SAXOPHONE ACOUSTICS: INTRODUCING A COMPENDIUM OF IMPEDANCE AND SOUND SPECTRA

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We introduce a web-based database that gives details of the acoustics of soprano and tenor saxophones for all standard fingerings and some others. It has impedance spectra measured at the mouthpiece and sound files for each standard fingering. We use these experimental impedance spectra to explain some features of saxophone acoustics, including the linear effects of the bell, mouthpiece, reed, register keys and tone holes. We also contrast measurements of flute, clarinet and saxophone, to give practical examples of the different behaviour of waveguides with open-open cylindrical, closed-open cylindrical and closed-open conical geometries respectively.

INTRODUCTION

Saxophones are made in many sizes. All have bores that are largely conical, with a small, flaring bell at the large end and a single reed mouthpiece, fitted to the truncation, that replaces the apex of the cone. The soprano (length 710 mm, including the mouthpiece) and smaller saxophones are usually straight. Larger instruments (the tenor has a length of 1490 mm) are usually bent to bring the keys more comfortably in the reach of the hands (Fig 1). The half angles of the cones are 1.74° and 1.52° for the soprano and tenor respectively. These are much larger than the angles of the orchestral woodwinds: the oboe and bassoon have half angles of 0.71° and 0.41° respectively, while the flute and clarinet are largely cylindrical. The relatively large angle of the cone gives saxophones a large output diameter: even the soprano saxophone has a considerably larger end diameter than oboe, bassoon and clarinet. Because of the geometry and other reasons [1], the saxophone is noticeably louder than these instruments, which was one objective of its inventor, Adolphe Sax.

Figure 1. The soprano (bottom) and tenor (top) saxophones used here, shown with a metre rule.

The playing of reed instruments produces coupled oscillations involving the reed and standing waves in the bore of the instrument [e.g. 2–5], and sometimes also in the vocal tract of the player [6].

Many of the important acoustical properties of the instrument's bore can be described by its acoustic impedance spectrum, Z, measured at the embouchure or input of the instrument. For each note, there is at least one configuration of closed and open tone holes, called a fingering, and the impedance spectrum for each fingering is unique. Impedance spectra for a small number of fingerings on the saxophone have been reported previously [7, 8], but measurement technology has improved considerably since then [9, 10]. This paper reports an online database comprising, for each standard fingering on both a soprano and a tenor saxophone, an impedance spectrum, a sound file of the note produced and the spectrum of that sound. It also includes such data for a number of other fingerings. It thus extends our earlier online databases for the flute [11] and clarinet [12]. Beyond its acoustical interest, this saxophone database will be of interest to players and teachers. Another application is intended for the future: The analogous database for the flute was used in the development of 'The Virtual Flute', an automated web service that provides fingering advice to flutists for advanced techniques [13, 14]. A similar service for saxophone could use these data.

We use our experimental measurements to show separately the effects of the bell, mouthpiece, reed, tone holes and register keys. We also include a number of comparisons to illustrate the different behaviours of cylindrical and conical waveguides.

MATERIALS AND METHODS

The saxophones were a Yamaha Custom EX Soprano Saxophone and Yamaha Custom EX Tenor Saxophone: both high-grade models from a leading manufacturer.

The impedance spectra were measured using a technique described previously [10, 12]: see [10] for a review of measurement techniques. Briefly, it uses two non-resonant calibrations and three microphones spaced at 10, 50 and 250 mm from the reference plane (Fig 2). The smallest microphone separation is a half wavelength and therefore measurements are unresolved around 4.3 kHz, which is well above the cut-off frequency of both instruments. Measurements were made between 80 Hz and 4 kHz, which includes the range of fundamental frequencies of both instruments.

The impedance head has a diameter of 7.8 mm, whose cross sectional area is smaller than the internal bore of the mouthpiece, but larger than the average opening between the reed and mouthpiece, through which air flows into the instrument. Using a rigid seal, it was fitted to the tip of the mouthpiece, with the reed removed (Fig 2).

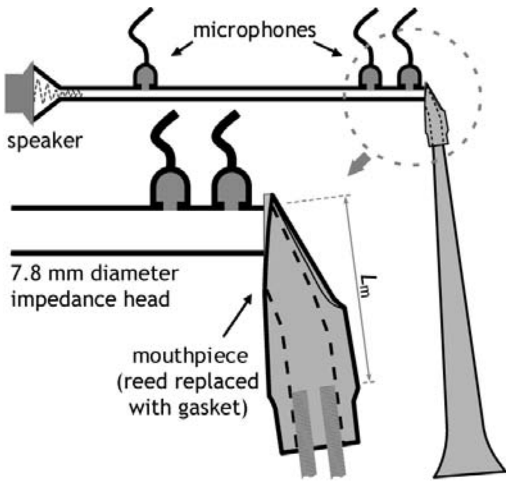

Figure 2. Schematic for measuring saxophone acoustic input impedance using three microphones. Not to scale.

Three different cones with the same half-angle as the soprano saxophone were made. The first was cast in epoxy resin with a length chosen to produce an impedance peak at the nominal frequency of C5 (523 Hz). The second cone was made (from Lexan) to replace the mouthpiece with an appropriate conical section. To allow input impedance measurements, both of these cones were truncated at 7.8 mm (the diameter of the impedance head) and extended with a cylindrical volume equivalent to that of the missing truncated section of cone. The third conical section was made (using a moulding compound) to fit inside the bell and to extend the conical bore over the full length of the instrument.

Sound files and the sound spectra files are not shown here but may be found at www.phys.unsw.edu.au/music/saxophone. They were recorded in the laboratory using two microphones, one placed about 30 cm directly in front of the bell, and the other one metre away. The player is a distinguished soloist who performs principally in jazz and contemporary concert styles. Sound spectra vary with distance from the instrument and direction, so these cannot be considered representative of all possible recordings.

RESULTS AND DISCUSSION

Fig 3 shows the impedance spectra measured on a soprano and a tenor saxophone for their lowest notes, sounding G#2 (tenor) and G#3 (soprano), both of which are written A#3 for these transposing instruments. In both cases, the first maximum determines the played pitch and the next several maxima closely match its harmonics. However, the first maximum is weaker than subsequent maxima, which is not the case for the clarinet [12]. This has the effect of making the lowest notes difficult to play softly, particularly on the tenor.

This weak first maximum of a cone is predicted by explicit models, but can be explained qualitatively if we use the geometric mean of the impedance of two adjacent extrema as an estimate of an effective characteristic impedance $Z_{0\mathrm{eff}}$ for a frequency between them. The characteristic impedance Z_0 associated with the bore cross-section decreases with distance down the bore. At sufficiently low frequencies, the air in the narrow section near the mouthpiece requires only small pressures to accelerate it, so $Z_{0\mathrm{eff}}$ is closer to the Z_0 of the larger bore downstream. The effect is stronger for the tenor saxophone, because its lowest resonances fall at lower frequencies. (Later we show that, for a first maximum at sufficiently high frequency, as in Fig 6 and 7, the first maximum is not much weakened.) The weak extrema at high frequencies are (in part) the results of increased viscothermal losses near the walls and increased radiation at the bell.

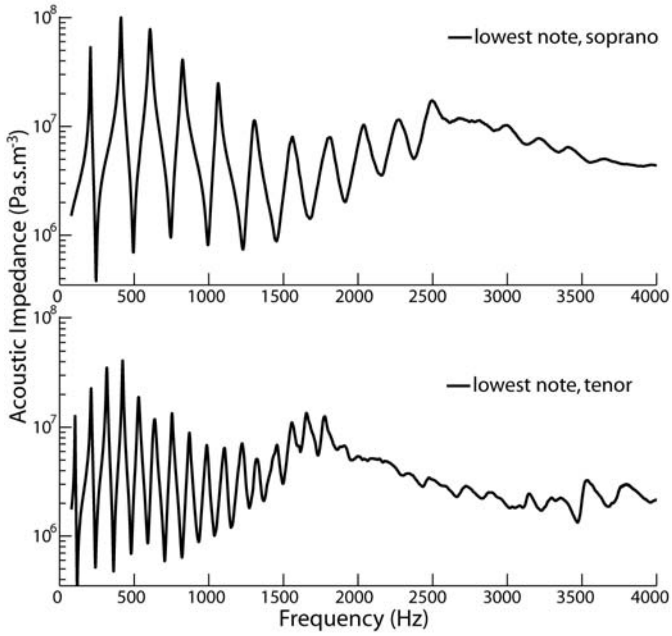

Figure 3. The measured impedance spectra of a soprano and a tenor saxophone for their lowest notes (G#3 and G#2 respectively, both written A#3 on these transposing instruments).

For the lowest note of the soprano, the maxima in Z occur at frequencies 207, 411, 606, 824, 1063, 1305, 1561, 1801, 2038 Hz, all ± 1 Hz. These are a good approximation to a (complete) harmonic series, f_1, $2f_1$, $3f_1$ etc. This contrasts with the case for the clarinet with cylindrical bore, whose impedance maxima occur at the odd members of the harmonic series, with the lowest having a wavelength about $4L$ and a frequency $c/4L$, where c is the speed of sound and L the length of the bore (~660 mm for the clarinet). Consequently, in spite of being 50 mm shorter than the soprano saxophone, the clarinet has a much lower lowest note (sounding D3 compared with G#3). The impedance spectra of the flute (also largely cylindrical, bore length of ~620 mm) are somewhat like that of the clarinet, but for the flute the minima rather than the maxima determine the playing regime, and so its lowest note is C4 – almost an octave higher than that of the clarinet [12]. We compare these three instruments below.

A completely conical bore of length L' (where L' includes the end correction of about 0.6 times the exit radius) would theoretically have maxima in Z at frequencies of about $c/2L'$ and all integral multiples of this. The frequency of the first maximum for the soprano saxophone, $f_1 = 207$ Hz, agrees well with this prediction. The cone of the saxophone, of course, is incomplete: if it were continued to a point at the mouthpiece, there would be no cross-section for air movement or place for a reed. The cone is truncated at a diameter of 9.2 mm, and the missing cone of length ~150 mm is replaced by a mouthpiece with a volume of 2.25 cm^3. When this is added to the effective volume due to the compliance of a reed (between 1.2 and 1.9 cm^3, discussed later), it is comparable with that of the missing cone (3.35 cm^3). This replacement has the affect of achieving resonances that fall approximately in the harmonic series expected for the complete cone [15, 16]. (The impedance peaks of a simple truncated cone are more widely spaced and not harmonically related.)

Effect of the bell

The loss of structure in Z above about 2.6 kHz for the soprano and above about 1.8 kHz for the tenor is due, in part, to the bell (Fig 3), which enhances radiation at high frequencies; the greater radiation means less reflection and therefore weaker standing waves. Fig 4 demonstrates this by plotting Z for the lowest note on the soprano with the bell replaced by a conical section of equal length and the same half-angle as the saxophone bore. The effective length with the cone is slightly greater, so the maxima appear at lower frequencies.

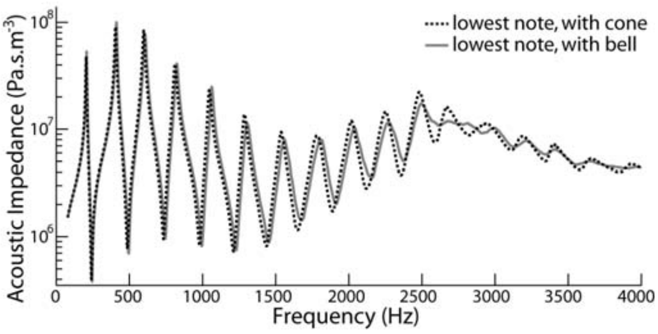

Figure 4. Measured impedance spectrum for G#3 (written A#3) on the soprano saxophone, its lowest note. The solid curve is for the normal saxophone (including mouthpiece and the compliance of the reed). In the dotted curve, the saxophone bell is replaced with a conical section of equal length and the same half-angle as the saxophone bore.

Mouthpiece and reed

The mouthpiece of the soprano has a volume of 2.25 cm^3 and an internal length, L_m, (see Fig 2) of 44 mm. At wavelengths very much longer than its length, it approximates a local compliance, in parallel with the rest of the bore. At low frequencies, the effective characteristic impedance of the bore is low, so the compliance will have only a modest effect, but will lower the frequencies of the maxima. At higher frequencies, it may lower the parallel impedance and, at still high frequencies, it is no longer appropriate to treat it as a simple compliance. The effect of the mouthpiece is shown in Fig 5. Neither the truncation nor the mouthpiece size scales exactly with the size of the instrument. Consequently this effect, which lowers the impedance in the range 1 – 2.5 kHz for soprano and 0.5 – 1.5 kHz for tenor, is not exactly scaled with the octave difference between the instruments (Fig 3).

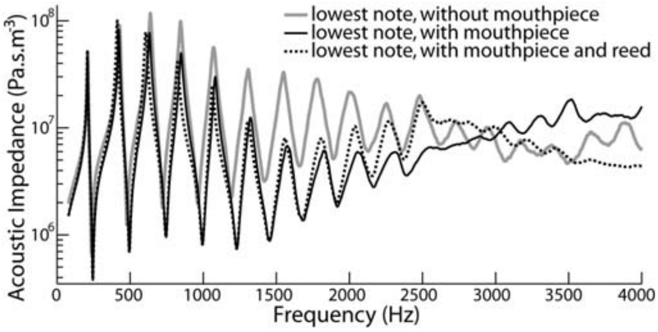

Figure 5. Effect of the saxophone mouthpiece and reed on the acoustic impedance of a soprano saxophone, shown for its lowest note. The dark curve is the measured impedance of a soprano saxophone with the mouthpiece attached, while the pale curve is measured with the mouthpiece replaced with a cone and cylinder. The dotted curve shows the effect of the reed on the acoustic impedance of a normal saxophone (with mouthpiece).

The reed has a mechanical compliance and may, to first order, be replaced by an acoustic compliance in parallel with the input. Its effect is also shown in Fig 5. Reed makers and saxophonists grade the reed according to hardness (a harder reed has a smaller compliance). The reed compliance here is for a number 3 reed. All else being equal, softer (more compliant) reeds lower the frequencies of the peaks in Z, and so play flatter. Of course, all else is not equal: the player may reduce the mouthpiece volume by sliding it further onto the instrument, or may increase the effective hardness of the reed by pushing it harder against the mouthpiece and reducing its effective length. S/he may also change the configuration of the vocal tract.

Bore comparisons

Musicians are generally puzzled by the fact that the (approximately) conical winds behave so differently from the (approximately) cylindrical clarinet. All have a reed at one end, which is therefore a region of large pressure variation, while the bell is open and approximates a pressure node. Yet the clarinet plays about an octave lower than a cone of the same length, its first two maxima in Z have a ratio of three instead of two, and its low notes have predominantly the odd harmonics, whereas the conical instruments have maxima in Z in the ratios 1:2:3 etc and all harmonics are present, even on low notes. Approximating the clarinet as a cylinder of length L with a pressure antinode in the mouthpiece ($x = 0$) and a pressure node at the bell ($x = L$), it is obvious that a pressure amplitude $p(x)$ of $\cos\{2(2n-1)\pi(x/4L)\}$ satisfies the boundary conditions for integers n, and that it gives a lowest note with a wavelength of approximately $4L$.

To acousticians, the explanation is simple: for the waveguide with constant cross section, solutions to the one dimensional wave equation describe propagating modes, and these are readily written in terms of sine and cosine functions. For a waveguide whose cross section goes as r^2, where r is the distance from the apex, the solution is a sum of spherical harmonics. These include a pressure term proportional to $(1/r)\sin\{2n\pi(r/2L)\}$, which has an antinode at the origin and a node at L for all integers n. Comparisons of the relevant functions are given elsewhere [17], but in this study we are able to give explicit experimental comparisons using the impedance curves.

Fig 6 presents the measured impedance spectra of several bores, each of which has an effective length of about 325 mm. One is a cylinder of that length, one a truncated cone, with the truncation replaced by a cylinder of equal volume. The others are a clarinet with the fingering for the note C4 (written D4) and a flute and soprano saxophone with the fingering for C5 (written D5 for saxophone), the latter using an alternative or

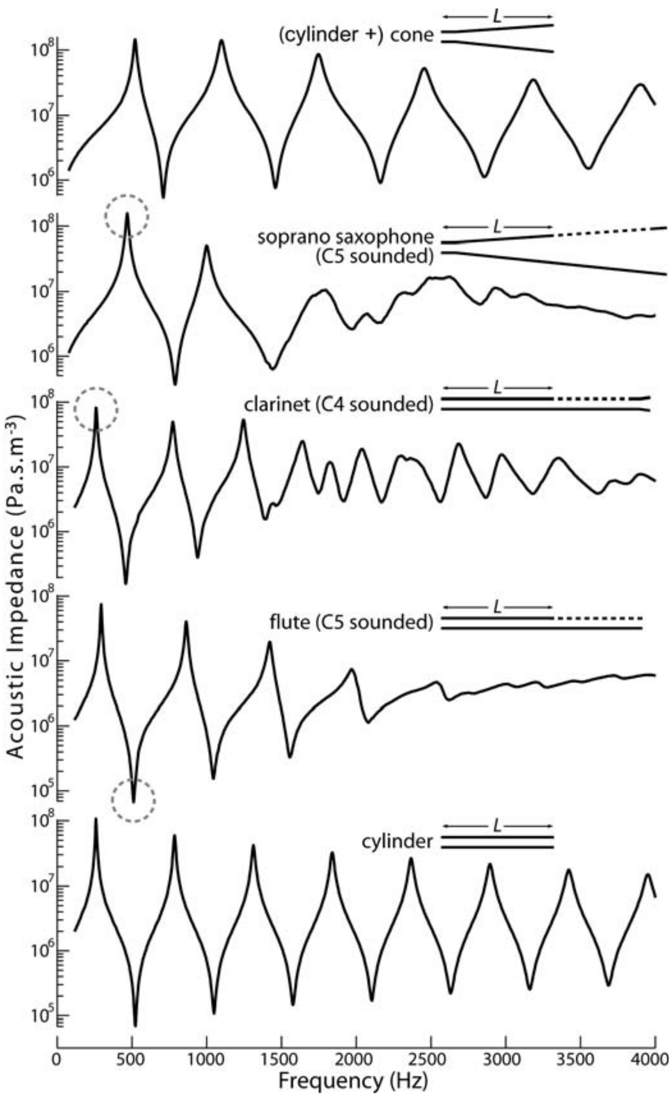

Figure 6. The acoustic impedance of (bottom to top) a simple cylinder, flute, clarinet, soprano saxophone and a truncated cone, all with an equivalent acoustic length. The circle indicates the maximum or minimum where each instrument operates. For the instruments, other notes are readily compared using the online databases reported here and in [11] and [12].

trill fingering that puts that note in the first register: i.e. the lowest frequency maximum in Z is used.

At low frequencies, the cylinder-cone combination has maxima at approximately $n(c/2L)$, which correspond to the minima for the cylinder. (At high frequencies, the replacement of the truncation of the cone becomes important for both the cylinder-cone and the saxophone.) So the saxophone and the flute can play C5 and C6 with these fingerings (although a saxophonist will usually use different fingerings for both notes). The maxima for the cylinder are at approximately $(2n-1)(c/4L)$, so the clarinet can play C4 and G5 with this fingering (although the player will usually use a register key for the latter, as explained below). The behaviour of the instruments at high frequencies illustrates several interesting effects, which we discuss below.

Cut-off frequency

At sufficiently high frequencies, the inertance of air in the tone holes also generates a non-negligible pressure difference between the bore and outside. An array of open tone holes and the short sections of bore that connect them thus resemble an acoustical transmission line comprising compliances and inertances. Above a cut-off frequency, tone holes can also effectively seal the bore from the air outside. Consequently, Z for many fingerings on a clarinet, flute and saxophone show, at sufficiently high frequency, a series of peaks spaced at frequencies corresponding roughly to standing waves in the whole length of the bore, irrespective of what tone holes are open, as shown, for example, in Figs 6 and 7.

The tone hole cut-off frequency, f_c, can be calculated for cylindrical waveguides using either the continuous transmission line approximation [2] or by assuming that the open tone holes approximate an infinite array [18]. Both give $f_c \sim 0.11\,(b/a)c(t_e s)^{-1/2}$, where a and b are bore and hole radii, s is half the separation, t_e is the effective tone hole length including end effects. No comparable expression currently exists for conical bores. Using this expression naïvely for the saxophone gives values of 1340 ± 240 Hz and 760 ± 250 Hz for soprano and tenor saxophone respectively, values that are similar to the frequencies at which there is a sudden change in the slope of the envelope of the sound spectrum [15 and the online database]. These values approximate the frequencies above which the broadly and regularly spaced maxima are replaced by the irregular and more narrowly spaced maxima (Figs 6 and 7, and others in the online database).

The periodic vibration of the air flowing past the reed (and also that of the reed itself) and that of the air jet exciting a flute, are both nonlinear processes, which therefore give rise to a spectrum with many harmonics [4, 5]. Only for notes in the low range of the instrument do several of these harmonics coincide with resonances in the bore. In that range, the spectrum of the clarinet contains predominantly odd harmonics while that of saxophone and flute have all harmonics. In the higher range, there is little systematic difference between even and odd harmonics [12].

Registers

For most standard fingerings in the first register, all tone holes are closed upstream of a point, which determines an effective length of the instrument for that fingering, and most of the tone holes downstream from that point are open. At low frequencies, these open holes approximate a short circuit between the bore and the air outside, effectively 'cutting-off' the bore at the first open tone hole, and producing the first two or more resonances seen in Fig 6.

A flutist can thus play C5 or C6 using the fingering whose impedance spectrum shown in Fig 6 by varying (chiefly) the speed of the jet of air. In clarinet and saxophone, the upper note corresponding to that fingering is selected using a register key.

To play in the second register of the saxophone, one of two register holes is opened. These are holes with small diameter (about 2 mm) and relatively long length (about 6 mm). At low frequencies, they allow air flow. At higher frequencies, the mass of the air in the hole can only oscillate substantially if there is a substantial pressure difference across it. Thus, at high frequencies, the air in a register key effectively seals it [2]. Consequently, when opened, a register hole weakens and changes the frequency of the first impedance peak, making it easier for the second peak to determine the playing frequency. Fig 7 shows the difference. This is how notes in the second register are produced in most reed instruments.

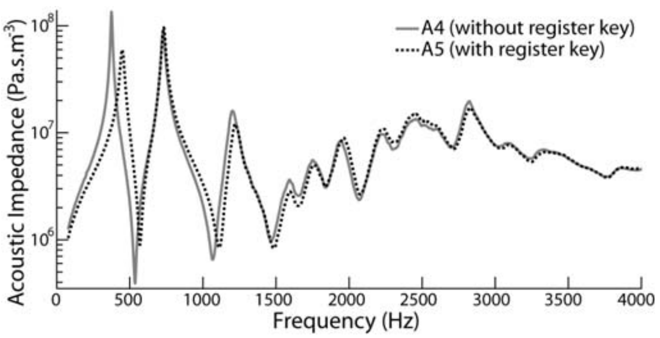

Figure 7. Effect of the register key, shown here for the fingerings for written A4 and A5 (sounding G4 and G5) on the soprano saxophone. The first impedance peak in the spectrum for the A4 fingering is weakened and detuned when the register key is engaged in the A5 fingering, so the reed now operates at the second peak. The high frequency impedance structure is less affected.

On the clarinet, with its largely cylindrical bore, notes in the first and second registers that use similar fingerings are separated by a frequency ratio of three (a musical twelfth) compared with a ratio of two (an octave) for the saxophone. The clarinet uses only one register hole to play the whole second register (which spans 13 semitones). Consequently, the position and dimensions of the register key appear to be less critical for the clarinet – so much so that it also uses this hole as a tone hole for the highest note in the first register.

In contrast, because its two registers are separated by only an octave, the saxophone requires two register holes to cover a second register (which spans 16 semitones). The register hole must be small enough so that it does not affect the second resonance too much. Therefore, to have a sufficiently large effect on the lower register, it must be located where the standing waves in that register have relatively high pressure. Consequently, the lowest seven notes are played using a register hole (2 mm diameter and 6 mm deep on the soprano), which is further from the mouthpiece than the register key used for higher notes. An automated octave key, operated by a system of mechanical logic, uses one key, for the left thumb, to open the appropriate register hole. Inspection of the *Z* plots for the highest notes using the lower register hole (G5 and G#5, sounding F5 and F#5: on the online database, not shown here) shows that it is rather less effective at reducing the first impedance peak than it is for lower notes.

The high range of the saxophone

For the saxophone, the combination of the cut-off frequency and the bore geometry strongly attenuate the magnitude and sharpness of peaks in *Z* above about 1.3 kHz for the soprano saxophone. The weak maxima in this range have important musical consequences. Traditionally, the range of the instrument comprises only two registers, playing notes corresponding to the first and second impedance peaks respectively. This range finishes (depending on the model) at written F6 or F#6 (for the soprano, this sounds D#6 or E6, about 1300 Hz; for the tenor, D#5 or E5, about 650 Hz). The higher resonances will not usually support notes on their own. However, with assistance of sufficiently large peaks in the acoustical impedance of the player's vocal tract, tuned to the appropriate frequency, expert players do achieve notes above the traditional range, in what is called the altissimo range [6].

Saxophone acoustics

Saxophone acoustics provides a range of physically interesting phenomena and musically interesting details [19], including subharmonics and multiphonics, which involve superpositions of standing waves, cross fingerings, and the relations between sound spectra, sound recordings and impedance spectra. These are best explored on-line. The compendium is located at www.phys.unsw.edu.au/music/saxophone.

ACKNOWLEDGEMENTS

We thank the Australian Research Council for support, Yamaha for the saxophones, and saxophonist Sandy Evans.

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This is the twelth in a series of regular items in the lead up to ICA in Sydney in August 2010.

The ICA 2010 will incorporate the AAS 2010 annual conference and so will provide an opportunity to showcase the work that is being undertaken in Australia on a wide range of acoustics and vibration topics. All AAS members are strongly encouraged to put ICA 2010 in their calendar for 23-27 August as well as recommending participation from their national and international colleagues .

If anyone knows of an opportunity for promotion of ICA 2010 either using electronic distribution or for posters and flyers etc at a conference or meeting please contact Marion Burgess (m.burgess@adfa.edu.au) and it can be followed up.

Information on the conference can be found on the web page: **www.ica2010sydney.org**

Marion Burgess, Chair ICA 2010