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Vibration transfer analysis based on characterization of vibration energy dissipation

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ABSTRACT

Reducing vibration energy of mechanical structures by controlling energy dissipation is important for improving structure-borne noise, such as interior noise in a vehicle. In this study, vibration transfer characteristics are investigated using vibration energy evaluation, when no particular vibration mode is dominant. A beam structure modeled by Finite Element approach is used as an example. We examine the efficient method of determining effective position to add damping. The method using distribution of dissipation power and that using damping addition sensitivity analysis of the vibration energy are examined. In the resonance excitation case, the optimum damping layout for vibration suppression can be determined by both methods. On the other hand, in the non-resonance excitation case, no obvious correlation is found between the optimum damping layout and the distribution of dissipation power. However, the analysis of damping addition sensitivity leads the optimum damping layout even if the excitation frequency is non-resonance.

Keywords: Vibration energy, Damping, Sensitivity I-INCE Classification of Subjects Number(s): 43.2

1. INTRODUCTION

To improve noise field such as interior noise in a vehicle, reduction of vibration propagation is important. Vehicle noise is influenced by many kinds of vibration transfer in structures such as body panels, chassis frames, suspension arms, drive shafts, exhaust pipes, and tires. Modal analysis is generally used to examine these phenomena (1). However, in mid-frequency, modal analysis has difficulty due to density of vibration modes. Furthermore, tire is made of polymer, and some structures have been replaced by plastic or FRP because of weight saving. So vibration of structures with high damping becomes important. High damping structure also has tendency to raise modal density. Therefore, vibration reduction in non-resonance excitation case becomes important in which plural modes contribute simultaneously.

To examine vibration transfer or vibration power flow, structural intensity has been proposed (2, 3). Vibration has been reduced using damping material effectively considering power dissipation (4, 5). The modal expansion technique of structural intensity has been proposed (6). However most of these studies target resonance condition.

In this paper, we investigate reduction of the structure vibration energy, focusing on the energy dissipation caused by damping. Not only resonance condition but also non-resonance condition is evaluated. We examine the efficient method of determining effective position to add damping. Two kinds of method are examined. The one is distribution of the dissipation power and the other is damping addition sensitivity analysis of the vibration energy. A beam structure modeled by Finite Element approach is used for simplicity.

2. THEORY OF BEAM VIBRATION

2.1 Finite Element Analysis of Bending Vibration of Beam

Fig. 1 shows a uniform beam along *x*-axis. For the bending vibration of the beam with harmonic excitation, vibration characteristic is evaluated using Finite Element analysis.

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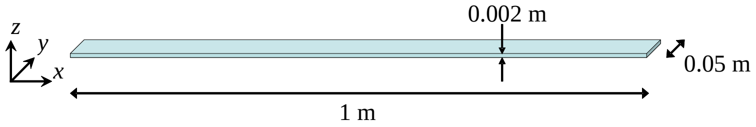


Figure 1 – A beam model

When the harmonic excitation force with amplitude **F** and angular frequency ω

$$\mathbf{F}_{ins}(t) = \mathbf{F}e^{j\omega t} \tag{1}$$

is applied to the beam, the steady-state response displacement can be expressed as follows, using complex displacement amplitude **u**:

$$\mathbf{u}_{ins}(t) = \mathbf{u}e^{j\omega t} \tag{2}$$

where j is an imaginary unit.

The displacement amplitude **u** can be expressed as follows:

$$\mathbf{u} = \left[-\omega^2\mathbf{M} + j\omega\mathbf{C} + \mathbf{K}\right]^{-1}\mathbf{F} \tag{3}$$

where **M** is the mass matrix, **C** is the damping matrix, and **K** is the stiffness matrix.

2.2 Steady-State Vibration Energy

In this paper, steady-state vibration energy of the beam is used as an index to express the strength of vibration. The period average $\bar{E}_U$ of the steady-state strain energy of the beam can be expressed as follows:

$$\begin{aligned}\bar{E}_U &= \frac{1}{T_c}\int_0^{T_c}\frac{1}{2}\mathrm{Re}\left[\mathbf{u}e^{j\omega t}\right]^T\mathbf{K}\,\mathrm{Re}\left[\mathbf{u}e^{j\omega t}\right] \\ &= \frac{1}{4}\left[\mathrm{Re}[\mathbf{u}]^T\mathbf{K}\,\mathrm{Re}[\mathbf{u}] + \mathrm{Im}[\mathbf{u}]^T\mathbf{K}\,\mathrm{Im}[\mathbf{u}]\right]\end{aligned} \tag{4}$$

where T_c is period, Re[] means real part, Im[] means imaginary part, and a transpose is denoted by T.

In the same way, the period average $\bar{E}_K$ of the steady-state kinetic energy can be expressed as follows:

$$\bar{E}_K = \frac{\omega^2}{4}\left[\mathrm{Re}[\mathbf{u}]^T\mathbf{M}\,\mathrm{Re}[\mathbf{u}] + \mathrm{Im}[\mathbf{u}]^T\mathbf{M}\,\mathrm{Im}[\mathbf{u}]\right] \tag{5}$$

The period average $\bar{E}$ of the total vibration energy is

$$\bar{E} = \bar{E}_U + \bar{E}_K \tag{6}$$

2.3 Variation in Vibration Energy Caused by Damping Addition

As the way of modifying structure to reduce vibration energy, damping material addition is supposed. The element damping matrix $\mathbf{C}_i$ of one element i of the beam can be modified to $(1+\alpha)\mathbf{C}_i$ by adding damping. The period average of the vibration energy with additional damping, calculated by Eq. (5), is expressed as $\bar{E}_i$. Then, the amount of change from initial condition

$$\delta E_i = \bar{E}_i - \bar{E} \tag{7}$$

means the variation of vibration energy of the beam caused by damping addition to the element i. By calculating this amount concerning each element of the beam, the distribution of influence of each element damping is obtained. This distribution shows the effective damping addition area to reduce vibration energy. In this paper, this evaluation method is called repeated dynamic analysis. The result of this method is used as semi-optimum solution.

3. EXAMINATION OF EFFICIENT METHOD OF DETERMINING THE EFFECTIVE POSITION TO ADD DAMPING

In this paper, the bending vibration of a beam with uniform material damping is investigated. The efficient method of determining the effective position to add damping in order to reduce the vibration energy is examined. “Efficient method” means that repetition of dynamic analysis is unnecessary.

3.1 Method using Distribution of Dissipation Power

3.1.1 Direct Method

We hypothesized that if the beam has uniform damping in the initial state, the addition of damping in areas of high dissipation power should be effective. So, the distribution of dissipation power is evaluated to determine the position to add damping.

At one element i, the period average of dissipation power due to damping can be expressed as the work of damping force:

$$\begin{aligned}\overline{D}_i &= \frac{1}{T_c}\int_0^{T_c} \mathrm{Re}\left[\mathbf{C}_i \frac{d}{dt}\left(\mathbf{u}_i e^{j\omega t}\right)\right]^T \mathrm{Re}\left[\frac{d}{dt}\left(\mathbf{u}_i e^{j\omega t}\right)\right]dt \\ &= \frac{1}{2}\left\{j\omega\mathbf{u}_i\right\}^* \mathbf{C}_i \left\{j\omega\mathbf{u}_i\right\} \\ &= \frac{\omega^2}{2}\mathbf{u}_i^{\ *}\mathbf{C}_i\mathbf{u}_i\end{aligned} \tag{8}$$

Where complex conjugates is denoted by *. By calculating it about all elements of the beam, the distribution of dissipation power is obtained.

Additionally, the period average of strain energy of the element i can be expressed as follows:

$$\begin{aligned}\overline{E}_{U,i} &= \frac{1}{T_c}\int_0^{T_c} \mathrm{Re}\left[\mathbf{u}_i e^{j\omega t}\right]^T \mathrm{Re}\left[\mathbf{u}_i e^{j\omega t}\right]dt \\ &= \frac{1}{4}\mathbf{u}_i^{\ *}\mathbf{K}_i\mathbf{u}_i\end{aligned} \tag{9}$$

So, in proportional viscous damping system, the distribution of dissipation power is in proportion to the distribution of strain energy. In this paper, Eq. (8) is called direct method.

3.1.2 Structural Intensity Method

The period average I of the structural intensity for the bending vibration of a uniform beam can be expressed as the passage of energy in a unit section in a unit time (6):

$$I = \frac{1}{2}\mathrm{Re}\left[-j\omega\left(\frac{Q}{A}w^* + \frac{M}{A}\theta^*\right)\right] \tag{10}$$

where Q is the shearing force, w is the translation displacement in the z direction, M is the bending moment, θ is the angular displacement of the y-axis circumference, and A is the cross-sectional area of the beam.

At each element, the difference between inflow intensity and outflow intensity means power dissipated in the element.

$$\Delta I_i = I_{i,in} - I_{i,out} \tag{11}$$

By calculating it about all elements of the beam, the distribution of dissipation power is obtained. In this paper, Eq. (11) is called intensity method.

3.2 Method using Sensitivity Analysis of Vibration Energy

We propose damping addition sensitivity analysis of vibration energy. At a structure shown in Fig. 2, damping $\alpha\mathbf{C}_i$ is added to element i, whose initial element damping matrix is $\mathbf{C}_i$.

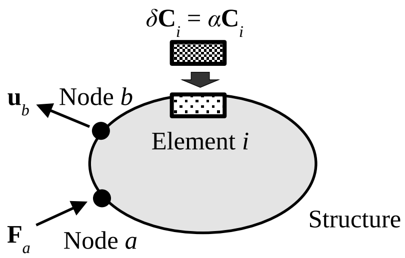


Figure 2 – A diagram of damping addition sensitivity analysis

When excitation force $\mathbf{F}_a$ is applied at node a, the response displacement can be expressed as follows:

$$\begin{Bmatrix} \mathbf{u}_a \\ \mathbf{u}_b \\ \mathbf{u}_i \end{Bmatrix} = \begin{bmatrix} \mathbf{G}_{aa} & \mathbf{G}_{ab} & \mathbf{G}_{ai} \\ \mathbf{G}_{ba} & \mathbf{G}_{bb} & \mathbf{G}_{bi} \\ \mathbf{G}_{ia} & \mathbf{G}_{ib} & \mathbf{G}_{ii} \end{bmatrix} \begin{Bmatrix} \mathbf{F}_a \\ \mathbf{0} \\ -\alpha\mathbf{C}_i j\omega\mathbf{u}_i \end{Bmatrix} \tag{12}$$

where $\mathbf{u}_a$ is the displacement of the excitation node a, $\mathbf{u}_b$ is the displacement of one node b, $\mathbf{u}_i$ is the displacement of damping added element i, and $\mathbf{G}_{xx}$ are the initial Compliance matrixes.

When Eq. (12) is differentiated with design variable α, the following equation is obtained.

$$\frac{d\mathbf{u}_b}{d\alpha} = -\mathbf{G}_{bi}\mathbf{C}_i j\omega\mathbf{u}_i + \mathbf{G}_{bi}\alpha\mathbf{C}_i j\omega\left[\mathbf{I} + \mathbf{G}_{ii}\alpha\mathbf{C}_i j\omega\right]^{-1}\mathbf{G}_{ii}\mathbf{C}_i j\omega\mathbf{u}_i \tag{13}$$

Sensitivity is Eq. (13) with α=0, so the damping addition sensitivity of displacement can be obtained as follows:

$$\left.\frac{d\mathbf{u}_b}{d\alpha}\right|_{(\alpha=0)} = -\mathbf{G}_{bi}\mathbf{C}_i j\omega\mathbf{G}_{ia}\mathbf{F}_a \tag{14}$$

The damping addition sensitivity S of the period average of the vibration energy can be calculated by differentiating the energy formulas Eq. (4)~(6) with variable α.

$$S = S_U + S_K \tag{15}$$

$$S_U = \frac{dE_U}{d\alpha} = \frac{1}{2}\left[\mathrm{Re}[\mathbf{u}]^T \mathbf{K}\,\mathrm{Re}\left[\frac{d\mathbf{u}}{d\alpha}\right] + \mathrm{Im}[\mathbf{u}]^T \mathbf{K}\,\mathrm{Im}\left[\frac{d\mathbf{u}}{d\alpha}\right]\right] \tag{16}$$

$$S_K = \frac{dE_K}{d\alpha} = \frac{\omega^2}{2}\left[\mathrm{Re}[\mathbf{u}]^T \mathbf{M}\,\mathrm{Re}\left[\frac{d\mathbf{u}}{d\alpha}\right] + \mathrm{Im}[\mathbf{u}]^T \mathbf{M}\,\mathrm{Im}\left[\frac{d\mathbf{u}}{d\alpha}\right]\right] \tag{17}$$

where S_U and S_K are the sensitivity of the period average of the strain energy and the kinetic energy, and d$\mathbf{u}$/dα is the column vector of the sensitivity of all node's displacement.

By calculating Eq. (15) with damping addition to each of all elements, the distribution of the damping addition sensitivity of the vibration energy can be obtained.

4. VALIDATION OF THE EVALUATIO METHOD OF DAMPING ADDITION

4.1 Description of the Beam Model

We use a beam structure shown in Fig. 1, modeled by Finite Element approach. The beam is constructed of brass and has a rectangular section. The boundary condition is free-free. The input force is a harmonic vertical excitation of 1 N at a position 0.4 m from the left. The model has proportional viscous damping. The damping matrix $\mathbf{C}$ is expressed as follows:

$$\mathbf{C} = \beta\mathbf{K} \tag{18}$$

The factor of proportionality is β=0.0003. To evaluate high damping condition, the factor is set to slightly higher value.

We consider the issue of determining the effective position to add damping in order to reduce the steady-state vibration energy $\bar{E}$. The methods of determining the effective position to add damping are validated.

4.2 Resonance Excitation Case

In the first case, the excitation frequency is 38.2 Hz, which is the third resonance frequency. The shape of vibration response is shown in Fig. 3. Because of the third resonance excitation, the third bending mode is dominant.

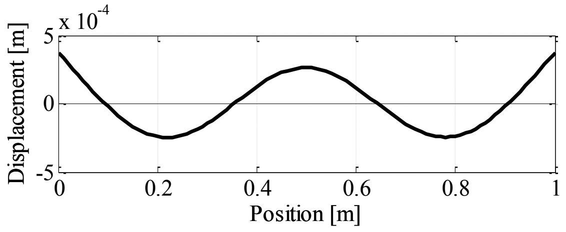


Figure 3 – Shape of vibration response at resonance condition

4.2.1 Calculation of Optimum Damping Addition (Resonance Case)

Fig. 4 shows the energy variation calculated using the repeated dynamic analysis, expressed in Eq. 7. Damping matrix of each element of the whole beam is increased by 1 % sequentially. The result shows that the steady-state vibration energy can be reduced effectively when damping is added at the 0.2 m, 0.5 m, or 0.8 m position. We treat it as the semi-optimum solution.

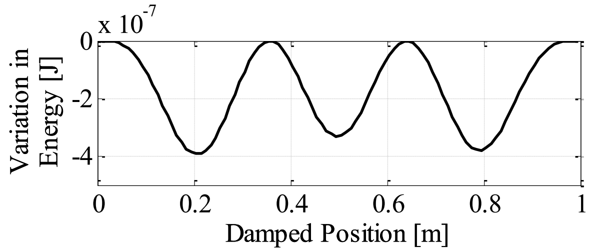


Figure 4 – Energy variation caused by adding damping

4.2.2 Validation of the Method using Distribution of Dissipation Power (Resonance Case)

First, the method using distribution of dissipation power is validated. Fig. 5 shows the result of the calculated structural intensity of the beam. The power flow in the direction of the x-axis is positive. The discontinuous position 0.4 m is the excitation point. The negative slope means that energy is dissipated in the elements.

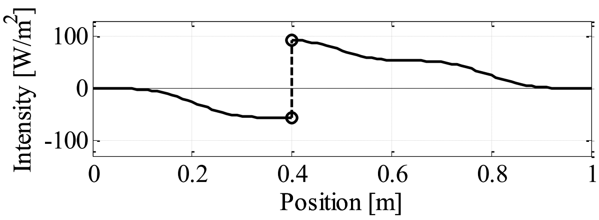


Figure 5 – Calculated structural intensity at resonance condition

Based on Eq. (11), the distribution of the dissipation power can be calculated from the intensity. The result is shown in Fig. 6 with thin line. The dissipation power is large at the 0.2 m, 0.5 m, and 0.8 m. This result is equal to the result of the direct method shown with bold line. Because these results seem to correlate the semi-optimum solution of damping addition, it is validated that the effective position to add damping can be determined using either dissipation power method - the direct method or the intensity method.

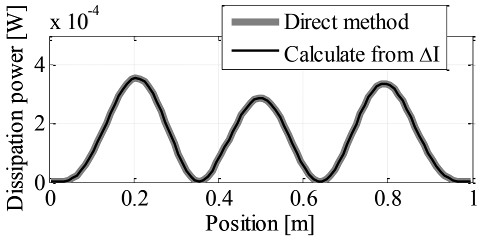
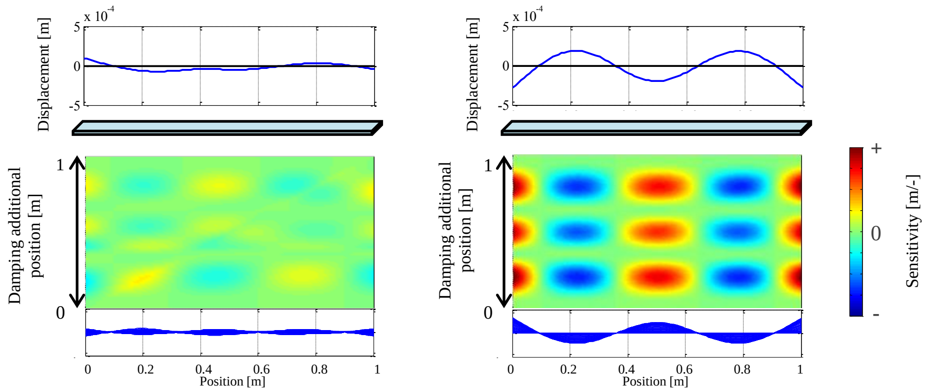


Figure 6 – Distribution of dissipation power at resonance condition

4.2.3 Validation of the Method using Sensitivity Analysis (Resonance Case)

Next, the method using sensitivity analysis is validated. The upper section of Fig. 7 shows the real part and imaginary part of the displacement amplitude. The middle section of this figure shows the damping addition sensitivity of displacement calculated from Eq. (14). The color contour expresses sensitivity. When damping is added at the positon expressed by the vertical axis, distribution of sensitivity of displacement appears, which is shown as one horizontal line of this color contour chart. Damping addition at 0.2 m, 0.5 m, and 0.8 m has reduction sensitivity for the imaginary part which is dominant in the complex displacement amplitude.



(a) Real part (b) Imaginary part

Figure 7 – Damping addition sensitivity of displacement

Based on these results, the damping addition sensitivity of the steady-state vibration energy is calculated as shown in Fig. 8, using Eq. (15). There is negative sensitivity at 0.2 m, 0.4 m, and 0.8 m. This result correlates to the semi-optimum solution. So it is validated that the effective position to add damping can be determined using this sensitivity analysis.

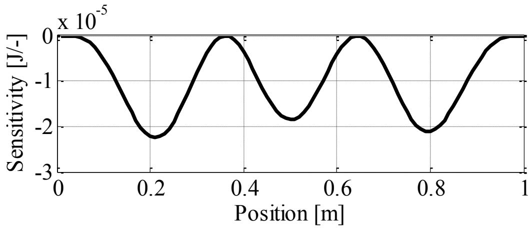


Figure 8 –Damping addition sensitivity of the steady-state vibration energy

From the above, it is clarified that the effective position to add damping can be determined using either the dissipation method or the sensitivity method.

4.3 Non-Resonance Excitation Case

In this section, the excitation frequency is set to 46 Hz which is a non-resonance frequency. We evaluate the effective damping position in the same manner as the resonance excitation case.

The shape of vibration response is shown in Fig. 9. Because of non-resonance excitation, no particular mode is dominant. Mainly the third and fourth bending modes contribute.

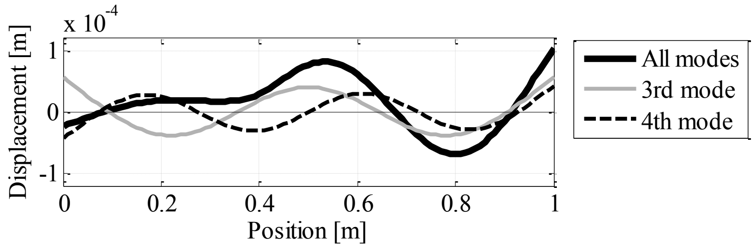


Figure 9 – Shape of vibration response at non-resonance excitation

4.3.1 Calculation of Optimum Damping Addition (Non-Resonance Case)

In the same manner as the previous section, the semi-optimum solution of damping addition position calculated using the repeated dynamic analysis is shown in Fig. 10. The vibration energy can be reduced effectively when damping is added at 0.2 m and 0.4 m position.

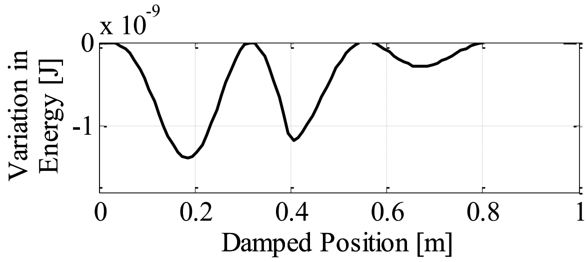


Figure 10 – Energy variation by damping addition, at non-resonance condition

4.3.2 Validation of the Method using Distribution of Dissipation Power (Non-Resonance Case)

The structural intensity and distribution of dissipation power are shown in Fig. 11 and 12. The results of dissipation power of the direct method and intensity method are the equal. However, no obvious correlation is found between the semi-optimum damping layout and the distribution of dissipation power.

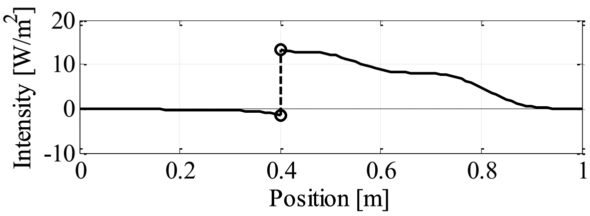


Figure 11 – Calculated structural intensity at non-resonance condition

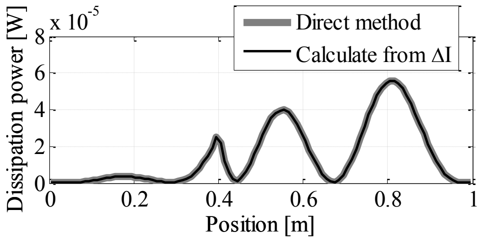
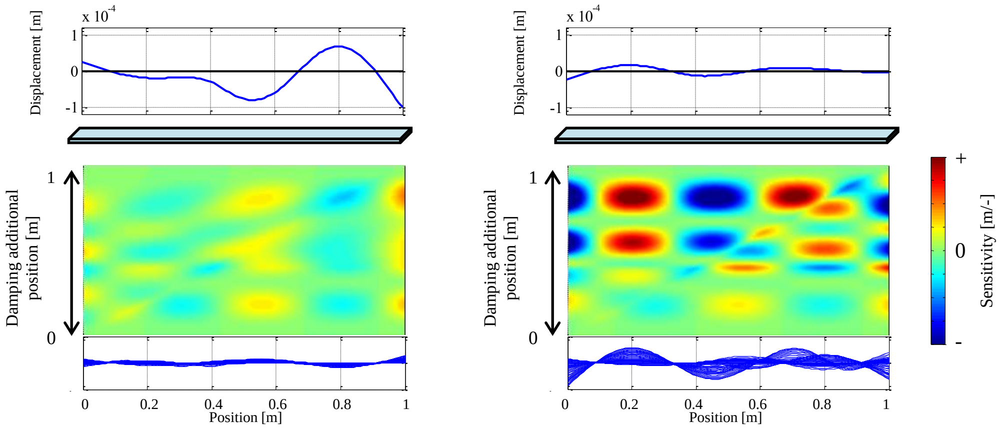


Figure 12 – Distribution of dissipation power at non-resonance condition

Therefore, in non-resonance excitation, the position of damping addition cannot be determined based on the distribution of dissipation power, which is different from the case of resonance excitation.

4.3.3 Validation of the Method using Sensitivity Analysis (Non-Resonance Case)

Figure 13 shows the damping addition sensitivity of displacement. Distribution of the sensitivity doesn't correlate to the displacement shape, and it change variously according to damping addition position.



(a) Real part (b) Imaginary part

Figure 13 – Damping addition sensitivity of displacement, at non-resonance condition

The damping addition sensitivity of the steady-state vibration energy is shown in Fig. 14. There is large negative sensitivity at 0.2 m and 0.4 m. This result correlates to the semi-optimum solution.

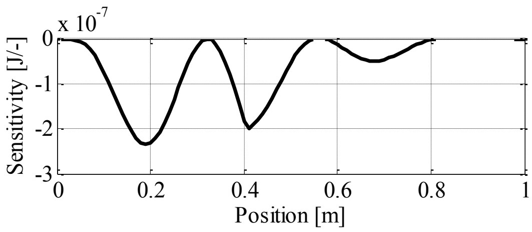


Figure 14 – Damping addition sensitivity of vibration energy, at non-resonance condition

Therefore, it is validated that the sensitivity method leads the effective position to add damping even if the excitation frequency is a non-resonance frequency.

5. DISCUSSIONS

We discuss the reason why the distribution of dissipation power is not a valid method at non-resonance condition.

The relationship between damping addition and variation of vibration energy is examined. The harmonic translational excitation force with amplitude F_n is applied to node n. The complex amplitude of translational displacement of node n can be expressed as follows:

$$w_n = |w_n|(\cos\phi + j\sin\phi) \tag{19}$$

where ϕ is the phase difference between input force and response, and $|\ |$ denotes the magnitude of complex number.

The period average of the input power caused by the excitation force can be expressed as follows:

$$\begin{aligned}\overline{P} &= \frac{1}{2}\mathrm{Re}\left[-j\omega F_n w_n^{\ *}\right] \\ &= \frac{\omega}{2}F_n\left|w_n\right|\sin\phi\end{aligned} \tag{20}$$

The period average of dissipation power of the whole structure can be expressed as follows, referring to Eq. (8):

$$\overline{D} = \frac{\omega^2}{2}\mathbf{u}^*\mathbf{C}\mathbf{u} \tag{21}$$

where **u** is the complex amplitude of displacement of the structure.

Because of steady-state vibration, the input power equals to the dissipation power.

$$\frac{\omega}{2}F_n\left|w_n\right|\sin\phi = \frac{\omega^2}{2}\mathbf{u}^*\mathbf{C}\mathbf{u} \tag{22}$$

In resonance condition, one vibration mode is dominant. So the vibration shape keeps similarity, not being dependently on damping addition, as shown in Fig. 7. Therefore, the vibration shape of damping added structure can be expressed as $\gamma\mathbf{u}$. Eq. (22) can be expressed as follows, using $\sin\phi$=1 because of resonance:

$$\begin{aligned}&\frac{\omega}{2}F_n\left|\gamma w_n\right| = \frac{\omega^2}{2}\{\gamma\mathbf{u}\}^*\mathbf{C}'\{\gamma\mathbf{u}\} \\ &\therefore\ \frac{1}{\gamma}\frac{\omega}{2}F_n\left|w_n\right| = \frac{\omega^2}{2}\mathbf{u}^*\mathbf{C}'\mathbf{u}\end{aligned} \tag{23}$$

where **C**’ is the damping matrix with additional damping.

It is obvious that the steady-state vibration energy becomes minimum value when γ is minimum. According to Eq. (23), γ becomes minimum value when the right side of the equation, the dissipation power, is maximum value. That means adding damping to the position of high dissipation is effective to reduce the vibration energy. Therefore, the method of the dissipation power is valid.

However, in non-resonance condition, the vibration shape does not keep similarity with damping addition, as shown in Fig. 13. Moreover, the phase difference ϕ between input force and response changes being dependent on damping addition. Therefore vibration cannot be expressed as Eq. (23). These are the reason why the effective position to add damping cannot be determined using distribution of the dissipation power, in non-resonance condition.

6. CONCLUSIONS

We investigated the structure vibration energy, focusing on the energy dissipation caused by damping. The main conclusions of this paper are as follows:

1) In the resonance excitation case, the optimum damping layout for vibration suppression can be determined using either method of the dissipation power or the sensitivity.
2) In the non-resonance excitation case, the optimum damping layout can be determined using sensitivity method. On the other hand, there is no correlation between the optimum damping layout and the distribution of dissipation power in non-resonance.
3) We propose the method of damping addition sensitivity of vibration energy for the evaluation method which is applicable to either resonance or non-resonance condition.

REFERENCES

1. Kido I. Research trend of the modal analysis. Proc Mechanical Engineering Congress (MECJ-10); 5-8 September 2010; Nagoya, Japan 2010. pp. 353-354 (in Japanese).
2. Noiseux D U. Measurement of power flow in uniform beams and plates. J Acoust Soc Am. 1970; 47(1(2)): pp. 238-247.
3. Pavić G. Measurement of structure borne wave intensity, Part I: Formulation of the methods. J SoundVib. 1976; 49(2): pp.221-230.
4. Nakagawa T, Furusu K, Asai M. Analysis of dissipated energy using vibration intensity. Trans Japan

Society of Mechanical Engineers, Series C. 1995; 61(589); pp. 3482-3488 (in Japanese).

5. Ohishi H, Yamakawa S, Ohno S. Evaluation of the effect of damping treatment using structural intensity and strain energy. Proc Dynamic and Design Conference (D&D '98); 17-20 August 1998; Hokkaido, Japan 1998. No.116 (in Japanese).
6. Yamazaki T. Modal expansion of structural intensity for flexural vibration on beams. Trans Japan Society of Mechanical Engineers, Series C. 2007; 73(731); pp. 1948-1954 (in Japanese).