DYNAMIC MODELLING AND APPLICATIONS FOR PASSENGER CAR POWERTRAINS

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ABSTRACT. Torsional finite elements for direct, geared, branched and grounded connections are presented. For a simple three-degrees-of-freedom powertrain model the finite elements are defined and the global system assembly is detailed. The appropriateness of the finite element method for powertrain systems is illustrated via examples for modelling manual, automatic and continuously variable transmissions. The use of custom elements is discussed for an element for toroid-roller contact and for a two-stage planetary gear set. A test rig is presented and model verification is discussed.

1. INTRODUCTION

Powertrain vibration analysis is an important area of research for the automotive industry. The goal of the research is to improve operating characteristics with the reduction of steady state and transient vibration. A particular focus is on vehicle powertrains in which the quality of the finished product, the motor vehicle, can be seriously diminished by unwanted noise and motion felt within the passenger compartment. This noise and motion is partly due to the torsional vibration of powertrain components.

The refinement in design of vehicle powertrain systems requires many complex phenomena to be analysed in the whole powertrain. Lumped mass models are used to represent the system and a simple way of developing their equations of motion is to use the finite element method. Wu and Chen [1] outlined the method for deriving 'so called' [1, 2] torsional finite elements. Using these elements they developed systems of equations of motion for geared systems and performed free vibration analysis for these systems. Crowther et al. [3] used the method for the dynamic modeling of a powertrain system fitted with an automatic transmission with the planetary gear set modelled with one degree of freedom. Zhang et al. [4] used the method for the dynamic modeling of the same powertrain system with the planetary gear set modeled with four degrees-of-freedom. Both Crowther and Zhang used the dynamic models for free vibration analysis and transient vibration simulations. [2]-[5] provide review of additional related literature.

In this paper the torsional finite elements for direct, geared, branched and grounded connections are presented. Using these elements the global system of equations is developed for a simple three-degrees-of-freedom powertrain model that is commonly used to represent vehicles fitted with manual transmissions. Dynamic modelling schematics are provided for systems with automatic and continuously variable transmissions. The appropriateness and usefulness of the finite element method for these systems is outlined. The use of custom finite elements is discussed with examples of a finite element representing the dynamics of toroid-roller contact and a finite element for a two-stage planetary gear set.

2. TORSIONAL FINITE ELEMENTS

Torsional finite elements simplify powertrain modelling. They represent inertias, their local coordinates and coupling within global dynamic systems. These elements are used to develop a global system of equations of motion via a simple matrix assembly [1], [3]. Model schematics are shown in figure 1 for five simple dynamic systems with lumped inertias and connecting damping and stiffness. The examples are for *direct*, *geared – rigid* and *elastic mesh, branched* and *grounded* systems. Stiffness and damping parameters are torsional except for the geared connection with elastic mesh were the tooth stiffness is normal to the plane of contact. For each system the required torsional finite elements are outlined. The matrices for inertia, stiffness and damping and the local coordinate vectors are given in table 1. The general finite element types presented can be used for quickly obtaining the equations of motion for large complicated systems. The method can be used for lumped inertia torsional systems and is particularly useful for vehicle powertrain applications. Coordinates can be also be grounded by removing them from the coordinate vector.

Matrix assembly for systems using these finite elements is a simple process. As an example a powertrain system dynamically modelled with three-degrees-of-freedom is shown in figure 2. This system has one *gear step*. It is *grounded* at one end via a damping element – representing absolute damping on the engine. It is *grounded* at the other end via stiffness and damping elements - powertrain systems can be grounded in this fashion when the models are to be used for free vibration analysis and the grounded end has a very large comparative inertia.

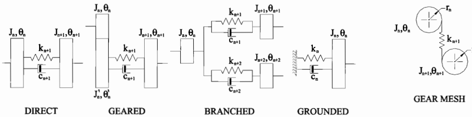

Figure 1. Model Schematics

Table 1. Torsional Finite Elements for Direct (1) Rigid and Elastic Geared (2)-(3) Branched (4) and Grounded Systems (5)

$$I_{e(n+1)}=\begin{bmatrix} J_n & 0 \\ 0 & J_{n+1}\end{bmatrix} \quad K_{e(n+1)}=\begin{bmatrix} k_{n+1} & -k_{n+1} \\ -k_{n+1} & k_{n+1}\end{bmatrix} \quad C_{e(n+1)}=\begin{bmatrix} c_{n+1} & -c_{n+1} \\ -c_{n+1} & c_{n+1}\end{bmatrix} \quad \theta_{e(n+1)}=\begin{Bmatrix}\theta_n \\ \theta_{n+1}\end{Bmatrix} \tag{1}$$

$$I_{e(n+1)}=\begin{bmatrix} n_G^2 J'_n & 0 \\ 0 & J_{n+1}\end{bmatrix} \quad K_{e(n+1)}=\begin{bmatrix} n_G^2 k_{n+1} & -n_G k_{n+1} \\ -n_G k_{n+1} & k_{n+1}\end{bmatrix} \quad C_{e(n+1)}=\begin{bmatrix} n_G^2 c_{n+1} & -n_G c_{n+1} \\ -n_G c_{n+1} & c_{n+1}\end{bmatrix} \quad \theta_{e(n+1)}=\begin{Bmatrix}\theta_n \\ \theta_{n+1}\end{Bmatrix} \tag{2}$$

$$I_{e(n+1)}=\begin{bmatrix} J_n & 0 \\ 0 & J_{n+1}\end{bmatrix} \quad K_{e(n+1)}=\begin{bmatrix} r_n^2 k_{n+1} & -r_n r_{n+1} k_{n+1} \\ -r_n r_{n+1} k_{n+1} & r_{n+1}^2 k_{n+1}\end{bmatrix} \quad \theta_{e(n+1)}=\begin{Bmatrix}\theta_n \\ \theta_{n+1}\end{Bmatrix} \tag{3}$$

$$I_{e(n+1)}=\begin{bmatrix} \frac{J_n}{2} & 0 \\ 0 & J_{n+1}\end{bmatrix} \quad K_{e(n+1)}=\begin{bmatrix} k_{n+1} & -k_{n+1} \\ -k_{n+1} & k_{n+1}\end{bmatrix} \quad C_{e(n+1)}=\begin{bmatrix} c_{n+1} & -c_{n+1} \\ -c_{n+1} & c_{n+1}\end{bmatrix} \quad \theta_{e(n+1)}=\begin{Bmatrix}\theta_n \\ \theta_{n+1}\end{Bmatrix} \tag{4A}$$

$$I_{e(n+2)}=\begin{bmatrix} \frac{J_n}{2} & 0 \\ 0 & J_{n+2}\end{bmatrix} \quad K_{e(n+2)}=\begin{bmatrix} k_{n+2} & -k_{n+2} \\ -k_{n+2} & k_{n+2}\end{bmatrix} \quad C_{e(n+2)}=\begin{bmatrix} c_{n+2} & -c_{n+2} \\ -c_{n+2} & c_{n+2}\end{bmatrix} \quad \theta_{e(n+2)}=\begin{Bmatrix}\theta_n \\ \theta_{n+2}\end{Bmatrix} \tag{4B}$$

$$I_{e(n)}=[J_n] \quad K_{e(n)}=[k_n] \quad C_{e(n)}=[c_n] \quad \theta_{e(n)}=\{\theta_n\} \tag{5}$$

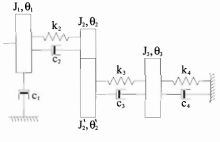

Figure 2. Three Degrees-of-Freedom Powertrain System

The finite element matrices are assembled into global system matrices by using local coordinate vectors and the global coordinate vector. The final equation of motion for the system will have the form:

$$I\ddot{\theta}+C\dot{\theta}+K\theta=0 \tag{6}$$

With global coordinate vector

$$\theta=\{\theta_1 \quad \theta_2 \quad \theta_3\} \tag{7}$$

The finite element inertia, stiffness and damping matrices and local coordinate vectors for this system are given in table 2. Also given in this table are the assembled global inertia, stiffness and damping matrices.

The grounded inertial finite elements used in this three-degrees-of-freedom system have been modified from the previously presented general grounded element (8). *The modification is the replacement of the inertia value with a zero.* This is as the inertia is accounted for in the direct and geared inertial elements and otherwise would be counted twice. This modification to inertia finite elements can be necessary in certain situations.

The example illustrates the simplicity of the finite element method when used for a typical torsional system. For the geared elements the displacement coordinates are absolute coordinates. It is common in dynamic analysis for powertrains for the coordinates downstream of gearing to be modelled with equivalent engine coordinates, the finite elements and local coordinate vectors can be modified to meet this requirement.

Table 2. Local and Global Matrices and Coordinate Vectors for three-degrees-of-freedom system

$$\mathrm{I}_{e1}=[0] \qquad \mathrm{C}_{e1}=[c_1] \qquad \theta_{e1}=\{\theta_1\} \tag{8}$$

$$\mathrm{I}_{e2}=\begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix} \qquad \mathrm{K}_{e2}=\begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \qquad \mathrm{C}_{e2}=\begin{bmatrix} c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \qquad \theta_{e2}=\begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} \tag{9}$$

$$\mathrm{I}_{e3}=\begin{bmatrix} n_G^2 J_2' & 0 \\ 0 & J_3 \end{bmatrix} \qquad \mathrm{K}_{e3}=\begin{bmatrix} n_G^2 k_3 & -n_G k_3 \\ -n_G k_3 & k_3 \end{bmatrix} \qquad \mathrm{C}_{e3}=\begin{bmatrix} n_G^2 c_3 & -n_G c_3 \\ -n_G c_3 & c_3 \end{bmatrix} \qquad \theta_{e3}=\begin{Bmatrix} \theta_2 \\ \theta_3 \end{Bmatrix} \tag{10}$$

$$\mathrm{I}_{e4}=[0] \qquad \mathrm{K}_{e4}=[k_4] \qquad \mathrm{C}_{e4}=[c_4] \qquad \theta_{e4}=\{\theta_3\} \tag{11}$$

$$\mathrm{I}=\begin{bmatrix} J_1 & 0 & 0 \\ 0 & J_2+n_G^2 J_2' & 0 \\ 0 & 0 & J_3 \end{bmatrix} \quad \mathrm{K}=\begin{bmatrix} k_2 & -k_2 & 0 \\ -k_2 & k_2+n_G^2 k_3 & -n_G k_3 \\ 0 & -n_G k_3 & k_3+k_4 \end{bmatrix} \quad \mathrm{C}=\begin{bmatrix} c_1+c_2 & -c_2 & 0 \\ -c_2 & c_2+n_G^2 c_3 & -n_G c_3 \\ 0 & -n_G c_3 & c_3+c_4 \end{bmatrix}$$

3. APPLICATIONS FOR DYNAMIC MODELLING OF POWERTRAIN SYSTEMS

The simplest model for a vehicle powertrain system with a *manual transmission* is the three-degrees-of-freedom model of Figure 1. The gear ratio, n, can be set for the particular gear and the model used for free vibration analysis. If the grounding on coordinate 3 is removed (stiffness and damping element 4) and a torque vector included in the equation of motion then the model can be used for forced vibration analysis. The model can be extended to include more degrees of freedom and branching to drive wheels if needed, such as for four-wheel drive versions with a differential between the differentials configuration.

Modelling powertrains fitted *automatic transmissions* can be complicated but the finite element method simplifies the task considerably. Crowther et al. [3] developed the global system of equations for a powertrain fitted with a transmission with a two-stage planetary gear set, four wet clutches, two one way clutches and two brake bands. Figure 3 provides a schematic for the dynamic model of this powertrain system. The schematic is for second gear and for second to third upshifts. For this system the elements connecting to the planetary gear set are modelled as geared elements and the gear ratios are sourced from a rigid body dynamic analysis. The gear set is modelled with equivalent ring gear coordinates and set as θ_4. The geared elements are k_2, k_3 and k_4. The differential requires geared and branched elements, k_6 and k_8. All other elements are direct. The finite element method is especially useful in this case for numerical simulations of shift transients, i.e. vibration due to gear shifts. For the shift from second to third gear the C1 clutch engages, connecting coordinate 2 and 3. *One degree of freedom drops out of the system*, so the global system of equations is reassembled where the *only modifications are to the local coordinate vector for element three, and the corresponding change for the global coordinate vector and the torque vector*. For the period of gear shifting the gear ratio parameter n was varied as per a ratio versus shift time data map.

Custom finite elements can be developed to suit various complexities within powertrains. The method is particularly appropriate for powertrain systems with planetary gear sets. In the models presented in figures 2 and 3 the gear ratios were predetermined and the gears are modelled as a single rigid body with one degree of freedom. They can be improved by using a custom element that has been developed for a Ravigneaux two-stage planetary gear set. The gearset has six degrees of freedom and consists of a forward sun gear, rear sun gear, three short and three long pinions or planet gears, and a planet gear carrier that holds the pinions and a ring gear. The forward sun gear, rear sun gear, planet carrier and ring gear connect through to the clutch drum/differential pinion via shaft stiffness and/or damping elements. Of the six degrees of freedom, two, the short and long pinions can be ignored – they are totally dependent, two are semi-independent and two are independent. The complete derivation for this element is provided by Zhang et al. [4]. Briefly, the element is derived from equations of motion for gear components that include the internal forces and external torques and from the constraining acceleration relationships between the components. The stiffness and damping element matrices includes gear inertias and radii. The element is general and can be modified for each gear state when placed in the surrounding powertrain system.

Geared systems require clearance between mating gears for smooth operation. The clearance is termed *lash* and the mating gears must separate across the lash when their relative directions of rotation change. The mating gears can be modelled with a mesh stiffness which is non-linear. It is set as zero across the lash zone. On torque reversals mating gears switch direction of rotation, this causes a 'clonk' (a term used in the automotive industry) when they impact. Transient dynamics from engine tip/in, gear shifts, etc. can produce a torque reversal (shuffle) thereby inducing clonk [5]. The finite element (3) for a gear pair and the custom planetary gear element both have elastic tooth meshing and the lash non-linearity can be included into numerical simulations.

The transmission has many states of operation – first through to fourth gears and torque converter lock-up, with clutches and bands controlling gear shifts and their states defining the motion of the gearset components. Using the general torsional finite elements and the planetary gear set element the global system can be quickly assembled for any of these states. The final set of equations includes the complete dynamics of the planetary gear set. This same methodology can be applied to five and six speed automatic transmissions.

Continuously Variable Transmissions (CVT) are the most recent type of transmission to be widely used in vehicle powertrains. Common types are toroidal, v-belt and hydromechanical CVTs. These systems can be even more complicated than automatic transmissions as some have multi-staging and some are used in tandem with planetary gear sets – then requiring clutches and/or brake bands. *The finite element method provides an appropriate tool for the dynamic modelling of these systems.* Figure 4 presents a model for a powertrain fitted with a half toroidal CVT and planetary gear set. There are two clutches, a high velocity clutch (HVC) which connects the toroid direct to the differential and a low velocity clutch (LVC) which connects the toroid to the differential via a single stage planetary gear set. In this system the power can flow either way depending on the clutch engagement. The connection between the LVC and the ring gear (via the sun gear), k_6 and c_6, are modelled as geared elements. Note the gear set is modelled with equivalent ring gear coordinates. The connections from the differential to the wheels, k_8 and c_8, and k_9 and c_9, are modelled as geared and branched elements.

Torque is transferred between the toroids and the roller via a thin film of oil that transiently acts like a solid. This film can be represented with a damping and stiffness. Custom finite elements have been derived to represent this connection. They are essentially the same as the elastic gear element (3). Connections k_2, c_2 and k_3, c_3 are considered as horizontal. With radii r_2, r_3 and r_4, torsional stiffness is introduced as (likewise for torsional damping):

$$k_2' = r_3^2 k_2 \quad \text{and} \quad k_3' = r_4^2 k_3$$

The derived elements are given in table 3. Note coordinates 2 and 4 (toroids) have positive rotation clockwise. Coordinate 3 (roller) has positive rotation anti-clockwise, if the signs of the stiffness/damping coefficients in the element are all made positive it will be clockwise. In either case in solution the roller rotates opposite to the toroids. All other connections in the system are direct elements. The global system can be quickly assembled from these elements with a global coordinate vector for either low velocity or high velocity clutch engagement.

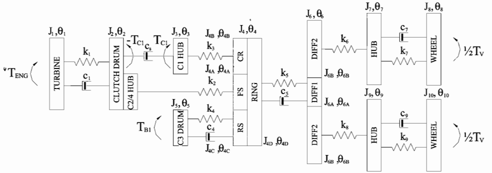

Figure 3. Dynamic Model for Powertrain Fitted with Automatic Transmission – Second Gear

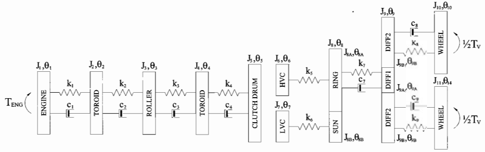

Figure 4. Dynamic Model for Powertrain Fitted with CVT and Planetary Gear Set

Table 3. Local Matrices and Coordinate Vectors for CVT

$$\mathrm{I}_{e2}=\begin{bmatrix} J_2 & 0 \\ 0 & \frac{J_3}{2} \end{bmatrix} \quad \mathrm{K}_{e2}=\begin{bmatrix} {n_2}^2 k_2' & -n_2 k_2' \\ -n_2 k_2' & k_2' \end{bmatrix} \quad \mathrm{C}_{e2}=\begin{bmatrix} {n_2}^2 c_2' & -n_2 c_2' \\ -n_2 c_2' & c_2' \end{bmatrix} \quad \theta_{e2}=\begin{Bmatrix} \theta_2 \\ \theta_3 \end{Bmatrix} \quad n_2=\frac{r_2}{r_3}$$

$$\mathrm{I}_{e3}=\begin{bmatrix} \frac{J_3}{2} & 0 \\ 0 & J_4 \end{bmatrix} \quad \mathrm{K}_{e3}=\begin{bmatrix} {n_3}^2 k_3' & -n_3 k_3' \\ -n_3 k_3' & k_3' \end{bmatrix} \quad \mathrm{C}_{e3}=\begin{bmatrix} {n_3}^2 c_3' & -n_3 c_3' \\ -n_3 c_3' & c_3' \end{bmatrix} \quad \theta_{e3}=\begin{Bmatrix} \theta_3 \\ \theta_4 \end{Bmatrix} \quad n_3=\frac{r_3}{r_4}$$

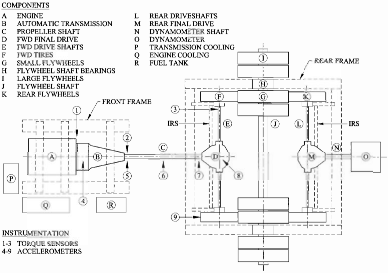

Figure 5. Powertrain Test Rig Schematic

4. EXPERIMENTAL VERIFICATION

Experimental verification is needed for industry to be able to rely on the analytical and numerical tools. For dynamics, typical test rig uses include, investigating component characteristics (engine, torque converter, clutch, tire, etc.), investigating free, steady state and transient response, and calibration for gear shifts. At the University of Technology, Sydney, a powertrain test rig has been constructed for the investigation of vibration response and gear shift quality assessment. The aim is to verify dynamic analysis using a model similar to that of figure 3 with an automatic transmission. The model is used for free vibration analysis and steady state and transient numerical simulations. Detailed information is available by correspondence, in brief:

The test rig includes all the components of the vehicle powertrain and has been designed to include a vehicle mass of 1500 kg (as inertia) and a dynamometer load (figure 5). For data acquisition the engine and transmission control systems are tapped and instrumentation added for pressures, torques and accelerations. Accelerometers are fixed on the transmission and differential case. Torque is measured via strain gauges on the flywheel, transmission output shaft and drive shaft. Radio telemetry is used to pass the strain gauge data from the rotating shaft to a non-rotating element. The gauge voltage is amplified, processed by an analogue to digital converter and then transmitted. Transceivers are used on both rotating and non-rotating sides. Data is recorded and post-processed with Lab View.

Various tests can be conducted with this test rig:

Free vibration: The transmission is placed in park (grounding the rigid body motion). A torque is applied to the tires and released. Accelerometers and torque sensors provide free vibration results. The purpose is to compare real system frequency response to a free vibration analysis of the driveline system. This allows a validity check for the stiffness and inertia parameters and driveline dynamic model.

Critical Speed: The engine is run within speed ranges that are calculated for resonance for given gear states. Shaft torque and case accelerations provide steady state response at test speeds. The purpose is to compare resonant modes for the powertrain system. This allows a validity check for the stiffness, inertia and damping parameters and the whole dynamic model.

Engine Tip in/out: The engine is run at a constant speed and the throttle is suddenly increased/decreased. Shaft torque and case accelerations provide transient response. The purpose is to investigate driveline shuffle and clonk (backlash). Case accelerometers should indicate high frequency transients from gear backlash.

Gear Shifting: Gear shifts are performed for various throttle settings. Shaft torque and case accelerations provide transient response. The purpose is to investigate transient torque from gear shifts and associated driveline shuffle as well as oscillations at higher modes. This allows a validity check for gearshift numerical simulations. Case accelerometers should indicate high frequency transients from any gear backlash.

4. CONCLUSIONS

The finite element method is a powerful tool for torsional vibration analysis, particularly for powertrain systems. Once an understanding of the dynamic system is gained and a lumped mass model devised then the general finite elements (1)-(5) can be assigned. In some situations custom elements can be developed to handle added system complexities, such as for single or multi-stage planetary gear sets and toroid-roller contact (6)-(9). Using a global coordinate vector, the finite elements for inertia, stiffness and damping and their corresponding local coordinate vectors can be assembled into the standard equations of motion for the global system (10). For systems that change state often, such as transmissions with clutch shifting, global assemblies can be quickly made that govern each state.
Once the global system has been assembled the equations of motion can be used for the typical investigations:

Free vibration analysis, with the torque vector set to zero, and the wheels either grounded or linked to the vehicle mass. Gear ratios are fixed or in the case of the system with the gear set element the clutch connections and held gear set components fix the gear ratio

Forced vibration analysis, analytical or numerical; analytical for fixed gear states and input torques that can be handled analytically, such as harmonic or stepped, numerical for the parametric condition of gear ratio change, for input torques from mapped data – such as engine torque and other non-linearities such as stick-slip, clutch judder and gear backlash.

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NOTATION

k_n component stiffness	K_n stiffness finite element
c_n component damping	C_n damping finite element
J_n lumped inertia	I_n inertia finite element
n_G gear ratio	$\theta_{e(n)}$ local coordinate vector
r radius	