DETERMINING INDIVIDUAL MEMBER STIFFNESS OF BRIDGE STRUCTURES USING A SIMPLE DYNAMIC PROCEDURE

Jianchun Li, Bijan Samali and Keith Crews

Centre for Built Infrastructure Research, Faculty of Engineering,
University of Technology, Sydney, NSW, Australia

Abstract. A reliable determination of the structural condition of timber bridges presently requires costly load testing. A new dynamic based testing method was developed by authors to reduce the cost and shorten the testing time. The method has been successfully used to undertake field-testing of more than 40 timber bridges across NSW. The dynamic testing procedure involves the attachment of accelerometers underneath the bridge girders. The bridge girders are then excited by a modal hammer. The method requires tests with and without extra mass, so that the overall flexural stiffness of the bridge can be obtained. However, in order to accurately estimate the load carrying capacity of the bridge, it is necessary to obtain the stiffness values of individual members from test results without complicating the current testing procedure. In this paper, the authors review the dynamic testing procedure and propose a method to determine individual member stiffness for a bridge structure based on the field dynamic testing data. The outcomes of this work not only enable more accurate prediction of the load carrying capacity of the bridge but will also identify defective members of the bridge structure.

1. INTRODUCTION

Local Government in Australia is responsible for the operational management and maintenance of over 20,000 bridges. More than 70% of these bridges comprise aging timber bridges, the load capacity and structural adequacy of many of which have been impaired over time. A major challenge facing Local Government nationally is to develop effective strategies for the maintenance and rehabilitation of the extensive timber bridge stocks which form a key component of the road network under its control. Raising the efficiency and reliability of bridge maintenance practices of local government has the potential not only to minimise costly unscheduled emergency repairs, but also to reduce the overall maintenance costs, whilst improving the operational effectiveness of its road network.

The field testing of over 40 timber bridges in NSW has been undertaken and forms part of the second phase of an earlier project sponsored by the Institution of Public Works Engineering Australia (IPWEA) in 1999. As part of that project, a new testing regime, based on dynamic measurements, was developed and a thorough pilot study on the single span Cattai bridge in Baulkham Hills Shire was undertaken to demonstrate the potential of the proposed procedure [1,3]. The second phase had as its principal goal the further development and implementation of the procedure and enabling equipment for the cost-effective determination of the load deformation characteristics and load carrying capacity of a wide variety of short-span bridges[2]. Coupled with specially developed analysis software, this provides a measure of the structural adequacy of the bridge and a reliable basis for devising appropriate maintenance or remedial measures.

In this paper, this new dynamic testing approach will be reviewed and a method based on modal analysis will be proposed to determine the stiffness of individual bridge members, which will enhance the dynamic testing approach.

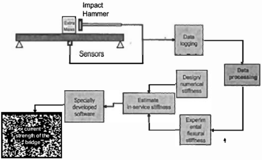

Figure 1 Schematic diagram of the proposed dynamic testing/analysis procedure for bridge assessment.

2. REVIEW OF THE NEW APPROACH TO THE MANAGEMENT OF BRIDGE ASSETS

Procedure

The new dynamic bridge assessment procedure involves the attachment of accelerometers underneath the bridge girders and the measurement of the vibration response of the bridge superstructure unloaded and with one or more loads (such as a truck, water tanker, grader, concrete blocks, etc, of known mass) applied at midspan. The excitation is usually generated by a modal impact hammer. The resulting dynamic responses are measured with low frequency and high sensitivity accelerometers, which are robust and simple to install. The data is logged and the bridge deck properties evaluated, using dynamic signal analyses on a standard computer with special software. Two sets of frequencies are measured for the bridge, 'as is', and when loaded by the extra mass. From the resulting frequency shift due to added mass, flexural stiffness of the bridge can be calculated. Figure 1 summarizes, schematically, the testing-analysis-assessment procedures which comprise the new dynamic method of bridge assessment. Effective field procedures have been developed to minimise costs of testing and disruptions to traffic. These procedures utilise instrumentation comprising readily available off-the-shelf items as well as in-house developed software. The test does not require the precise measurement of deformations as is the case for static load tests.

Analytical models

For a structure which can be modeled as a beam, closed form solutions, describing the transverse vibration of flexure beams, were developed. The governing equation of motion for simple beams under free vibration is

$$EI\frac{\partial^4 v}{\partial x^4} + \overline{m}\frac{\partial^2 v}{\partial t^2} = 0 \tag{1}$$

By adding mass at mid-span of a simple beam, the first natural frequency of a simple beam can be expressed as [1]:

$$\omega_2 = \left[\frac{48EI}{\alpha L^3(\Delta M + \beta M)}\right]^{1/2} \tag{2}$$

where M is self mass of the beam and ΔM is the added mass. In the above equation, α and β are constraint factors owing to different boundary conditions and modal mass coefficients, respectively.

Stiffness Prediction by Adding Mass

When a structure is considered as a dynamic system, it is possible to calculate the stiffness of the structure through its natural frequency changes. This method involves two identical dynamic tests but with different modal masses. First, one conducts a simple dynamic test on the structure 'as-is' and then conducts the same dynamic test with a lumped mass added at the appropriate location to directly increase the structural modal mass by this added lumped mass. Under a Single Degree of Freedom (SDOF) assumption and from equation (2) the flexural stiffness of the structure can be expressed as:

$$k = \frac{48EI}{\alpha L^3} = \frac{\omega_1^2\omega_2^2}{\omega_1^2 - \omega_2^2}\Delta M \tag{3}$$

where ΔM is the additional mass and α is the constraint factor; ω_1 and ω_2 are natural frequency of the bridge before and after added mass.

From equation (3), the relationship between mass ratio (ratio of added mass to original mass) and frequency changes can also be obtained:

$$\frac{\Delta M}{\beta M} = \frac{\omega_1^2}{(\Delta\omega + \omega_1)^2} - 1 \tag{4}$$

by simplifying and rearranging equation (4), we have:

$$\xi = 1 - \frac{1}{\sqrt{1 + \frac{\mu}{\beta}}} \tag{5}$$

$$\text{where } \xi = \frac{\Delta\omega}{\omega_1} \quad \text{and} \quad \mu = \frac{\Delta M}{M} \tag{6}$$

Figure 2 shows the graphical representation of equation (5). For in-service boundary conditions the value of b lies between those for fully pinned and fully fixed cases.

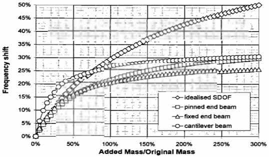

Figure 2 Frequency changes versus mass ratio

By rearranging Equation (5), one can obtain an explicit relationship between predicted stiffness and the natural frequency of the structure as well as the amount of mass added to the structure:

$$k = \omega^2\Delta M\left[\frac{1}{(2-\xi)\xi} - 1\right] \tag{7}$$

where frequency ratio is defined in Equation (6).

Strength Prediction of Timber Bridge Girders

Using a probabilistic approach, with a large database of timber properties from testing, a relationship was established and used in a reliability-based model to predict the load capacity of a deck from the stiffness data obtained from the new dynamic method, with acceptable and transparent degrees of uncertainty. However, since the new dynamic method only provides the global flexural stiffness of the bridge, in order to enhance the accuracy of prediction of bridge load carrying capacity, the determination of flexural stiffness of individual members is necessary.

3. DETERMINATION OF INDIVIDUAL MEMBER STIFFNESS

General Formulation

For a general linear time-invariant structural system, the equation of motion can be expressed as follows:

$$M\ddot{x} + C\dot{x} + Kx = Ef(t) \tag{8}$$

where **M** = n x n mass matrix; **C** = n x n damping matrix; **K** = n x n stiffness matrix; **E** = r x n location matrix;

f = excitation force; **x** = displacement vector. Equation (8) can be expressed in state space form as:

$$\dot{z}(t) = Az(t) + Hf(t) \tag{9}$$

where $z(t)$ is a 2n state vector; A is a $(2nx2n)$ system matrix; B is a (nxr) location matrix; and H is a $2n$ excitation matrix as follows:

$$z(t)=\begin{bmatrix} X(t) \\ \dot{X}(t) \end{bmatrix}; \qquad H=\begin{bmatrix} 0 \\ M^{-1}E \end{bmatrix} \tag{10}$$

$$A=\begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \tag{11}$$

In the meantime, if the given modal parameters (ie, frequency, damping and mode shapes) of the system are known, system matrix A can be reconstructed as [4] :

$$\hat{A}=\begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} \\ \hat{A}_{21} & \hat{A}_{22} \end{bmatrix} \tag{12}$$

Comparing matrix $\hat{A}$ to matrix A of equation (11), it is obvious that :

$$\hat{A}_{21} = -M^{-1}K \tag{13}$$

When additional mass (ΔM) is added to the structure, repeating the procedure above, results in equation (14):

$$\hat{A}^{*}_{21} = -(M+\Delta M)^{-1}K \tag{14}$$

The asterisk indicates that matrix $\hat{A}^{*}_{21}$ has been reconstructed from modal parameters with added mass.

With Equations (13) and (14), mass matrix can be eliminated and stiffness matrix K can be obtained:

$$K=\Delta M(\hat{A}^{-1}_{21} - \hat{A}^{*-1}_{21})^{-1} \tag{15}$$

where ΔM is the added mass matrix $\hat{A}_{21}$ and $\hat{A}^{*}_{21}$, are sub-matrices of reconstructed system matrices without and with added mass, respectively.

Bridge Applications

Considering that superstructure of bridges consists of n girders, especially timber bridges, the main structural elements which carry loads are girders. Depending on the design/construction, generally speaking the transverse / longitudinal planks contribute much less to the flexural stiffness of the bridge. For a given bridge with n girders, when flexural stiffness is the main concern, the structural system can be simplified as a n DOF spring mass system (Fig. 3).

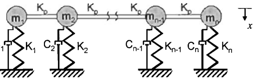

Figure 3 A bridge simplified as a n DOF spring mass system

In the model above, Ki (i=1, 2,...n) represents the flexural stiffness of girder i; Ci (i=1, 2,...n) represents the flexural damping of girder i; and K_p represents the flexural stiffness of planks (combining transverse / longitudinal). The governing equation of motion is again :

$$M\ddot{x} + C\dot{x} + Kx = Ef(t) \tag{8}$$

where

$$M=\begin{bmatrix} m_1 & 0 & \ldots & 0 & 0 \\ 0 & m_2 & \ldots & . & . \\ . & . & \ldots & . & . \\ . & . & \ldots & m_{n-1} & 0 \\ 0 & 0. & \ldots & 0 & m_n \end{bmatrix} \quad \text{and}$$

$$K=\begin{bmatrix} k_1+k_p & -k_p & \ldots & 0 & 0 \\ -k_p & k_2+2k_p & \ldots & . & . \\ . & . & \ldots & . & . \\ . & . & \ldots & k_{n-2}+2k_p & -k_p \\ 0 & 0 & \ldots & -k_p & k_n+k_p \end{bmatrix} \tag{16}$$

It is obvious that if the stiffness matrix K is reconstructed from modal parameters, with Equation (15), the girder and deck stiffnesses can be obtained.

Case study

To demonstrate the proposed methodology in obtaining individual stiffnesses, first span of a two span bridge from Cabonne Council in NSW was chosen. The chosen bridge has been field tested in the second phase of the project and is a four girder bridge in newly constructed condition (See Figures 4).

Figure 4 A two span bridge from Cabonne Council in NSW

The modal parameters of the bridge with and without added mass are given in Tables 1 and 2. Figures 5(a) to 5(d) show the mode shapes of the bridge at midspan with and without added mass.

Using the modal parameters and applying equation (15), the stiffness matrix K can be obtained (equation 17). Comparing equation 17 with equation (16), the flexural

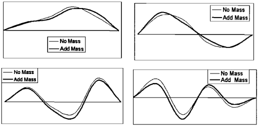

Figure 5 comparison of mode shapes with and without extra mass

Table 1. Frequencies of the bridge with/without extra mass

	Frequency (Hz)			
	mode 1	mode 2	mode 3	mode 4
no mass	7.451	8.176	8.850	9.218
add mass	6.005	6.451	6.943	7.439

Table 2. Mode shapes of the bridge with/without extra mass

no mass				add mass			
mode 1	mode 2	mode 3	mode 4	mode 1	mode 2	mode 3	mode 4
0.514	-1.521	0.615	-1.872	0.444	-1.723	0.626	-1.952
0.946	-1.165	-0.199	1.786	0.624	-0.903	-0.325	3.214
1.668	0.609	-0.464	-1.085	1.203	0.331	-0.703	-1.82
1.	1.	1.	1.	1.	1.	1.	1.

stiffness of girder and deck of the bridge are obtained. The flexural stiffness of girders 1 to 4 are 3665, 5264, 4323, 3513, kN/m respectively and deck flexural stiffness is 600kN/m.

$$K=\begin{bmatrix} 4265 & -600 & 0 & 0 \\ -600 & 5864 & -600 & 0 \\ 0 & -600 & 4923 & -600 \\ 0 & 0 & -600 & 4113 \end{bmatrix} \tag{17}$$

4. CONCLUSIONS AND FUTURE WORKS

A new method, based on dynamic response of timber bridges to an impact load has been proposed to measure the in-service flexural stiffness of timber bridges. Utilising a statistically based analysis, the knowledge of flexural stiffness can be converted into an estimate of the load carrying capacity of the bridge. The reliability and simplicity of the proposed methodology has been demonstrated by testing 40 bridges covering a wide range of single and multi-span timber bridges.

To further refine the method and enhance the accuracy of prediction of load carrying capacity of bridges, a new method is proposed to determine the member stiffness of bridges without complicating the testing procedure. Through modelling, the results of a case study involving a two span bridge demonstrated the potential of the proposed method. The further verification of the proposed method is planned to carried out one different timber bridges. However, field noise and signal processing are likely to be challenging when the method is applied to field testing.

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