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# Vibration rendering on a thin plate by actuator array on the boundary

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## ABSTRACT

Generation of localized vibration on the display panel in the mobile electronic devices is needed for transferring the tactile sensation to the users. When the actuators cannot be positioned at the central part of the panel due to the interference with electronic components, the only choice for the actuator locations is the edge of the panel. To generate a rendered distribution and pattern of the vibration, eigenfunction expansion and superposition can be used as the basic principle. The target vibration field consists of the hot and cold zones. In the hot zone, high amplitude vibration should be felt by the user, and, in the cold zone, the vibration amplitude should be tranquil not to incur any tactile sensation. A general inverse method to determine the proper weighting for each actuator in the array is pursued. Experiments are conducted on a thin rectangular panel and employing small moving-coil actuators. Performance indices for evaluating the control results are suggested based on the threshold and difference limen of tactile feeling. Performances of vibration localization varying the location of hot zone and the number of involved modes are evaluated, and the measured and rendered vibration targets match very well.

Keywords: Generation of localized vibration, inverse rendering, tactile perception
I-INCE Classification of Subjects Number(s): 42

## 1. INTRODUCTION

To control the vibration on a structure like plate or beam, actuators are usually placed at the dominant positions for the excitation considering the modal characteristics [1]. In audio-visual equipments and mobile devices, one can supply various information effectively by feeding a strong vibrotactile sensation only at a certain limited area. For practical applications in such devices, actuators cannot be placed at the advantageous positions of the plate for exciting the vibration due to the interference with electronic components occupying the central part of the device interior. In this work, a pattern of vibration which is rendered initially is to be generated by using the eigenfunction expansion and superposition method as the basic principle, on the condition that the actuator array are placed at the periphery of the panel.

## 2. THEORETICAL BACKGROUND

### 2.1 Vibration of a thin rectangular plate

For a thin, uniform plate, the out-of-plane displacement $w(x, y, t)$ can be described as [2]

$$D\nabla^4 w + \rho h \frac{\partial^2 w}{\partial t^2} = \sum_{i=1}^{N} F_i\left(x_i, y_i, t\right). \tag{1}$$

Here, $D$ denotes the flexural rigidity of the plate in bending, $i$ indicates the excitation position, $\rho$ is the density of plate, $h$ the thickness of plate, and $F(x_i, y_i, t)$ the exciting force at the $i$ position. Equation (1) can be rewritten in velocity terms to express the response of periodic vibration as

$$D\nabla^4 v - m'\omega^2 v = j\omega p\left(x_i, y_i\right). \tag{2}$$

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Here, $v$ is the velocity, $\omega$ the exciting frequency, $m'$ the mass density, and $p$ the exciting pressure distribution. The solution of Eq. (2) can be expressed in terms of a sum of eigenfunctions, called mode shapes, as

$$v(x,y)=\sum_{n=1}^{\infty} v_n \psi_n(x,y)\,. \tag{3}$$

Here, $v_n$ and $\psi_n$ are velocity phasor and mode shape of the n-th mode, respectively.

By applying the orthogonality of mode shapes, the general solution of velocity as functions of modal parameters is given by

$$v(x,y)=\sum_{n=1}^{\infty} \frac{\psi_n(x,y)}{\Lambda_n\left(\omega_n^2-\omega^2\right)}\int_S j\omega p(x,y)\psi_n(x,y)\,dxdy\,, \tag{4}$$

where

$$\Lambda_n=\int_S m'\psi_n^2(x,y)\,dxdy\,. \tag{5}$$

Equation (4) represents the eigenfunction expansion theorem, which states that the response of a system can be expressed by the weighted superposition of mode shapes and natural frequencies. Considering of the effect of damping on the system, that yields the complex natural frequencies, Eq. (4) can be rewritten as

$$v(x,y)=\sum_{n=1}^{\infty} \frac{\psi_n(x,y)}{\Lambda_n\left\{\omega_n^2(1+j\eta_n)-\omega^2\right\}}\int_S j\omega p(x,y)\psi_n(x,y)\,dxdy\,, \tag{6}$$

in which $\eta$ is the loss factor.

## 2.2 Generation of localized vibration

To realize the localized vibration field in a steady form, proper weightings in both excitation amplitude and phase for each actuator in the array should be identified to meet the rendered pattern and constraints. Here, array actuators are being operated in the same excitation frequency.

If a point excitation at ($x_i$, $y_i$) position is given, the following can be obtained from Eq. (6) as

$$v(x,y)=A_i\sum_{n=1}^{\infty}\psi_n^i\frac{j\omega\psi_n(x_i,y_i)\psi_n(x,y)}{\Lambda_n\left\{\omega_n^2(1+j\eta_n)-\omega^2\right\}}\equiv A_i\sum_{n=1}^{\infty}\psi_n^i\gamma_n(x,y)\,, \tag{7}$$

where

$$A_i=\int_S p(x,y)\,dxdy\,,\quad \psi_n^i=\psi_n(x_i,y_i)\,,\quad \gamma_n(x,y)=\frac{j\omega\psi_n(x,y)}{\Lambda_n\left\{\omega_n^2(1+j\eta_n)-\omega^2\right\}}\,. \qquad \text{(8a, b, c)}$$

Here, $A_i$ is the complex weighting for the *i*th actuator, and $\psi_n^i$ is the n-th mode shape at the *i*th actuator position. In this work, the rational fraction polynomial method [3], which uses mobility of structure in frequency domain, is adopted for the curve fitting and estimation of the modal parameters.

To define the relationship between modes and actuators, Eq. (7) can be further modified in terms of participating modes as follows:

$$v(x,y)=\sum_{n=1}^{\infty}\left[\left(\sum_{i=1}^{N}A_i\psi_n^i\right)\gamma_n(x,y)\right]. \tag{9}$$

The target vibration field, $v_d$, can be expressed by using the superposition of weighted modes as

$$v_d(x,y)=\sum_{n=1}^{\infty}W_{n,d}\gamma_n(x,y)\,, \tag{10}$$

where $W_{n,d}$ denotes the complex modal weighting.

Considering the actual finite number of actuators, Eqs. (9) and (10) should be considered in deriving proper actuator weightings to obtain an appropriate coefficient of each mode, which approximately fulfills the desired target vibration field as follows:

$$\sum_{i=1}^{N}\left(A_i\psi_n^i\right) = W_{n,d}, \quad n = 1,2,\ldots,m. \tag{11}$$

Here, $m$ is the number of included modes. $N$ is the number of actuators. Equation (11) can be now rewritten in a matrix form as

$$\mathbf{\Psi}_{m\times N}\,\mathbf{A}_{N\times 1} = \mathbf{W}_{m\times 1}. \tag{12}$$

By taking the inverse operation, the matrix **A**, that contains proper amplitude and phase weightings, can be calculated as

$$\mathbf{A}_{N\times 1} = \mathbf{\Psi}^{\dagger}_{N\times m}\,\mathbf{W}_{m\times 1}, \tag{13}$$

where $\mathbf{\Psi}^{\dagger}$ is the pseudo-inverse of $\mathbf{\Psi}$ and $^{\dagger}$ denotes the Hermitian operator.

# 3. EXPERIMENT

## 3.1 Rendered target field and measurement set-up

To validate the present methodology, a thin glass panel used as the display medium of a tablet PC is adopted. It is intended to render the vibration amplitude distribution in a designated pattern. To define the target vibration field, the information on the vibrotactile sensation is utilized. At the "hot zone", the vibration should be felt by the fingertip, while the vibration should not be felt outside the hot zone, which is called the "cold zone". In the experiment, the rendered pattern is specified as a 2x2 grid on the plate, in which one module is designated as the hot zone.

We adopted a tempered glass panel (Corning Gorilla Glass) with 268 mm in length, 170 mm in width, and 0.7 mm in thickness which is employed in commercial tablet PC. Small moving-coil type actuators which are placed on the plate with free mounting condition are located at the boundary of the glass panel. The spacing between adjacent actuators is 20 mm to avoid the spatial aliasing. Observation points are uniformly selected with 20 mm spacing. Total number of actuators is 34, and the number of observation points is 77 as illustrated in Fig. 1(a). To obtain the mobility and velocity response from observation points, a laser scanning vibrometer is used, of which phase is provided with reference to the excitation signal. Figure 1(b) shows the measurement set-up.

Pacinian corpuscle which is one of the psychophysical receptors for tactile sensation is known to take a principal role about vibration perception at 250 - 300 Hz as the most sensitive frequency range [4]. In this experiment, actuators are excited with 300 Hz as to generate the vibrotactile sensation at the hot zone. At 300 Hz, the velocity amplitude corresponding to the threshold of vibration sensation is given as 0.2 mm/s [4].

(a)

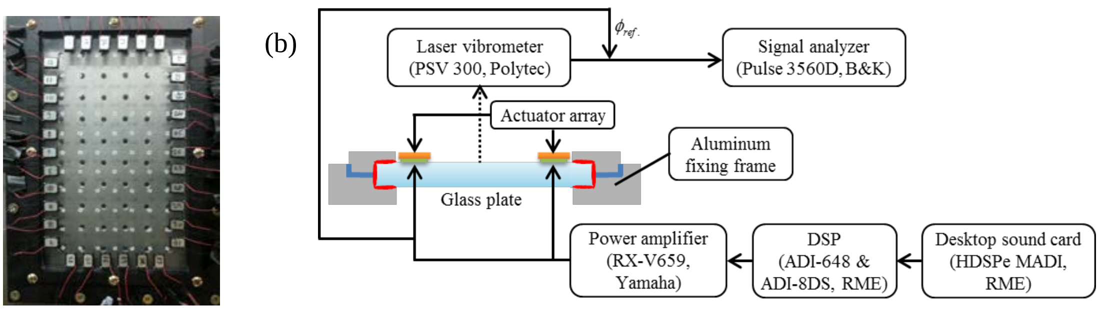


Figure 1 – (a) Top-view photo of the system, (b) experimental set-up.

## 3.2 Performance evaluation for vibration control

To evaluate the extent of fulfilment of control object, performance indices are developed. The ratio of perceptible area to the rendered hot zone area, and the ratio of tranquil area, at which the vibration is imperceptible, to the rendered cold zone area are two indices. However, these indices are valid as far as the generated vibration pattern in space is kept as the rendered one. Success or failure in each zone is defined with the aid of difference limen of tactile feel intensity [5]. Figure 2 illustrates the suggested constraints of the target vibration field in the viewpoint of energy. The difference between maximum energy at cold zone and haptic threshold is should be at least larger than 2 dB, which is the difference limen at non-vibration state. The same 2-dB difference rule is also true for the hot zone vibration larger than haptic threshold. Total 4 dB demarcates the hot and cold zones, which is used as a buffer to tolerate any unexpected error in control.

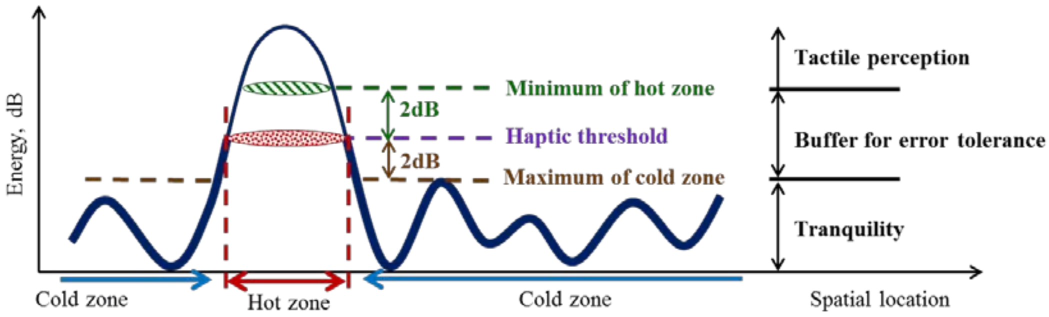


Figure 2 – Constraints of target energy field for generating the localized vibration with consideration of the difference limen for tactile perception: , actually controlled area for hot zone; , strongly perceptible area; , actually controlled area for cold zone.

### 3.3 Experimental result

Modal coefficients, $W_{n,d}$, to achieve the target velocity field distribution and pattern are obtained as shown in Fig. 3(a). Owing to the use of 34 actuators, 34 modes can be controlled satisfactorily. Actuator weightings, $A_i$, for realizing the rendered vibration distribution and pattern are calculated by employing the suggested inverse technique specified in Eq. (13), and the result is illustrated in Fig. 3(b).

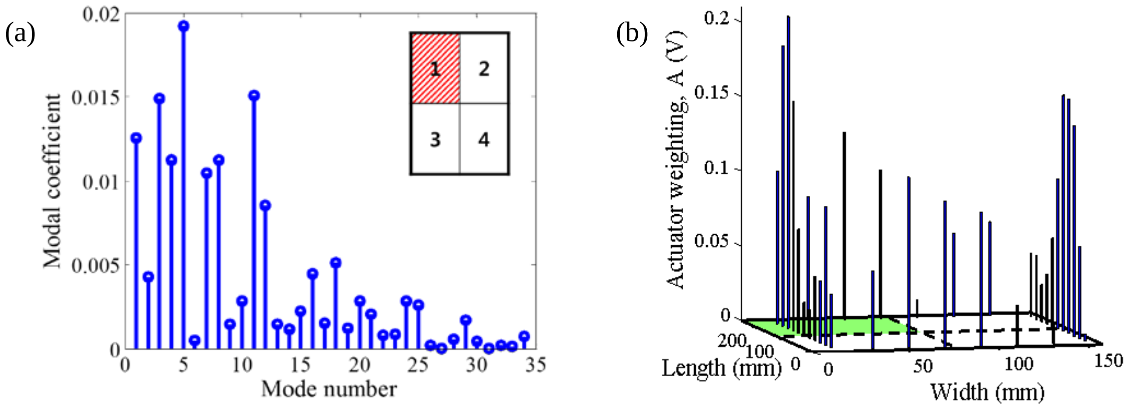


Figure 3 – Modal coefficients for low order 34 modes and actuator weightings for 34 actuators determined to achieve a specific target field for a 2x2 grid division on a plate: (a) modal coefficient, (b) actuator weighting. Shaded zone 1 indicates the hot zone.

In Fig. 3(b), one can find that the actuator weightings with large magnitudes are not crowded near the hot zone but are distributed in various positions. In this regard, one should recall that the control target is not just generating the high amplitude vibration at the hot zone, which is zone 1 in Fig. 3(b), but also the vibration at the other zones, which are zones 2, 3, 4 in Fig. 3(b), should be suppressed simultaneously. A final performance of 90.2 % is obtained in this test.

## 4. CONCLUSIONS

In this paper, multiple actuators located at the boundary of a thin plate is employed to generate the rendered target patterns and distribution of vibration using the inverse weighting technique based on the modal expansion. In the control, the human perception characteristics on the tactile feeling at the fingertip is considered to facilitate the vibration sensation at the point of fingertip touching. Performance of vibration localization in the experimental result is compared with the rendered velocity field, and it is found that the success ratio is 90.2%. Because the present method is very simple in its concept, it can be implemented as a useful practical method for generating a complicated, localized vibration field in a structure.

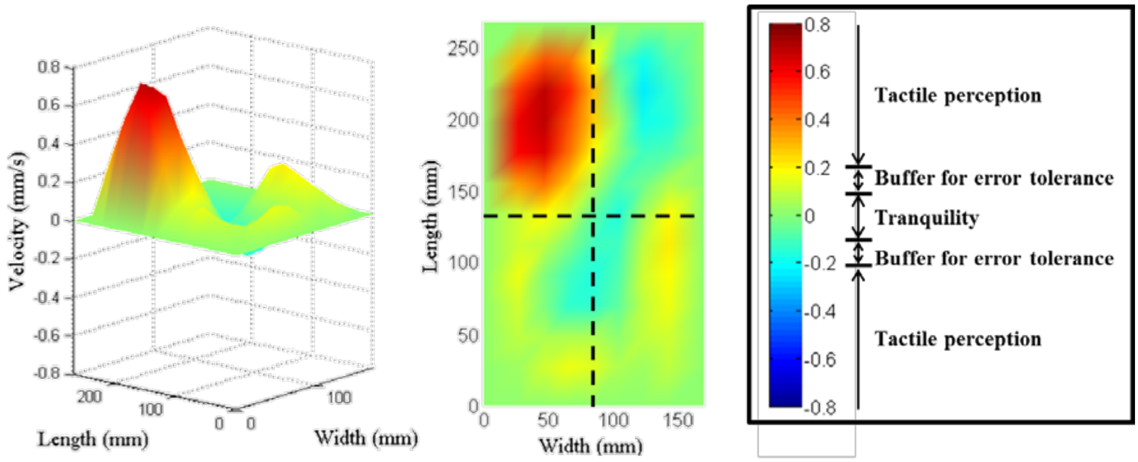


Figure 4 – Experimental result achieving a specific target field shape rendered in a 2x2 grid pattern.

## ACKNOWLEDGEMENTS

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