Modal floor parameters and their correlation with footfall vibration

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ABSTRACT

Excessive floor vibration in buildings can make occupants uneasy and many prediction methods of varying degree of sophistication have been developed over the years to assist engineers in the prediction of footfall vibration. This paper presents a statistical analysis of footfall vibration measurements on a series of relatively stiff floors which exhibit impulse response behaviour. The correlation of dynamic floor properties (such as dynamic and static stiffness, fundamental frequencies, damping and functional combinations) with measured footfall vibration is discussed. The discussion considers correlation with time-domain measurement data and one-third octave data. More than thirty result sets are presented and subjected to statistical analyses.

Keywords: Footfall Vibration, I-INCE Classification of Subjects Number: 42

1. INTRODUCTION

The response of structures to footfall vibration is of considerable interest and a large body of research has been devoted to this topic over the decades.

Key input parameters of footfall prediction models depend on the type of approach and usually include the modal mass of the floor, the static stiffness, the walker mass and stepping frequency and/or the fundamental floor frequency. Particularly the fundamental floor frequency plays a pivotal role as the separation of floor frequency and stepping frequency (and its harmonics) determines the appropriate forcing and response model. This distinction is also often referred to as low frequency and high frequency floors (according to Pavic et al. (14) this term was first coined by Wyatt and Dier in 1989 (18)) or resonant response floors and impulse response floors (Willford et al. (16)). There is no clearly defined frequency cut-off frequency. Willford et al. (16) report a range of 5 Hz (1) to 19.2 Hz (9) but state that 'it is the current authors' experience that the practical significance of resonance above the fourth harmonic is very small'. Pavic et al. (14) report cut-off frequencies of 7 Hz (18), 8 Hz (13) and 9 Hz (12). Ellis (6) suggests that damping may actually be the critical factor following observed resonant build up on stiff but very lightly damped floors (having a fundamental floor frequency greater than 8 Hz).

In this paper a large body of measured footfall data is presented. More than 70 result sets were reviewed and condensed to a total of 31 sets that are presented and discussed in this paper. For the purpose of footfall vibration the chosen floors can be accurately approximated as Single Degree of Freedom (SDOF) systems. The chosen data can be classified as stiff floors as the maximum footfall response coincides with the frequency of the dominant floor mode.   .

Actual test data is used as input into different prediction models to determine the accuracy and applicability of the models. In addition, the dependence of modal parameters on footfall vibration is also discussed in terms of the square of Pearson's product-moment correlation coefficient, referred to as the coefficient of determination.

2. DATA SETS

All measurements were conducted on suspended floors in office buildings with concrete columns. The floors were either reinforced concrete or composite floors. At the time of testing all floors were bare ie no fit-out, furniture, floor treatments (eg carpet) or partitioning walls were installed.

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2.1   Footfall Vibration

Footfall vibration was measured in the centre of each test bay with an accelerometer. The sampling frequency was 256 Hz and continuous, unweighted acceleration time traces were recorded.

A walker would start from close to the centre of the adjacent bay, walk through the centre of the test floor passing by the accelerometer, continue to walk towards the centre of the next bay, turn around and return to the start point in a similar fashion. Only the maximum measured vibration descriptors of both walk-throughs are used. Results are presented for two walkers; Walker Red (represented by red diamonds in all Figures) with a mass of 75 kg and Walker Blue (represented by blue circles in all Figures) with a mass of 115 kg.

Naturally, there is a large range of stepping frequencies spanning from as low as 60 beats per minute (bpm) or 1 Hz to perhaps 150 bpm (2.5 Hz). In this publication all results are presented for a nominal stepping frequency of 120 (bpm) or 2 Hz. While footfall results were available for a number of discrete stepping frequencies, results for only one stepping frequency are presented as the focal point of this paper is the influence of modal parameters.

All results are presented for a nominal stepping frequency of 120 beats per minute (bpm) or 2 Hz. The actual stepping frequencies were acquired from the measurements through inspection of the narrowband spectra, and ranged from 2.01 Hz to 2.12 Hz. The mean of the actual stepping frequencies was calculated as 2.05 Hz which is very close to the targeted stepping frequency.

Throughout this paper, all quoted frequencies have been determined from spectra with 0.2 Hz resolution and subsequently interpolation procedures were applied (Gasior (8)).

2.2   Dynamic Floor Properties

The footfall measurements were collected on 27 different floors[3]. The dynamic floor properties of each floor were determined through impact tests. A floor was impacted with a rubber tipped hammer at midbay (the geometric centre of the bay) and the acceleration response at midbay was recorded using an accelerometer. Results were averaged over (typically) five impacts. Five second time windows were used giving a frequency resolution of 0.2 Hz.

All floors exhibit a dominant, vertical structural mode. Typical floor characteristics of a floor that has been included in this study is Floor 1 presented in Miller et al. (11). The frequency of the dominant mode is referred to as the fundamental floor frequency. Each floor is characterized in terms of the vertical drivepoint mobility at midbay. In addition for each floor the underlying modal mass, damping and dynamic stiffness were estimated by curve-fitting a single degree of freedom system (de Silva (5)). The static stiffness was estimated by extrapolating the receptance spectrum towards zero Hertz (Amick et al.(2)).

All impact test data exhibited excellent coherence at the fundamental and the estimates on modal mass, damping and dynamic stiffness are therefore judged as accurate. The estimates on the static stiffness required extrapolation at low frequencies where the coherence was poor due to 1/f noise and therefore some uncertainty is associated with these estimates.

2.3   Data Correlation

The square of Pearson's product-moment correlation coefficient is used to gauge the strength of a relationship between two sets of variables (Kreyszig (10)). This value is referred to as the coefficient of determination (Taylor (15)). This coefficient indicates how well the data sets can be approximated with a linear relationship. A perfect correlation exists when the coefficient of determination is one. A coefficient of determination of close to zero indicates that there is a weak linear relationship between the two variables.

The coefficient of determination quantifies the percentage of the variation of the dependent variable that can be attributed to the variations in the independent variable. While this coefficient provides a means of directly comparing the influence of modal parameters on the maximum footfall vibration levels in terms of a percentage, it does not imply that the independent variable is the cause of the change.

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[3] Two data sets (Walker Blue and Red) were available on four floors.

3. RESULTS

3.1   Velocity Metrics and Dominant Frequencies

Three velocity descriptors are used: the maximum instantaneous velocity or peak particle velocity (PPV) of the recorded walk-by, the peak hold, root-mean-square 1/3 octave band velocity and a running 1s root-mean-square (RMS) overall velocity. The left graph in Figure 1 shows the measured 1/3 octave velocities and the measured PPVs versus the measured 1s RMS velocities. An almost linear dependence can be observed which is reflected in high coefficients of determination of 0.96 and 0.97 for PPV to 1s RMS and 1/3 octave velocities to 1s RMS, respectively. The measured PPVs range from 0.35 mm/s to 2.5 mm/s. The PPVs are approximately 1.75 times the 1s RMS velocities and the 1/3 octave velocities are generally very close to the 1s RMS velocities due to the tonal floor vibration characteristics.

On the right graph in Figure 1 the fundamental floor frequencies are plotted versus the dominant floor response frequency (all frequency estimates were interpolated from 0.2 Hz spectra). The coefficient of determination between these two variables is 0.996. The floor fundamentals range from 5.7 Hz to 11.1 Hz with approximately half of the floors having a fundamental frequency in the 8±0.5 Hz bandwidth.
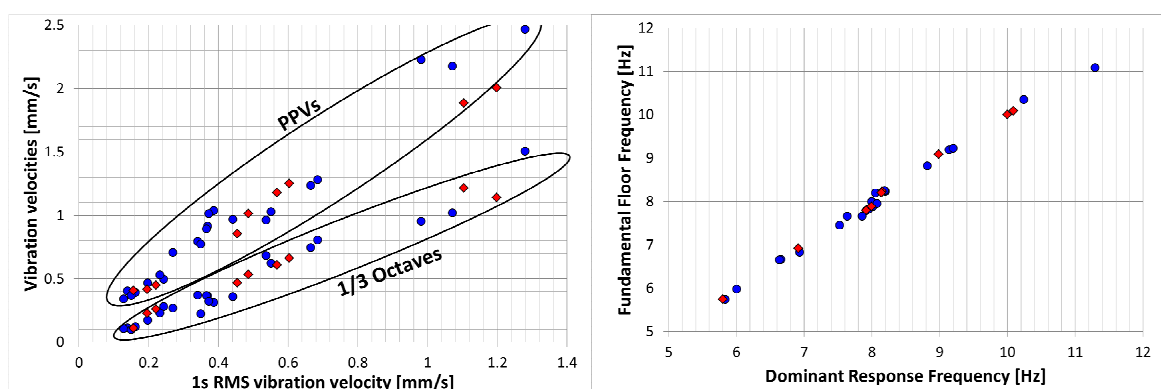

Figure 1 – Velocity descriptors (left) and frequencies (right)

3.2   Modal Parameters

The six graphs in Figure 2 demonstrate the dependence of selected modal properties on PPVs. While this data is showing scatter, some trends can be clearly identified. Coefficients of determination of the discussed modal properties and footfall vibration are presented in Table 1. PPVs generally provide the best data correlation while correlation with the 1/3 octave velocity data is always poorest.

Fundamental floor frequencies versus PPVs are plotted in the graph on the top left. Floor vibration generally decreases with increasing floor fundamental. The dependence is strong for the floors with fundamental frequencies greater than 8.5 Hz. However, a large PPV range is observed over the small frequency range of 8±0.5 Hz.

The floors' mobilities at their fundamental frequencies versus measured PPVs are shown in the graph on the top right, Figure 2. The mobilities range from 0.009 mm/s per N to 0.09 mm/s per N and increase with increasing PPV.

PPVs decrease with increasing modal mass (center left graph, Figure 2). The modeshapes of the investigated floors have modal masses of 4 t to 42 t associated with their fundamental mode.

Damping (center right graph, Figure 2) generally ranges from 1% to 3% which is comparatively high for bare concrete floors as Willford et al. (17) specify a 1% to 2% critical damping for bare reinforced concrete floors. No influence of damping on the PPVs is apparent. Linear regression actually indicates that floor vibration increases with damping which is counter-intuitive. Of all plotted parameters, damping and PPVs exhibit the lowest coefficient of determination (Table 2).

The two graphs in the bottom row of Figure 2 show the dependence of the static stiffness (left) and the dynamic stiffness (right) on the floors' PPVs. For each floor the dynamic stiffness is on average 2.5 times greater than the static stiffness.
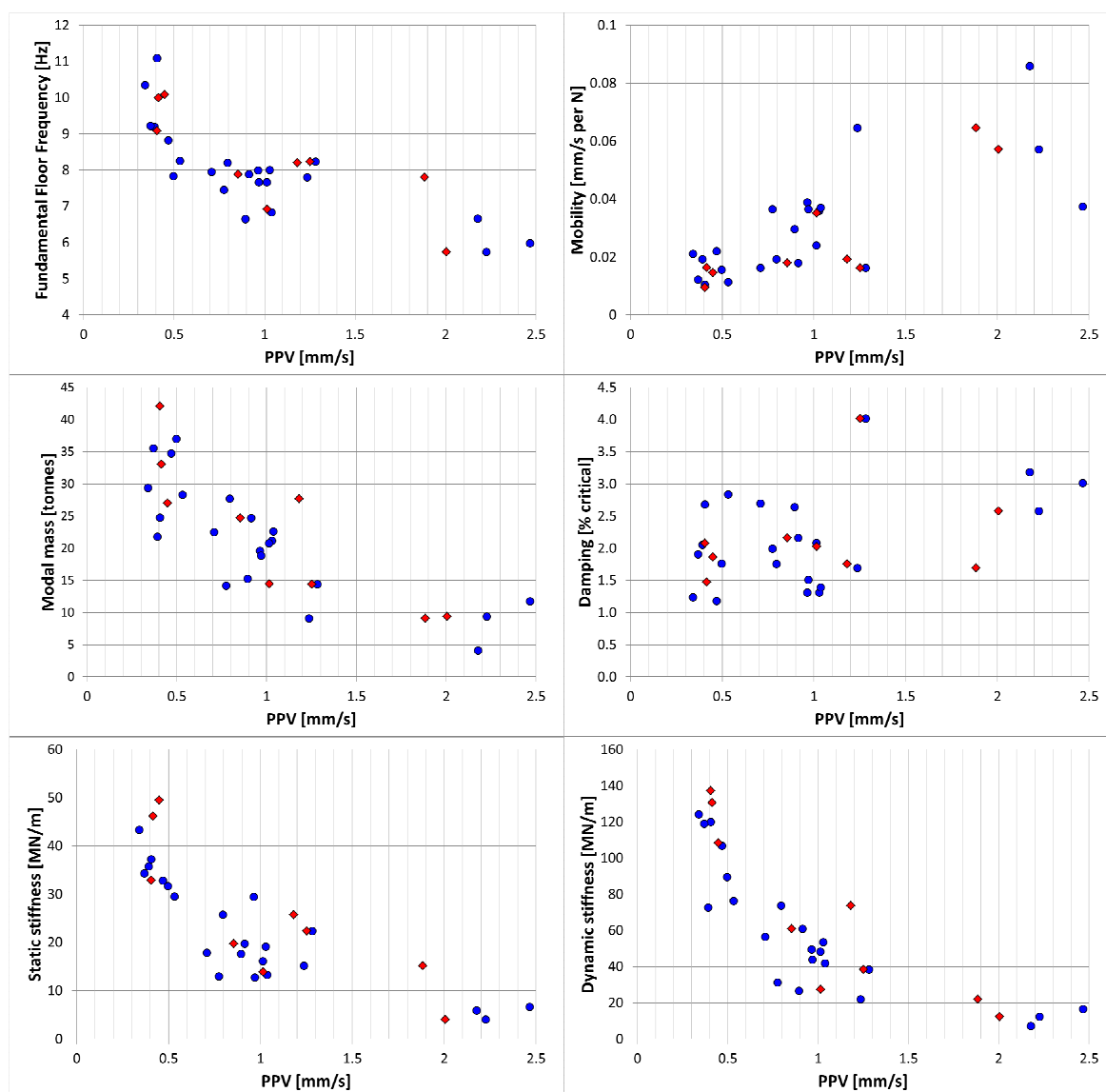
Figure 2 – Modal Floor Properties

In addition to the modal parameters, Table 1 also presents the coefficients of determination of some selected functional combinations which are commonly used for predicting footfall vibration. Correlation can be increased and there may be potential for a more rigorous statistical analysis of the used data sets.

The dynamic stiffness exhibits the best correlation. Surprisingly the mobility (which can be viewed as a combination of dynamic stiffness, damping and modal mass) shows comparatively poor correlation. The static stiffness correlates very well justifying approaches utilizing the static stiffness. From a design perspective, the static stiffness is often the only parameter that can be provided with some degree of accuracy.

The authors note that correlation with scaled velocities (ie Walker Blue vibration reduced by 53% - 75 kg/115 kg) always yields poorer correlation than using the actual measured velocities. Some prediction approaches for stiff floors have the walker mass factored in (eg resonant response methods Willford et al. (17) and Allen et al. (12)) and the data set presented in this paper does not support this.

Table 1 – Coefficients of Determination

Parameter	Functional Representation	PPV	1s RMS	1/3 Octave
Modal Mass m, Eq. (1)	$1/m$	0.58	0.56	0.46
Floor Fundamental f_n	$1/f_n$	0.67	0.58	0.49
Static Stiffness, k_0	$1/k_0$	0.64	0.58	0.43
Dynamic Stiffness, k_{dyn}	$1/k_{dyn}$	0.70	0.65	0.53
Mobility at Fundamental, M	M	0.58	0.59	0.49
Damping, ζ	$1/\varsigma$	0.14	0.12	0.10
Scaled Floor Fundamental (part of Equations 1, 2, 3)	$1/f_n^{1.30}$	0.67	0.59	0.49
Scaled Floor Fundamental (part of Equations 1, 2)	$f_{step}^{1.43}/f_n^{1.30}$	0.69	0.61	0.51
Equations 1, 2, 3	$\dfrac{1}{m \times f_n^{1.30}}$	0.68	0.64	0.52
Equations 5, 6	$\dfrac{1}{f_n \times k_0}$	0.69	0.60	0.46

4. PREDICTION METHODS

4.1 Methods based on modal mass

For a SDOF floor it is found that the amplitude of the velocity of the floor is proportional to the effective impulse and inversely proportional to modal mass. This is shown in Eq. (1), where *I* is the impulse and *m* is the modal mass.

$$PPV \approx I/m \tag{1}$$

Eq. (1) can be used for continuous systems where the system is approximated as a series of SDOFs and a summation over all contributing modes (up to twice the fundamental) is required.   If the floor is similar to a single degree of freedom, there's no summation and results can simply be expressed at the fundamental frequency, depending on modal mass only.

Within this framework, different ways of implementing the effective impulse have been reported. Willford et al. (17) use Eq. (2) where the impulse is dependent on the stepping frequency of the walker and the fundamental floor frequency. The coefficient of 54 in Eq. (2) is based on the upper quartile of the database used in Willford (17), and is intended to be used for design purposes. Hicks et al. (9) provide impulse estimates independent on the stepping frequency (Eq. (3)). Both formulas do not account for the mass of the walker. For a nominal stepping frequency of 2 Hz, Eq. (3) gives 30% greater effective impulses than Eq. (2).

$$I = 54 \times \frac{f_{step}^{1.43}}{f_n^{1.30}} \qquad (2)$$

$$I = 190 \times \frac{1}{f_n^{1.30}} \qquad (3)$$

The left graph in Figure 3 shows the predicted PPVs (using Eqs. (1, 3) and the actual modal mass, stepping frequencies and fundamental floor frequencies) versus the measured PPVs. With the Hicks et al. (9) impulse footfall vibration is generally under-predicted for the considered data set. The Willford et al. (17) method under-predicts footfall vibration even more so since the associated effective impulse is 30% smaller. Initially, contributions from higher order modes with lower modal mass were believed to be a possible cause of this under-prediction. However, inspection of narrowband FFTs of the floor response did not support this. The discrepancy may also arise from the fact that effective impulses may have partially been derived based on nominal mass (eg approximated as the mass of one quarter of the bay) rather than actual modal mass.
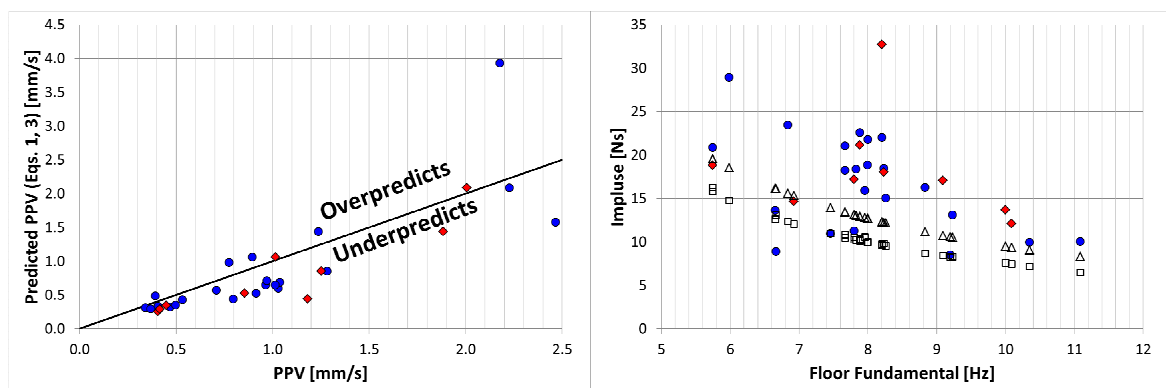

Figure 3 – Predicted PPV versus measured PPV (left) and back-calculated impulse versus floor fundamental (right)

Eq. (1) has been used to calculate the actual impulse force by multiplication of the measured PPVs and measured modal mass. The right graph of Figure 3 shows the impulse force back-calculated from the data versus fundamental floor frequency. The same graph also shows the theoretical impulse forces as per Eq. (2) and Eq. (3) (black squares and triangles, respectively). The average back-calculated impulse is 17.2 Ns compared to 10.4 Ns for Eq. (2) and 13.1 Ns for Eq. (3).

A point of note is the cluster of high impulses calculated where the fundamental floor frequency is approximately 8 Hz. As the stepping frequency is nominally 2 Hz, an intuitive conclusion is that the high impulses are actually a resonant response to the fourth harmonic of the stepping frequency.

The left graph in Figure 4 illustrates a ratio of an integer multiple of the stepping frequency (with the integer chosen so the stepping frequency is multiplied to correspond with the floor fundamental as close as possible) divided by the floor fundamental, which is plotted against the measured impulse. If the floors were undergoing a harmonic response, one would expect that when an integer multiple of the stepping frequency approaches the floor fundamental, that the measured impulse would increase dramatically. Figure 4 contradicts this assumption. Additionally, if the floors were undergoing a harmonic response, one would also expect that at frequencies distant from integer multiples of the stepping frequency the response would be comparatively low. Examining the results on the right in Figure 3 at 7 Hz and 9 Hz (both of which are distant from integer multiples of the stepping frequency) also contradicts this assumption.

In addition, it is found that this cluster does not have particularly low damping values (right graph, Figure 4). Ellis (6) suggests that this resonant buildup can occur at stiff floors which are lightly damped (less than 1%).
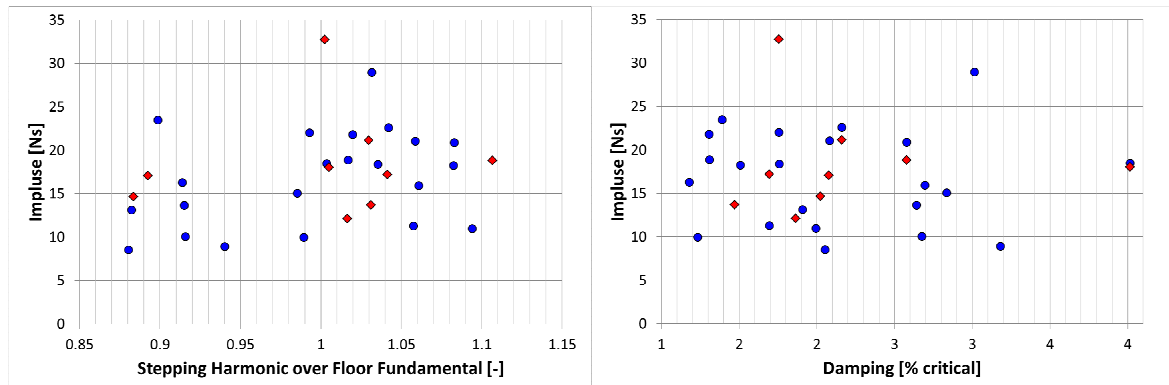
Figure 4 –Ratio of ith-harmonic over floor fundamental closest to unit (left) and impulse versus damping (right).

4.2   Methods based on static stiffness

Brownjohn et al. (4) present a simple formula (Eq. 5) which calculates one-third octave band velocity based on the static floor stiffness, k_0, and the fundamental frequency, f_n. The constant C_w equals 70,000. The predictions do not depend on walker mass or stepping frequency and are very simple to use.

$$v_{RMS} = \frac{C_w}{f_n \times k_0}$$ (5)

Eq. (6) is based on Murray et al. (12) and used the static stiffness of the floor, the fundamental frequency and the dependence of maximum force on the stepping frequency from Galbraith et al. (7). The maximum RMS floor velocity is expressed as

$$v_{RMS} = 1.75^{-1} \times \frac{\pi \times F_m \times f_0^2}{f_n \times k_0}$$ (6)

In Eq. (6), f_0 is the inverse of the footfall rise-time and F_m is the maximum force.   Murray et al. (12) states that equations pertain '*to the instantaneous maximum or "peak" vibration*' and a 1.75^{-1} peak-RMS scaling factor (refer to Section 3.1) was introduced by the authors to match the crest factor of the data set. While f_0 depends on stepping frequency only, F_m depends on the stepping frequency and the walker mass. For a nominal stepping frequency of 2 Hz, f_0 equals 7 (upper and lower bound range from perhaps 3 to 15) and F_m equals $1.9 \times W \times 9.81$, where W is the walker mass in kilograms.

The left graph in Figure 5 compares the Brownjohn et al. (Eq. 5) predictions with the measured 1s RMS levels. The right graph in Figure 5 compares the (modified) AISC chapter 6 (Eq. 6) predictions with the measured 1s RMS levels.

The Brownjohn et al. method generally shows comparatively good agreement and over-predicts floor vibration on average by 1.3. The over-predictions generally increase with increasing absolute vibration levels and the maximum over-prediction of this method is 3. The maximum under-prediction is 0.5.

The AISC chapter 6 method is found to over-predict vibration, particularly for Walker Blue. On average, this method over-predicted measured vibration levels by 3.2. The maximum over-prediction is greater than 8 and similarly to the Brownjohn et al. method, over-predictions increase with increasing absolute vibration level. The conservatism of this method may be due to the dependence of the walker mass which actually may not be an influence factor on stiff floors (ie as assumed by the impulse approach methods (previous section) and the Brownjohn et al. approach).
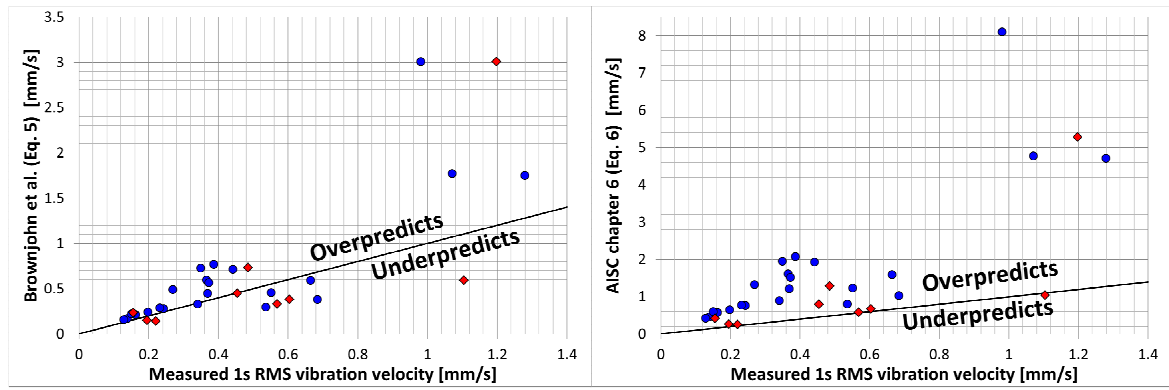
Figure 5 – Predictions versus measured – Left: Brownjohn et al., Right: AISC chapter 6.

4.2.1 Discussion

Eqs. (5) and (6) are similarly structured and the C_w constant can be expressed as a function of walker force (F_m) and the rise time ($1/f_0$) as shown in Eq. (7). Note that F_m and f_0 depend on the walker mass and the stepping frequency.

$$C_w = \frac{\pi \times F_m \times f_0^2}{1.75} \qquad (7)$$

The dashed lines in Figure 6, left and right, show the C_w factor (left term in Eq. (7)) which is 70,000 according to Brownjohn et al. and independent of the stepping frequency and walker mass.

The solid lines in Figure 6 left and right represent the right hand side of Eq. (7) for 75 kg and 115 kg walkers, respectively. The thick, solid lines are the mean[4] value and the thin lines represent the upper and lower bounds based on the inverse of the rise time f_0.

The symbols in Figure 6 show a back-calculated C_w value (ie the measured walker vibration (1s RMS) multiplied by the measured floor fundamental, f_n, and the measured static stiffness, k_0).

The back-calculated C_w values range from 22,800 to 130,000 and the average C_w value was 65,800. More than half of the back-calculated C_w values are less than 70,000 which means that the Brownjohn et al. approach over-predicts floor vibration in these cases.

Figure 6 shows that the AISC chapter 6 predictions depend heavily on the stepping frequency and the walker mass. Additional measurements at different stepping frequencies would be required to confirm the dependence on the stepping frequency.

For a 75 kg walker, the use of the mean AISC chapter 6 curve predicts lower estimates than the Brownjohn et al. method for walking slower than 100 bpm and higher estimates for walking faster than 100 bpm. This over/under prediction cut-off frequency is shifted to approx. 92bpm for a 115 kg walker.
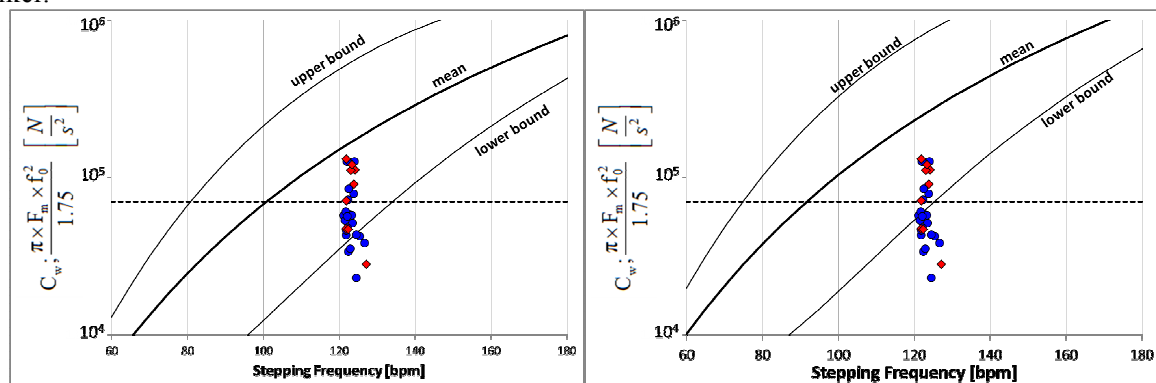

Figure 6 –Comparison of Eqs. (5) and (6) and measurements for 75kg walker (left) and 115kg walker (right).

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[4] The authors could not establish whether it is an average value or a certain percentile.

5. CONCLUSIONS

A set of 31 footfall vibration measurements on floors of known dynamic properties were discussed. All floors had dominant, fundamental modes which were excited by the walkers and which dominated the floor vibration spectra.

It was found that the modal parameter that correlated best with footfall vibration was dynamic stiffness. However, no commonly used methods that use dynamic stiffness are known to the authors and from a design perspective this parameter may be difficult to predict reliably.

A combination of fundamental floor frequency and static stiffness was found to correlate well with footfall vibration. Conversely to dynamic stiffness, the static stiffness and fundamental frequency can usually be determined (reasonably) accurately during the design phase.

The employed static stiffness based methods were found to over-predict footfall vibration levels at the investigated discrete stepping frequency of 2 Hz. The static stiffness based methods were found to match better when the predicted footfall vibration was low. The higher the predicted footfall vibration, the greater the over-prediction.

Impulse based approaches dependent on modal mass and scaled floor fundamental were found to generally under-predict footfall vibration. The accuracy of the used approaches may prove more reliable at different stepping frequencies and more work is required to ascertain this finding.

For the considered floors, damping was found to correlate very poorly with the measured footfall vibration. While this is not surprising for the PPVs, a better dependence was anticipated for the 1 s RMS velocities. However, this is not supported by the coefficient of determination which is lower for 1 s RMS than for PPVs.

Finally, the authors wish to point out that all results presented are for only two walkers and one discrete stepping frequency. A greater representative sample (comprising various walker styles, stepping frequencies and walker masses) is clearly required to further substantiate the conclusions.

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