)^T \boldsymbol{K}^p \boldsymbol{\phi}_i^R$, respectively. Since steel panel is generally much stiffer than damping material, $\varepsilon_s^d \varepsilon_t^d$ can be negligible compared with 1, ε_s^d and ε_t^d. Modal loss factor η_i is then approximately expressed as follows:

$$\eta_i \approx \sum_{s=1}^{n_d} \eta_s \left(1 - \frac{1+\varepsilon_t^d}{(1+\varepsilon_s^d)(1+\varepsilon_t^d)} \right) = \sum_{s=1}^{n_d} \eta_s \left(1 - \frac{1}{1+\varepsilon_s^d} \right) = \sum_{s=1}^{n_d} \eta_s \left(1 - \frac{(\boldsymbol{\phi}_i^R)^T \boldsymbol{K}^p \boldsymbol{\phi}_i^R}{(\boldsymbol{\phi}_i^R)^T (\boldsymbol{K}^p + \boldsymbol{K}_s^d)\boldsymbol{\phi}_i^R} \right) \tag{11}$$

We have to note that the approximation in the equation above is exactly correct when only one damping material is applied because ε_t^d becomes zero for $n_d = 1$.

Now we consider the i-th real eigenmode $\boldsymbol{\phi}_i^p$ when damping materials are removed, and the i-th real eigenmode $\boldsymbol{\phi}_{si}^d$ when damping material s with zero mass density is applied on thin steel panel. Then, $\boldsymbol{\phi}_i^p$ and

$\boldsymbol{\phi}_{si}^{d}$ satisfy the following eigen equation, respectively:

$$\lambda_i^p (\boldsymbol{\phi}_i^p)^T \boldsymbol{M}^p \boldsymbol{\phi}_i^p = (\boldsymbol{\phi}_i^p)^T \boldsymbol{K}^p \boldsymbol{\phi}_i^p, \tag{12}$$

$$\lambda_{si}^d (\boldsymbol{\phi}_{si}^d)^T \boldsymbol{M}^p \boldsymbol{\phi}_{si}^d = (\boldsymbol{\phi}_{si}^d)^T (\boldsymbol{K}^p + \boldsymbol{K}_s^d) \boldsymbol{\phi}_{si}^d, \tag{13}$$

where λ_i^p and λ_{si}^d are corresponding real eigenvalues as: When both $\boldsymbol{\phi}_i^p$ and $\boldsymbol{\phi}_{si}^d$ are expressed approximately by real eigenmode $\boldsymbol{\phi}_i^R$, Eqs. (**??**) and (**??**) can be written, respectively, as:

$$\lambda_i^p (\boldsymbol{\phi}_i^R)^T \boldsymbol{M}^p \boldsymbol{\phi}_i^R \approx (\boldsymbol{\phi}_i^R)^T \boldsymbol{K}^p \boldsymbol{\phi}_i^R, \tag{14}$$

$$\lambda_{si}^d (\boldsymbol{\phi}_i^R)^T \boldsymbol{M}^p \boldsymbol{\phi}_i^R \approx (\boldsymbol{\phi}_i^R)^T (\boldsymbol{K}^p + \boldsymbol{K}_s^d) \boldsymbol{\phi}_i^R. \tag{15}$$

Substituting these relationships to Eqs. (11), we finally obtain the approximate formulation of η_i expressed by the eigenvalues of λ_i^p and λ_{si}^d:

$$\eta_i \approx \sum_{s=1}^{n_d} \eta_s \left(1 - \frac{\lambda_i^p}{\lambda_{si}^d}\right). \tag{16}$$

As expected from Eqn. (11), (14) and (15), the approximate accuracy is getting worse as the stiffness or the thickness of damping material becomes large. However, the approximation can be valid practically as demonstrated in Section 4.

3. MAXIMIZATION OF MODAL LOSS FACTORS BY TOPOLOGY OPTIMIZATION OF DAMPING MATERIAL

3.1 Density method

The key idea of topology optimization is the introduction of a fixed extended design domain Ω^D that includes the original design domain Ω^d and the utilization of the characteristic function χ defined as

$$\chi(\boldsymbol{x}) = \begin{cases} 1 & (\boldsymbol{x} \in \Omega^d) \\ 0 & (\boldsymbol{x} \in \Omega^D \backslash \Omega^d) \end{cases}, \tag{17}$$

where $\boldsymbol{x}$ denotes a position in the extended design domain. This function signifies that material exists at locations where $\chi(\boldsymbol{x}) = 1$ and does not exist at locations where $\chi(\boldsymbol{x}) = 0$. Using this function, the topology optimization problem is defined in terms of finding the optimal distribution of the function $\chi(\boldsymbol{x})$ under several constraints including equilibrium equations. Since the characteristic function $\chi(\boldsymbol{x})$ has a binary value at every position in the domain Ω^D, an infinite number of discontinuities may appear in the design domain, which makes practical solutions impossible. To avoid such difficulties, the binary characteristic function $\chi(\boldsymbol{x})$ must be relaxed to a continuous function $\mu(\boldsymbol{x})$ that takes a real value between 0 and 1. In this study, we adopted the density method () for relaxation of the characteristic function due to its simplicity of formulation and implementation in numerical programs. In the density method, material characteristics are simply interpolated using a continuous density function that is regarded as a design variable. The solid isotropic material with penalization (SIMP) model is widely applied for optimization of elastic structures. The elasticity tensor $E_{ijkl}^0(\boldsymbol{x})$ is interpolated using a simple exponential expression of a design variable $\mu(\boldsymbol{x})$ that takes a value between 0 and 1 as follows:

$$E_{ijkl}^d(\boldsymbol{x}) = \mu^q(\boldsymbol{x}) E_{ijkl}^0, \tag{18}$$

where the exponential index q represents penalization powers and E_{ijkl}^0 is the elasticity tensor of the material used. Volume of the material in the extended design domain V is supposed to be proportional to the design variable $\mu(\boldsymbol{x})$ and expressed by

$$V = \int_{\Omega^D} \mu(\boldsymbol{x}) d\Omega. \tag{19}$$

3.2 Maximization of modal loss factors

In order to maximize *i*-th modal loss factor η_i, its negative value should be minimized under a prescribed volume constraint. We can formulate the optimization problem as follows:

$$\begin{aligned} \text{Minimize}\;\; & F = -\eta_i, \\ \text{Subject to}\;\; & Eigenvalue\ equation, \\ & \int_{\Omega^D} \mu(\boldsymbol{x}) d\Omega \leq V_0, \\ & 0 < \mu_{\min} \leq \mu(\boldsymbol{x}) \leq 1, \end{aligned} \tag{20}$$

where V_0 is the upper bound of the volume constraint for the damping material applied and $\mu_{\min}$ is a small positive value for the lower bound of the design variables set to avoid numerical instabilities. Eigenvalue equation is given by

$$\left(\boldsymbol{K}^R - \lambda_i^R \boldsymbol{M}\right) \boldsymbol{\phi}_i^R = \boldsymbol{O}, \tag{21}$$

when modal loss factors are evaluated by MSE method, and is given by

$$\left(\boldsymbol{K}^p + \boldsymbol{K}_s^d - \lambda_{si}^d \boldsymbol{M}^p\right) \boldsymbol{\phi}_{si}^d = \boldsymbol{O}, \;\; (s = 1, \cdots, n_d), \tag{22}$$

when modal loss factors are evaluated by real eigenvalues.

For maximization of multiple modal loss factors simultaneously, a linear weighted summation of the negative values η_i can be minimized. The objective function is taken as follows :

$$\text{Minimize } F = \sum_i -\beta_i \eta_i \tag{23}$$

where β_i is a prescribed positive value to account for the degree of the contribution of *i*-th mode to the objective function.

3.3 Discretization by continuous approximation of material distribution

The optimization problem in Eq. (**??**) can be solved numerically by discretizing the design variables. Considering the relaxation of the solution space from a binary function $\chi(\boldsymbol{x})$ to a continuous function $\mu(\boldsymbol{x})$, the material distribution should also be continuous over the design domain Ω^D, and this condition must hold true even after the discretization. When the relaxed optimization problem is solved in conjunction with a finite element analysis for solving the equilibrium equations, the design variables are usually discretized using finite element meshes prepared for the purpose of solving equilibrium equations. If the design variables are assigned to elements, they are usually set to piecewise constant values within each element. Sufficiently fine discretization is required in this setting, because a continuous material distribution is assumed through the relaxation of the solution space. However, using a such fine mesh may exceed the bounds of practical calculation resources. Furthermore, as pointed out by Sigmund and Petersson (8), numerical instabilities such as checkerboard patterns and mesh-dependencies also occur. To mitigate these numerical problems, several methods have been proposed such as filtering schemes by Sigmund (9), perimeter control by Haber (10), and the use of a local gradient constraint by Niordson (11). However, trial-and-error processes may be required in the above schemes to define appropriate bounds for the perimeter or gradient.

To overcome such problems, Matsui and Terada (12), and Rahmatalla and Swan (13) proposed that discretized design variables should be assigned to the nodes of the elements and interpolated by a continuous function within each element. This method ensures at least C^0 continuity of the design variables over the design domain, even if the adopted finite element mesh is not fine. Now, we can approximately express the design variable $\mu_i(\boldsymbol{x})$ by the following discretized formulation using the vectors $\boldsymbol{N}_t$ and $\boldsymbol{\mu}_i$ composed of interpolation functions N_{tj} and nodal design variables μ_{ij}, respectively:

$$\mu_i(\boldsymbol{x}) \simeq \boldsymbol{N}_t^T \boldsymbol{\mu}_i = \sum_{j=1}^{n_d} N_{tj} \mu_{ij}, \tag{24}$$

where n_d is the number of design variables. In this study, the bi-linear interpolation function is applied for N_{tj} due to its simplicity when quadrilateral elements are used. Consequently, the design variable $\mu_i(\boldsymbol{x})$ can preserve C^0 continuity throughout the design domain due to the partition-of-unity property.

3.4 Design sensitivities

When modal loss factors are calculated by MSE method, the design sensitivity of η_i with respect to design variable μ_j are given by

$$\frac{\partial \eta_i}{\partial \mu_j} = \frac{\dfrac{\partial f_i}{\partial \mu_j} g_i - f_i \dfrac{\partial g_i}{\partial \mu_j}}{g_i^2}, \tag{25}$$

where $f_i = (\boldsymbol{\phi}_i^R)^T \boldsymbol{K}^I \boldsymbol{\phi}_i^R$ and, $g_i = (\boldsymbol{\phi}_i^R)^T \boldsymbol{K}^R \boldsymbol{\phi}_i^R$. Sensitivities of f_i and g_i with respect to μ_j are written as

$$\frac{\partial f_i}{\partial \mu_j} = (\boldsymbol{\phi}_i^R)^T \frac{\partial \boldsymbol{K}^I}{\partial \mu_j} \boldsymbol{\phi}_i^R + 2(\boldsymbol{\phi}_i^R)^T \boldsymbol{K}^I \frac{\partial \boldsymbol{\phi}_i^R}{\partial \mu_j}, \tag{26}$$

$$\frac{\partial g_i}{\partial \mu_j} = (\boldsymbol{\phi}_i^R)^T \frac{\partial \boldsymbol{K}^R}{\partial \mu_j} \boldsymbol{\phi}_i^R + 2(\boldsymbol{\phi}_i^R)^T \boldsymbol{K}^R \frac{\partial \boldsymbol{\phi}_i^R}{\partial \mu_j}. \tag{27}$$

The design sensitivity of $\boldsymbol{\phi}^R$ with respect to design variable μ_j can be calculated by a method proposed by Nelson (14).

When modal loss factors are calculated by the method proposed in this study, the design sensitivity of η_i with respect to design variable μ_j are given by

$$\frac{\partial \eta_i}{\partial \mu_j} \approx \sum_{s=1}^{n_d} -\eta_s \frac{\lambda_i^p}{(\lambda_{si}^d)^2} \frac{\partial \lambda_{si}^d}{\partial \mu_j}. \tag{28}$$

The design sensitivity of eigenvalues λ_{si}^d with respect to μ_j ca be derived by differentiating the eigen equation $\left(\boldsymbol{K}^p + \boldsymbol{K}_s^d - \lambda_{si}^d \boldsymbol{M}^p\right) \boldsymbol{\phi}_{si}^d = \boldsymbol{O}$ and is written as

$$\frac{\partial \lambda_{si}^d}{\partial \mu_j} = \frac{(\boldsymbol{\phi}_{si}^d)^T \dfrac{\partial \boldsymbol{K}_s^d}{\partial \mu_j} \boldsymbol{\phi}_{si}^d}{(\boldsymbol{\phi}_{si}^d)^T \boldsymbol{M}^p \boldsymbol{\phi}_{si}^d}. \tag{29}$$

The design sensitivity of effective volume in the extended design domain V with respect to μ_j is given by

$$\frac{\partial V}{\partial \mu_j} = \int_{\Omega^D} \frac{\partial \mu}{\partial \mu_j} d\Omega. \tag{30}$$

3.5 Design sensitivity filter

In topology optimization for minimizing mean compliance of an elastic structure, various filters have been proposed to avoid mesh dependencies that fine discretization of design domain gives different topologies (8, 15, 16).

A local filter that is proposed by Sigmund (8) and is widely applied in many topology optimization problems modifies design sensitivity at node i by averaging design sensitivities at node j in a neighborhood domain with a weighting function of distance between node i and node j:

$$\widehat{\frac{\partial \eta_i}{\partial \mu_j}} = \frac{1}{\sum_{k=1}^{N} (R - r_{jk}) \mu_k} \sum_{k=1}^{N} \frac{\partial \eta_i}{\partial \mu_k} (R - r_{jk}) \mu_k, \;\; r_{jk} < R \tag{31}$$

where R is radius of the local filter, r_{jk} is the distance between node i and node j, and N is the number of nodes inside radius R. When R is smaller than element size, $\widehat{\dfrac{\partial \eta_i}{\partial \mu_j}}$ is identical to the original design sensitivity $\dfrac{\partial \eta_i}{\partial \mu_j}$. Although this filter is purely heuristic and is not proved mathematically, it has generated mesh-independent optimal solutions in many optimization problems.

3.6 Algorithm of optimization process

An optimization algorithm to solve the problem defined in Eq. (20) is explained in the flowchart shown in Fig. 1.

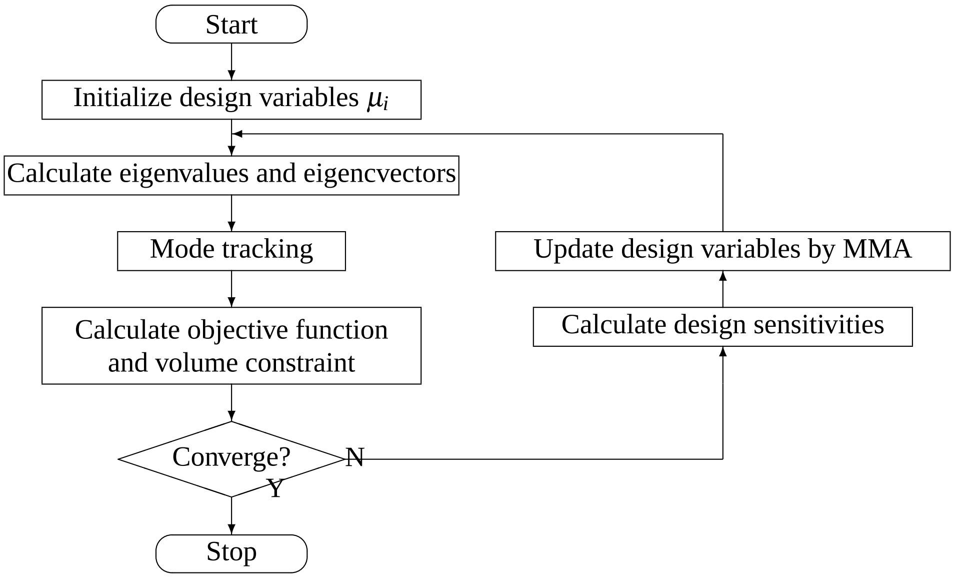


Figure 1 – Flowchart of optimization process

In the beginning of the optimization calculation, the design variable $\boldsymbol{\mu}$ is initialized to a prescribed value. It consists of five steps in an iteration loop. In the first step of the iteration count k, the elasticity tensor of the damping material is calculated by using the design variable $\boldsymbol{\mu}^k$ and Eq.(18), and the eigenvalues and the eigenvectors for the system of interest are calculated by finite element method. In the second step, the mode tracking based on Modal Assurance Criterion (MAC) is conducted and the eigenvectors that coincide with the prescribed eigenvectors to be suppressed are identified. The objective function F of the corresponding eigenvalues and the volume V of damping material in the extended design domain are then calculated. If the objective function converges, the optimal material distribution is obtained and the iteration of the optimization calculation terminates. Otherwise the design sensitivity of the objective function and the design sensitivity of the volume with respect to the design variables $\boldsymbol{\mu}^k$ are computed in the third step. In the final step, the design variables are updated using Method of Moving Asymptotes (MMA)(17). Then the optimization calculation goes back to the first step and the iteration count is incremented by 1. These procedures are iterated until the objective function converges.

4. NUMERICAL VERIFICATION

In this section, to verify the methodology described above, modal loss factors calculated by the proposed method by using only real eigenvalues are compared with those by the conventional MSE method. Then the optimal layout of damping material on a steel flat panel to maximize the modal loss factor for prescribed eigenmodes is compared with strain energy distribution of the panel, which is conventionally utilized to define the layout of damping material.

4.1 Verification of modal loss factors by real eigenvalues

Figure 2 shows a finite element model of a thin flat steel panel with a damping material to be applied for numerical verification. The panel has the size of 365 mm $\times$ 250 mm and the thickness of 0.8 mm with all edges fixed, and is discritized by 72 $\times$ 48 first order hexahedral elements. To avoid shear locking, a bubble function is added for interpolation function of elements. Young's modulus, Poisson's ratio, mass density and loss factor of the steel panel are 210 GPa, 0.29, 7860 kg/m^3, and 0.00, respectively. Young's modulus, Poisson's ratio, mass density and loss factor of the damping material are 1.00 GPa, 0.40, 1500 kg/m^3, and 0.40, respectively. The damping material of the 2mm thickness is assumed to be applied on the panel. Extended fixed design domain is defined where the damping material is allowed to be attached, and the layout of the damping material is optimized by the proposed method.

A damping material is attached on the steel panel with uniform thickness, and modal loss factors calculated by the proposed method are compared with those by the conventional MSE method. Table 1 gives modal loss factors for 1st to 5th eigenmode. The difference between these two values are less than 3%. Predictions by

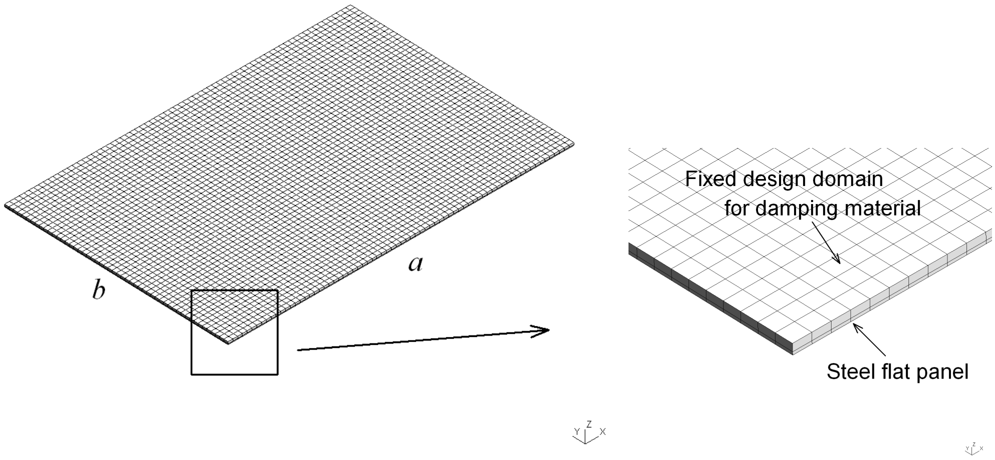


Figure 2 – Finite element model of flat panel with fixed design domain for damping material.

Table 1 – Comparisons of modal loss factors for uniform distributions of damping material by MSE method and by the proposed method.

	MSE	Proposed	Difference [%]
Mode 1	0.0988	0.1011	2.336
Mode 2	0.0995	0.1016	2.141
Mode 3	0.0985	0.1008	2.319
Mode 4	0.0998	0.1018	2.023
Mode 5	0.0986	0.1008	2.195

the proposed method can be practically valid to evaluate damping characteristics of panels.

4.2 Verification of optimized layout of damping material

The damping material is allowed to use by 50% of the volume of the fixed design domain. All the design variables are initialized to 0.5 before starting the optimization process. The lower bound of design variables is set to 1.0×10^{-6} to avoid numerical instability.

We consider the optimal design of damping material to maximize the modal loss factor for 1st eigenmode. Figure 3 shows (a) displacement of panel at 1st eigenmode, (b) strain energy distribution of damping material, (c) optimal distribution of damping material based on modal loss factor by MSE method, and (d) optimal distribution of damping material based on modal loss factor by the proposed method. In the optimal design, damping material distributes mainly where the strain energy is concentrated. This result agrees well with the conventional design methodologies. Moreover, the optimal design of damping material shown in Fig. 3(c) is almost the same as one shown in Fig. 3(d).

Modal loss factors for optimal distributions of damping material by MSE method and by the proposed method are 0.0841 and 0.0866, respectively. The difference between them is less than 3%, which ensures that the proposed method gives a practical approximated solution.

4.3 Practical design of damping material

As shown in Fig. 3, the optimal distributions of damping material are separated into several domains. However, in the manufacturing point of view, those distributions are not appropriate in terms of required process and cost. Thus, the sensitivity filter defined in Eq. (31) is applied to obtain material distribution in a single domain.

Figure 4 presents the optimal material distributions to maximize modal loss factor for 1st eigenmode. The radius of sensitivity filter R is varied as $0.05a$, $0.10a$ and $0.20a$. Damping material forms collectively as

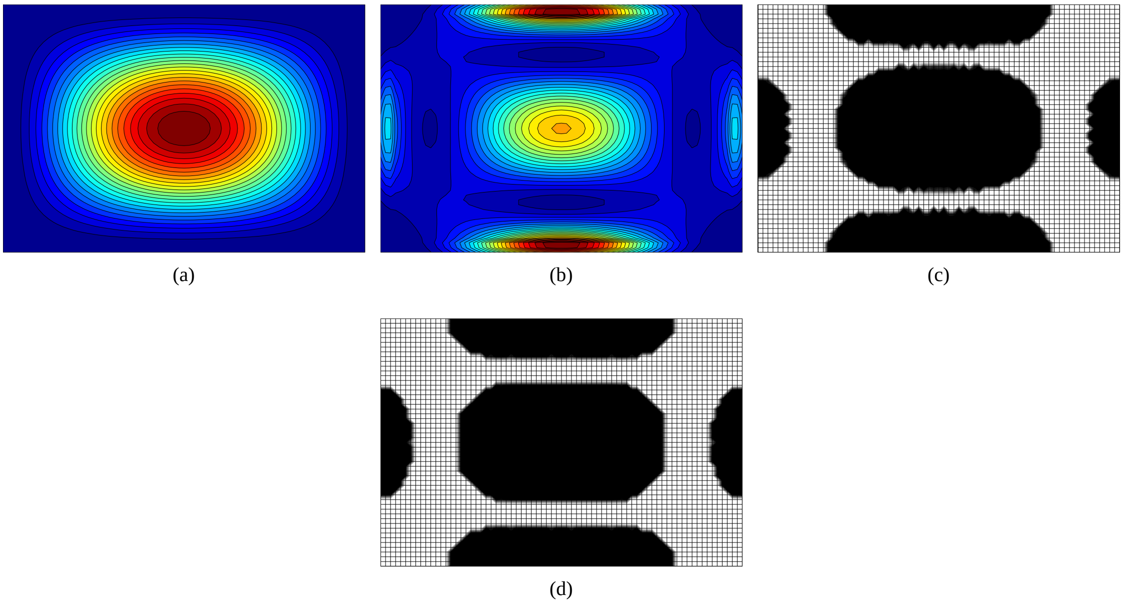
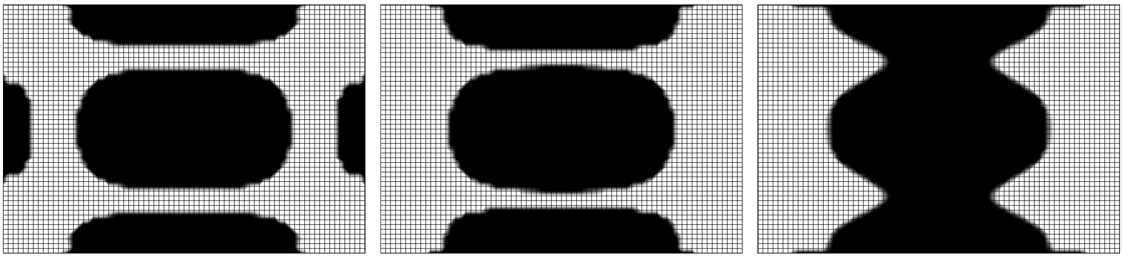


Figure 3 – (a) Displacement of panel at 1st eigenmode, (b) strain energy distribution of damping material, and optimal distribution of damping material using the modal factor (c) by MSE method and (d) by the proposed method.



(a) $R = 0.05a$ (b) $R = 0.10a$ (c) $R = 0.20a$

Figure 4 – Optimal distributions of damping material for 1st eigenmode with sensitivity filter.

the radius becomes large and is located in a single domain when R equals $0.20a$. Modal loss factors when the radius R is $0.05a$, $0.10a$ and $0.20a$ are 0.0864, 0.0846 and 0.808, respectively. As expected, modal loss factors become small as the radius R is large. By applying a sensitivity filter that utilizes a weighted average of design sensitivities over local area, damping material can be distributed collectively in a single domain to meet practical requirements for manufacturing,

5. CONCLUSIONS

This paper proposes an optimization method to maximize modal loss factors by optimizing the layout of damping material. In this method, the eigenvectors are assumed to be almost the same as the eigenvectors when damping material is removed, and the modal loss factors can then be expressed by using a corresponding eigenvalue where the mass density of the damping material is ignored whereas the stiffness is taken into account. Moreover, by applying a sensitivity filter utilizing a weighted average of design sensitivities over local area, damping material can be formed collectively in a single domain, which meets practical requirements for manufacturing.

REFERENCES

1. Yuge K, Takahashi Y, Saito K. Optimal layout of damping material using homogenization method. Transactions of the Japan Society of Mechanical Engineers, Series A. 1999;65(634):1224–1231.

2. Kang Z, Zhang X, Jiang S, Cheng G. On topology optimization of damping layer in shell structures under

harmonic excitations. Structural and Multidisciplinary Optimization. 2012;46:51 – 67.

3. Zheng H, Cai C, Tan XM. Optimization of partial constrained layer damping treatment for vibrational energy minimization of vibrating beams. Computers and Structures. 2004;82:2493 – 2507.
4. Lassila T. Optimal damping of a membrane and topological shape optimization. Structural and Multidisciplinary Optimization. 2009;38:43 – 52.
5. Johnson CD, Kienholz DA. Finite element prediction of damping in structures with constrained viscoelastic layers. American Institute of Aeronautics and Astronautics Journal. 1982;20:1284 – 1290.
6. Yamaguchi T, Kurosawa Y, Matsumura S, Nomura A. Finite element analysis for vibration properties of panels in car bodies having viscoelastic damping layer. Transactions of the Japan Society of Mechanical Engineers, Series C. 2003;69:297 – 303.
7. Ling Z, Ronglu X, Yi W, Sabbagh AE. Topology optimization of constrained layer damping on plates using Method of Moving Asymptote (MMA) approach. Shock and Vibration. 2011;18:221 – 244.
8. Sigmund O, Petersson J. Numerical instabilities in topology optimization: A survey on procedures dealing with checkerboards, mesh-dependencies and local minima. Structural Optimization. 1998;16:68–75.
9. Bendsøe MP, Sigmund O. Topology Optimization – Theory, Methods, and Applications –. Springer; 2003.
10. Haber RB, Jog CS, Bendsøe MP. A new approach to variable-topology shape design using a constraint on perimeter. Structural Optimization. 1996;11:1–12.
11. Niordson F. Optimal design of elastic plates with a constraint on the slope of the thickness function. International Journal of Solids and Structures. 1983;19:141–151.
12. Matsui K, Terada K. Continuous approximation of material distribution for topology optimization. International Journal for Numerical Methods in Engineering. 2004;59:1925–1944.
13. Rahmatalla SF, Swan CC. A Q4/Q4 continuum structural topology optimization implementation. Structural and Multidisciplinary Optimization. 2004;27:130–135.
14. Nelson RB. Simplified calculation of eigenvector derivatives. American Institute of Aeronautics and Astronautics Journal. 1976;14:1201 – 1205.
15. Guest JK, Prévost JH, Belytschko T. Achieving minimum length scale in topology optimization using nodal design variables and projection functions. International Journal for Numerical Methods in Engineering. 2004;61:238–254.
16. Sigmund O. Morphology-based black and white filters for topology optimization. Structural and Multidisciplinary Optimization. 2007;33:401–424.
17. Svanberg K. The method of moving asymtotes – a new method for structural optimization. International Journal for Numerical Methods in Engineering. 1987;24:359–373.