Adaptive feedback noise control with leaky FeLMS algorithm

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ABSTRACT

In this paper, an adaptive feedback control algorithm for ANC (Active Noise Control) with leaky FeLMS (Filtered-error Least Mean Square) algorithm is proposed. A shaping function is used in the updating equation, which selects the frequency band of the noise attenuation for the ANC system. Compared with the adaptive feedback ANC structure based on traditional FxLMS (Filtered-x Least Mean Square) algorithm, the effects of the proposed algorithm are focused only on the selected band and interference from other frequency bands, which can induce instability, can be reduced significantly. The RF is only applied on the updating process for the coefficients of the control filter rather, so the proposed algorithm does not introduce any extra delay. In this paper, the proposed algorithm is analyzed and it is proven to be suitable for applications where broadband noise is present, but the targeted band is limited to a narrow band.

Keywords: Active noise control, adaptive feedback control, FeLMS algorithm

I-INCE Classification of Subjects Number(s): 37.7

1. INTRODUCTION

ANC (Active Noise Control) is a technique to cancel the noise in a selected low-frequency band and in certain areas by estimating the practical sound field [1, 2]. Compared with the traditional passive method to mitigate noise, it is flexible and has the ability to attenuate the noise in the low frequency effectively. Over the last three decades, ANC has received a lot of attention from academics and industry.

Feedback control is an important strategy for the ANC. It works in a closed loop and takes into account the coupling of the acoustic sound and the electrical signals[3-5]. It is quite different from a feed-forward ANC system in that the system structure is simple, and includes only one error sensor and one control source for a single-channel system. There is no reference sensor and the reference signal for the control is obtained from the signal sampled from the error sensor. It is intended to use the inherent periodicity of the acoustic signal to cancel the unwanted noise [6]. One important advantage is that there is no need to worry about the relative position between the noise source and the reference sensor in the feedback ANC. On the other hand, it is an important issue in feed-forward ANC.

The technique of feedback ANC implemented in the form of an analog circuit is sophisticated. Some commercial products with feedback ANC have been commercialised for a long time. However, the widespread use of fixed feedback control is limited due to its inflexibility. AFANC (Adaptive Feedback ANC) is the feedback ANC implemented in digital platforms e.g. DSP (Digital Signal Processor) and FPGA (Field Programmable Gate Array) [7]. It involves an adaptive filter to match the practical sound field to cancel the offending noise flexibly. Although AFANC has been proposed for a long time [5], its use is still limited because of the problem of instability and unpredictable effect on the noise attenuation [8, 9]. Firstly, the stability of the system is limited by the Nyquist stability criterion. In the AFANC system, there must be some existent internal delays which are caused by analog-to-digital conversion, digital-to-analog conversion and processing time. This delay actually increases the phase of the open-loop frequency response, which makes the phase meet the unstable

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condition at some frequency. Besides, the performance of AFANC largely depends on the practical noise distribution in the frequency domain. When the practical noise is a pure periodic signal, the system works much better. However, when the noise contains any broadband signal, the system's performance can be degraded, and it can even become unstable.

The schematic diagram of an AFANC system is given in Figure 1. $s(n)$ is the practical secondary path, $\hat{s}(n)$ is the previously estimated secondary path, $w(n)$ is the control filter, $c(n)$ is the internal transfer function used in constructing the reference signal, which is always regarded as containing some delay, $x(n)$ is the reference signal, $d(n)$ is the practical noise and $e(n)$ is the error signal sampled by the error sensor. In order to achieve the cancellation effect, the control filter $w(n)$ is updated by the online FxLMS (Filtered-x Least Mean Square) algorithm to minimize the error signal $e(n)$.

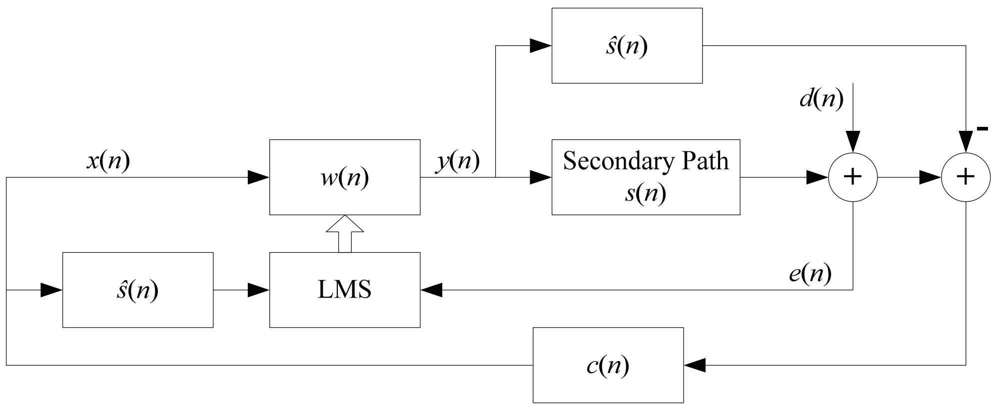

Figure 1 Diagram of fundamental AFANC structure.

FeLMS(Filtered-error Least Mean Square) algorithm is an effective method to reshape the spectrum of the residual noise for ANC[10]. FeLMS algorithm is widely used in feed-forward system to concentrate the effect of noise attenuation in a certain frequency range. In this paper, the optimization criterion for adaptive feedback ANC is improved. Based on the new criterion, an AFANC with leaky FeLMS algorithm is developed. The improved algorithm employs one SF (Shaping Function) to change the strategy of the adaptive filtering. Although the improved algorithm is implemented in the time domain, it is quite flexible to adjust the local frequency range by changing the SF. The stability and the performance of the system are also improved.

2. ADAPTIVE FEEDBACK ANC

The AFANC structure is derived from the feed-forward control system. The main difference is that its reference signal is from the summation of the error signal and the estimated control signal from the error sensor, not from the reference sensor directly. In the AFANC structure, the delay from the error sensor of the signal which feeds the control actuator is unavoidable, which is the $c(n)$ in Figure 1. The main purpose of the AFANC is also to optimize the control filter to minimize the signal at the error sensor. Next, the traditional AFANC is reviewed.

2.1 Feedback ANC with FeLMS algorithm

Assuming that the secondary path $s(n)$ is a linear system the relationship between the signals in Figure 1 can be expressed as

$$\begin{cases} e(n) = d(n) + y(n) * s(n) \\ y(n) = x(n) \cdot w_k(n) \\ x(n) = c(n) \cdot \left[e(n) - y(n) * \hat{s}(n) \right]. \end{cases} \tag{1}$$

Here the symbol $*$ denotes convolution in the time-domain, $w_k(n)$ is the control filter at the k-th iteration. The length of the estimated secondary path, $\hat{s}(n)$, is M and the length of the control filter, $w_k(n)$, is N.

In FxLMS algorithm, the criterion of the traditional adaptive feedback ANC actually covers the entire frequency range, which can cause the instability problem. However, in practical applications, the frequency range of the disturbing noise is usually limited to the low frequency range and some narrow bands. Based on this assumption, an additional filter is proposed to limit the error signal and the new criterion is defined as

$$J = \min_{W(\omega)} \left\{ e_R(n)^2 + \lambda \cdot \sum_{i=0}^{N-1} w(i)^2 \right\}, \tag{2}$$

where $e_{\mathrm{R}}(n)$ is the filtered error as

$$e_{\mathrm{R}}(n) = e(n) * r(n) = \sum_{i=0}^{T-1} e(n-i) \cdot r(i), \tag{3}$$

and $r(i)$ is the SF, T is the length of the SF, and λ is a leakage factor with a small positive value. The SF $r(i)$ can be regarded as a series of weighting coefficients in the frequency domain: the error in the pass-band of this filter is enhanced and much more important for optimization than that in the filter's stop-band. Besides, the summation of the control filter coefficient is limited in Eq.(2). It is in fact a form of the leaky algorithm and it will make the algorithm stable. The gradient of Eq.(2) with respect to the coefficients of the control filter is

$$\frac{\partial J}{\partial w_k(m)} = 2\left[e_R(n) \cdot v_R(n-k) + \lambda \cdot w_k(m) \right], \tag{4}$$

where

$$v_R(n) = v(n) * r(n) = x(n) * s(n) * r(n) \tag{5}$$

and

$$v(n) = x(n) * s(n). \tag{6}$$

Then the updating equation of the control filter in the improved algorithm is simply

$$\begin{aligned} w_{k+1}(m) &= w_k(m) - \frac{1}{2} \cdot \frac{\partial J}{\partial w_k(m)} \\ &= \delta \cdot w_k(m) - 2\mu \cdot e_R(n) \cdot \hat{v}_R(n-m), 0 \le m \le M-1, \end{aligned} \tag{7}$$

where

$$\delta = 1 - 2\lambda\mu. \tag{8}$$

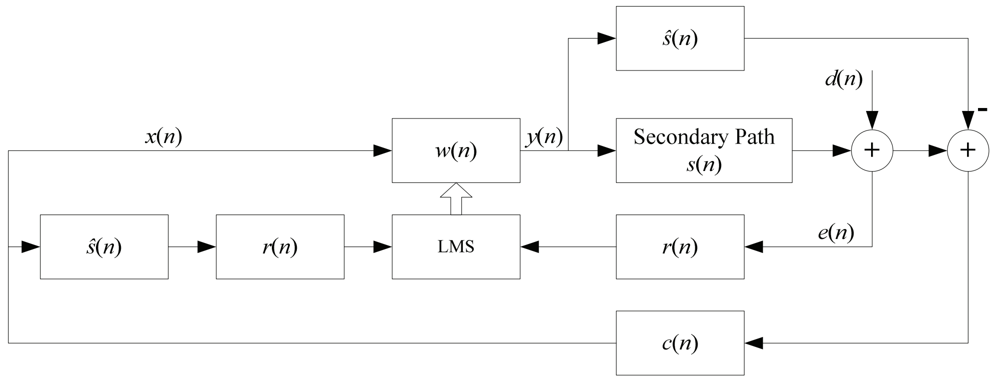

Figure 2 Schematic of the improve AFANC structure.

Figure 2 depicts a schematic of the improved AFANC. It should be noted that the SF $r(n)$ is not included in the feedback path. It means that there is no extra delay caused by the $r(n)$ and the correlation between the artificial reference signal and the practical noise does not change.

2.2 Expression in the frequency domain

The updating equation in the time domain can be rewritten in the form of block processing as

$$\mathbf{w}_{k+1}(m) = \delta \cdot \mathbf{w}_k(m) - 2 \cdot \mu \cdot e_{\mathrm{R}}(n) \cdot \hat{\mathbf{v}}_{\mathrm{R}}(n), \tag{9}$$

where

$$\mathbf{w}_k = \left[w_k(0), w_k(1), \cdots, w_k(N-1) \right]^T, \tag{10}$$

and

$$\hat{\mathbf{v}}_{\mathrm{R}}(n) = \left[\hat{v}_{\mathrm{R}}(n), \hat{v}_{\mathrm{R}}(n-1), \cdots, \hat{v}_{\mathrm{R}}(n-L+1) \right]^T. \tag{11}$$

It is possible to define $E_{\mathrm{R}}(\omega)$ and $V_{\mathrm{R}}(\omega)$ as the expressions of $e_{\mathrm{R}}(n)$ and $v_{\mathrm{R}}(n)$ in the frequency domain. $e_{\mathrm{R}}(n) \cdot \hat{\mathbf{v}}_{\mathrm{R}}(n)$ in Eq.(9) is actually the cross-correlation between $e_{\mathrm{R}}(n)$ and $v_{\mathrm{R}}(n)$, so its frequency-domain form is $E_{\mathrm{R}}(\omega) \cdot V_{\mathrm{R}}^*(\omega)$. Then the expression for the updating equation (9) in the frequency domain is

$$W_{k+1}(\omega) = \delta \cdot W_k(\omega) - 2 \cdot \mu \cdot E_{\mathrm{R}}(\omega) \cdot V_{\mathrm{R}}^*(\omega). \tag{12}$$

Taking the frequency-domain forms of Eq.(3) and Eq.(5) leads to

$$E_{\mathrm{R}}(n) = E(\omega) \cdot R(\omega), \tag{13}$$

$$V_{\mathrm{R}}(\omega) = X(\omega) \cdot S(\omega) \cdot R(\omega) \tag{14}$$

where, $E(\omega)$, $R(\omega)$, $X(\omega)$ and $S(\omega)$ are the expressions of $e(n)$, $r(n)$, $x(n)$ and $s(n)$ in the frequency domain.

Substituting Eq.(14) into Eq.(12) the final frequency-domain form of the improved algorithm can be obtained as

$$W_{k+1}(\omega)=\delta\cdot W_k(\omega)-2\cdot\mu\cdot E(\omega)\cdot X^*(\omega)\cdot S^*(\omega)\cdot\left\|R(\omega)\right\|, \tag{15}$$

where $\|\bullet\|$ denotes the squared modulus of certain complex number.

2.3 Convergence to optimum

Suppose that the optimal value of the control filter in the frequency domain is $W_{\text{opt}}(\omega)$. Substituting the control filters in the updating equation Eq.(15) with $W_{\text{opt}}(\omega)$ to get

$$W_{\text{opt}}(\omega)=\delta\cdot W_{\text{opt}}(\omega)-2\cdot\mu\cdot E(\omega)\cdot X^*(\omega)\cdot S^*(\omega)\cdot\left\|R(\omega)\right\|. \tag{16}$$

It is easy to get the solution as

$$W_{\text{opt}}(\omega)=-\frac{P_D(\omega)\left\|R(\omega)\right\|C^*(\omega)S^*(\omega)}{P_D(\omega)\left\|C(\omega)S(\omega)R(\omega)\right\|+\lambda}. \tag{17}$$

Then the ideal level of the noise attenuation is

$$\frac{E(\omega)}{D(\omega)}=\frac{\lambda}{P_D(\omega)\left\|C(\omega)S(\omega)R(\omega)\right\|+\lambda}, \tag{18}$$

when the control filter converges its optimal solution.

There are several points to be noted from the above expression:

Define

$$\Delta W_k(\omega)=W_k(\omega)-W_{\text{opt}}(\omega), \tag{19}$$

which is the difference of the *k*-th control filter from the optimal solution. Rearranging the updating equation with the optimal control filter as given in Eq.(17), gives

$$\Delta W_{k+1}(\omega)=\left(\delta-2\mu P_D(\omega)\left\|C(\omega)S(\omega)R(\omega)\right\|\right)\cdot\Delta W_k(\omega). \tag{20}$$

So the control filter $W_k(\omega)$ can converge to the optimal solution step by step as long as

$$\mu<\frac{1}{2\left(\lambda+P_D(\omega)\left\|C(\omega)S(\omega)R(\omega)\right\|\right)}, \tag{21}$$

It is important to note that the convergence rate is also determined by the response of the SF. That implies that the parts of the control filter in the pass-band of the SF have more priority to converge with higher rate to its optimal value than those in its stop-band.

Rewrite the optimal Eq.(17) to get

$$W_{\text{opt}}(\omega)\cdot C(\omega)\cdot\hat{S}(\omega)=-\frac{\left|D(\omega)\cdot R(\omega)\cdot C(\omega)\cdot\hat{S}(\omega)\right|^2}{\left|C(\omega)\cdot D(\omega)\cdot S(\omega)\cdot R(\omega)\right|^2+\lambda} \tag{22}$$

The right part of the above equation is an ideal system without any delays. Since $W_{\text{opt}}(\omega)$ and $\hat{S}(\omega)$ are both causal systems and $C(\omega)$ is purely a delay system, the left term of Eq.(22) is also a causal system with delays. Nevertheless, the right term of Eq. (22) is a system with zero phase, so it is impossible to get a real $W_{\text{opt}}(\omega)$ to meet the requirement. Thus, Eq.(17) is actually an ideal solution and it can only be approached. According to Eq. (22) and the discussion in [1], the approximation has the following rule: at the frequency where the value of $\left|C(\omega)\cdot D(\omega)\cdot S(\omega)\cdot R(\omega)\right|^2$ is considerable compared with λ, the condition of Eq. (22) should be met firstly. And at the frequency where the value of $\left|C(\omega)\cdot D(\omega)\cdot S(\omega)\cdot R(\omega)\right|^2$ is much less than λ, the practical solution converges slowly to a small value but arbitrary phase. Because the value of $C(\omega)$, $D(\omega)$ and $S(\omega)$ are known, the weight filter $R(\omega)$ actually determines the optimal solution of the possible $W_{\text{opt}}(\omega)$.

Since $\lim_{\lambda\to 0}\left[E(\omega)/D(\omega)\right]=0$, the noise in the whole frequency range can be cancelled completely when $\lambda=0$. However, this complete noise attenuation is impossible for the feedback control. This trend of unlimited decrease will lead to some instability at some frequency mentioned previously. So the parameter λ is used to keep the algorithm stable. It is also with the same purpose to include the leakage factor in most leaky algorithm.

$R(\omega)$ can be regarded as an artificial limitation for the products of the secondary path response and the spectrum of the practical noise. In the pass-band of $r(n)$, ANC has a strong attenuation ability, and in its stop-band the ratio between the error signal $E(\omega)$ and the practical noise $D(\omega)$ is close to 1.

Besides, since there is only the modulus of $R(\omega)$ in Eq.(15) and Eq.(17), the phase information of $R(\omega)$ has no influence for the system theoretically at all. Thus, it is more important to focus on the amplitude of the SF.

The leakage factor plays an important role in the improved algorithm. Firstly, it guarantees the convergence of the algorithm's stability. Secondly, combined with the SF, it decides the level of the noise attenuation. Thirdly, its value also affects the convergence rate of the control filter towards to the optimum solution. A larger value of the leakage factor can make the system converge faster with more stability, but the effect on the noise cancellation is reduced. Therefore, the value of the leakage factor should be chosen carefully according to practical requirements. Generally, in order to distinguish the selected frequency range from others, the value of λ should be set as

$$\begin{cases}\lambda \ll \left|C(\omega)\cdot P_D(\omega)\cdot S(\omega)\cdot R(\omega)\right|^2 & \omega\in\text{pass-band of } R(\omega)\\ \lambda \gg \left|C(\omega)\cdot P_D(\omega)\cdot S(\omega)\cdot R(\omega)\right|^2 & \omega\in\text{stop-band of } R(\omega),\end{cases} \tag{23}$$

which also determines the range of the λ's value.

2.4 Mismatch and waterbed effect

In the following section, the case when the estimated secondary path deviates from the real secondary path is discussed. Assume that the closed-loop system is stable and can converge to the ideal solution. The optimal solution of the criterion can be expressed as

$$W_{\text{opt}}(\omega)=-\frac{C^*(\omega)\cdot\hat{S}^*(\omega)\cdot\left\|D(\omega)\cdot R(\omega)\right\|}{\lambda\cdot\left\|1-C(\omega)\cdot W_{\text{opt}}(\omega)\cdot\Delta_S(\omega)\right\|+\left\|C(\omega)\cdot\hat{S}(\omega)\cdot D(\omega)\cdot R(\omega)\right\|} \tag{24}$$

In the above equation, the denominator of the right part still includes the optimal solution. With Eq.(24), it is easy to get the relationship between the practical noise and the error signal as

$$\frac{E(\omega)}{D(\omega)}=\frac{\lambda}{\lambda+\dfrac{\hat{S}^*(\omega)\cdot S(\omega)\cdot\left\|C(\omega)\cdot D(\omega)\cdot R(\omega)\right\|}{\left\|1-C(\omega)\cdot W_{\text{opt}}(\omega)\cdot\Delta_S(\omega)\right\|}}, \tag{25}$$

In the stop-band of the SF, because the value of $R(\omega)$ is quite small, the influence caused by $\Delta_S(\omega)$ in the whole feedback system can be reduced. In a feedback system mismatch always happens because there is not enough excitation in the high frequency during system modeling or there is some tiny displacement during the runtime. So the mismatch usually appears in the high frequency range. Because the working frequency for noise attenuation is located in the low frequency range, the mismatch in the feedback control system has little influence in the proposed algorithm in practical applications.

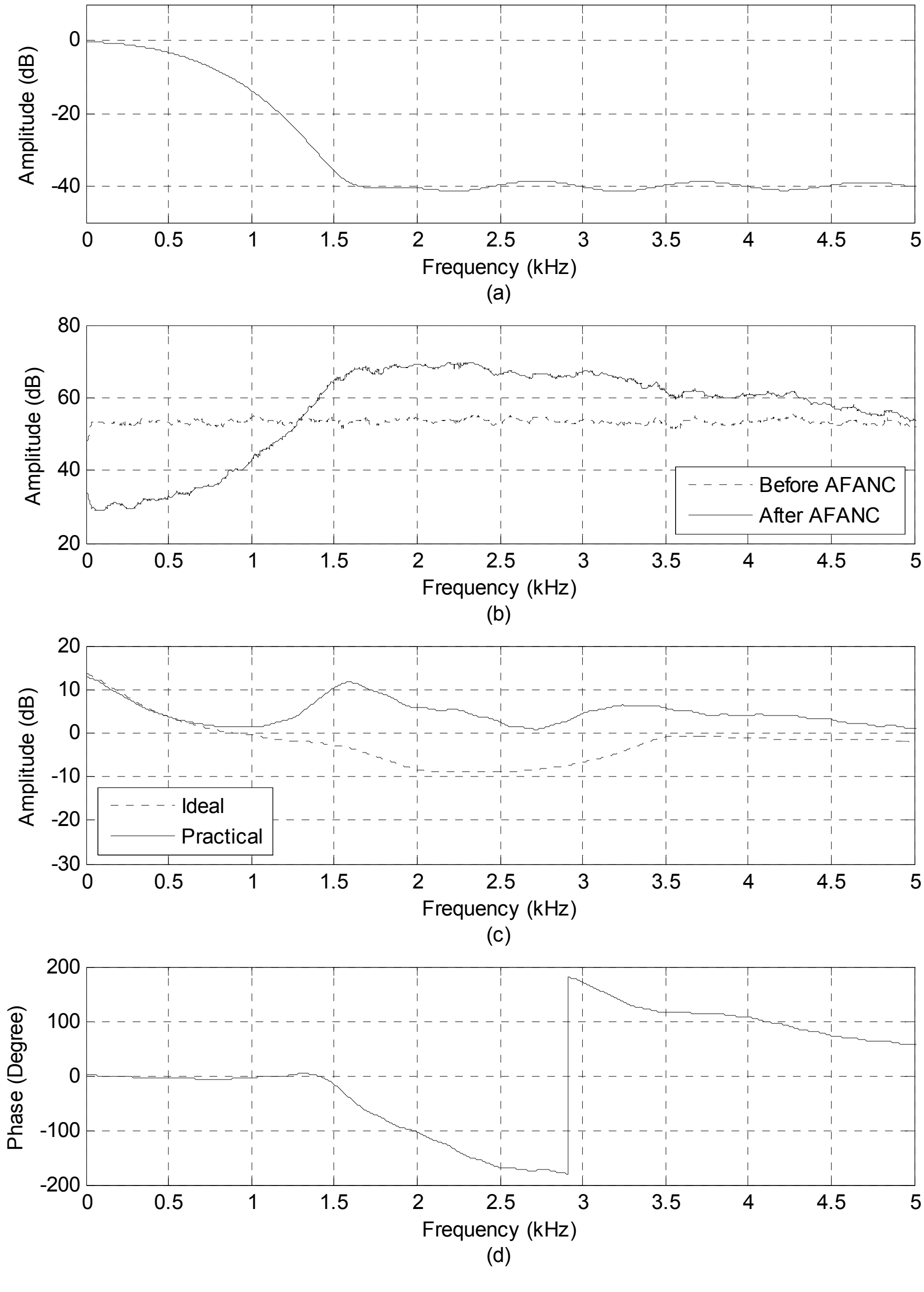

Figure 3 The results with a low-pass shaping function: (a) The frequency response of the shaping function. (2) The attenuated noise compared with the original noise. (c) Comparison of the frequency response of the ideal solution and the practical solution of the control filter. (d) The phase response of the term $-w(n)*c(n)*s(n)$

The waterbed effect is a specific phenomenon which exists in the feedback system. For a feedback system, it was shown in the Bode Integral Formulae that if the closed-loop system is stable then the integration of the logarithm of the system's sensitivity over the whole frequency range equals to zero or a positive value. This value is actually related to the summation of the poles in the corresponding open-loop system. It means that if there is noise suppression in some frequency range, then there must be increased noise level in some other frequency range. Thus, the waterbed effect cannot be avoided completely, but it can be reduced or the frequency range where the waterbed effect happens can be shifted properly. In the leaky feedback algorithm, the desired frequency range can be adjusted by changing the frequency response of the SF. Then the waterbed effect range in the frequency range of interest can also be moved in this way.

3. SIMULATION RESULTS

In this section, some simulation results are given to show the efficiency of the proposed algorithm. The sampling rate is set to 100 kHz. The secondary path was measured from an earphone system. The data were captured from the loudspeaker to the error microphone synchronously. The transfer function of the secondary path is calculated by the offline Wiener optimal filtering method. In order to reduce the influence of its deviation, the length of the control filter is set to 2000. The secondary path impulse and frequency responses are measured from the control loudspeaker to the error microphone in the real system. The length of the secondary path model is set to 500.

One result with a low-pass SF is given in Figure 3. The length of the SF is set to 200 samples and the length of the control filter is set to 500 samples. The frequency response below 10 kHz of the SF is shown in Figure 3(a), which is a typical low-pass filter. The leakage factor is set to 1e-5 and the step-size is set to 0.01. A white noise is used as the practical noise. For the result shown in Figure 3(b), there is obviously noise attenuation in the pass-band of the SF. The waterbed effect appears after 1.3 kHz. The amplitude and phase of the converged control filter strictly comply with the ideal solution in the pass-band of the SF. Beyond its pass-band, the amplitude increases initially and then decreases slowly with increasing frequency. The increase of the control filter can be explained as a continuation of the trend at the edge of the attenuation range where the system wants to keep the attenuation effect.

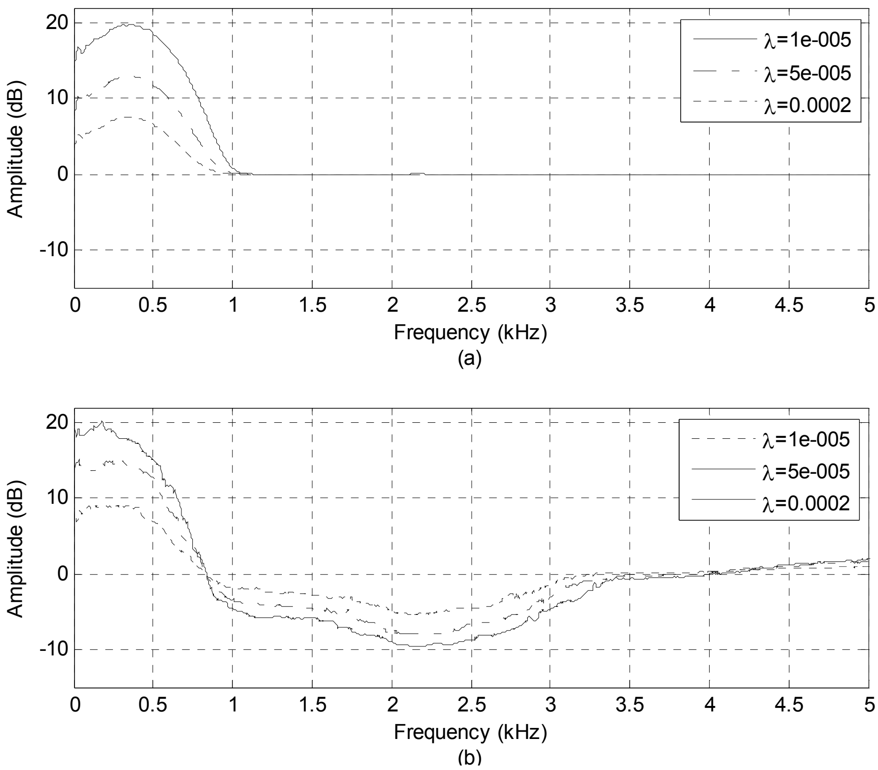

Figure 4 The comparison of the noise attenuation level in the frequency for three different values of λ, shaping function is a low-pass shaping function: (a) The results of the ideal solution, (b) The results of the practical solution.

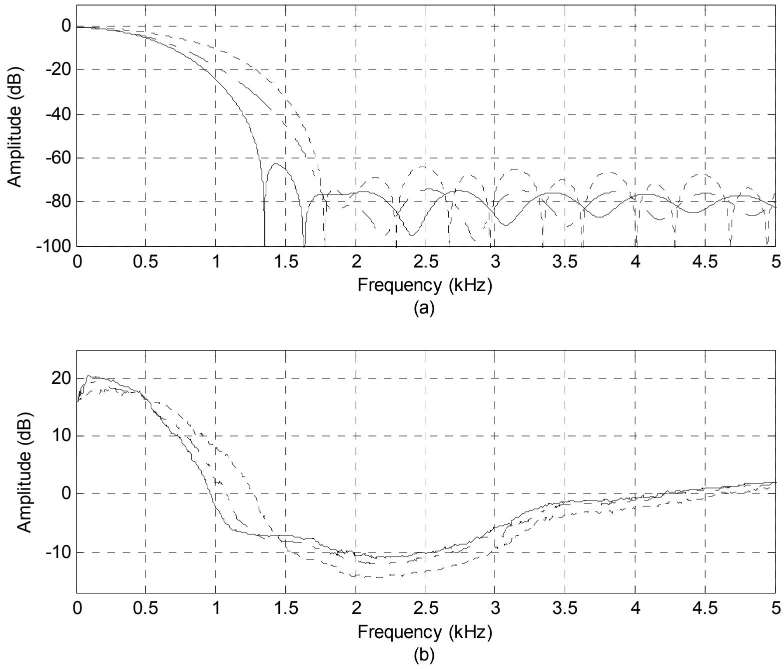

Figure 5 The results with different low-pass RFs: (a) The frequency response of the RFs. (b) The noise attenuation level in the frequency domain. The result curves in (b) of the solid, dash-dot and dotted line corresponds to the response curves of the same line type.

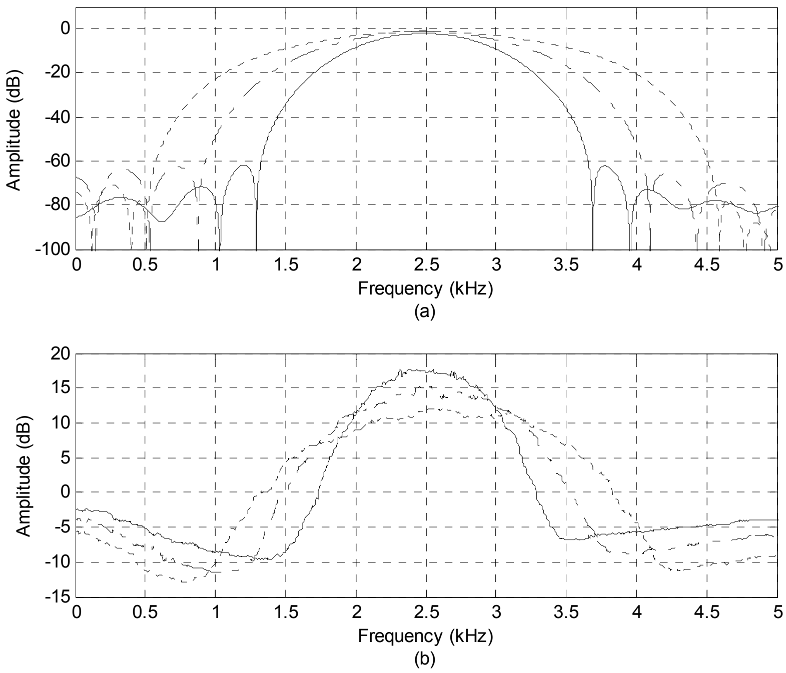

Figure 6 The results with different band-pass RFs: (a) The response of the RFs. (b) The attenuation level in the frequency domain. The result curves in (b) of the solid, dash-dot and dotted line corresponds to the response curves of the same line type.

For different leakage factors, the attenuation level of the noise in the desired frequency range is different. The simulation results are given in Figure 4 to show this property. The SF is the same for the three curves. It can be seen that the practical value is similar to the ideal case apart from the waterbed effects. The distribution of the noise attenuation and the waterbed effect is similar due to the same SF.

The larger the value of λ, the less the noise attenuation level is in the selected frequency range. The level of the waterbed effect also increases as the noise attenuation level increases. Thus the value of the leakage factor should be carefully chosen to trade off the performance between the attenuation and the waterbed effects.

The form of the SF filter affects the performance of the AFANC significantly. Figure 5 and Figure 6 show two simulation results. Both these simulations use the same leakage factor of 1e-5. In each simulation, the SF has the same function but different pass-band width. It is obvious that the location of the waterbed effect is shifted. The more the pass-band is, the further the waterbed effect appears in the frequency domain. The wider band-pass enlarges the desired range of the noise attenuation, but it may decrease the noise control performance. It can be argued that a wider frequency range increases the requirement of the control filter. For the same length of the filter, higher performance requirements can lead to a lower performance.

4. CONCLUSIONS

In this paper, an adaptive feedback control algorithm based on FeLMS algorithm for the ANC is proposed. In the proposed algorithm, a reshaping filter is used in the updating equation. Compared with the traditional adaptive feedback ANC structure, the proposed algorithm has the ability to select the band of noise attenuation by properly setting the coefficients of the weighted filter. It reduces the influence caused by the estimated mismatch of the secondary path as well, which often leads to the instability of the ANC systems in practice. A leakage factor used in the proposed algorithm not only makes the convergence stable, but also controls the noise attenuation level. Furthermore, because the RF is only used when the coefficients of the control filter are updated, there is no extra delay is introduced in the feedback ANC system. Thus, the new algorithm does not reduce the causality of the original system

ACKNOWLEDGEMENTS

This work was supported by National Science Foundation of China Grant Nos. 11204130, 11374156 and 11104141.

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