CLARINET ACOUSTICS: INTRODUCING A COMPENDIUM OF IMPEDANCE AND SOUND SPECTRA

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This paper introduces a web-based database that contains details of the acoustics of the clarinet for all standard fingerings and some others. It includes the acoustic impedance spectra measured at the mouthpiece and sound spectra recorded for each note. The data may be used to explain a number of the playing characteristics of the instrument, both in general and in detail. In this paper we give an overview, and highlight some interesting phenomena. The clarinet has, very approximately, a cylindrical bore, which is acoustically closed at one end and open at the other. Because it is so often used as an example of closed-open pipe, we show several phenomena that can be clarified by comparing measurements on a clarinet with those on a cylinder of equivalent acoustic length. We also compare these data with analogous data for the flute, an example of an open-open pipe.

INTRODUCTION

The acoustic behaviour of wind instruments is largely determined by their acoustic impedance spectrum measured at the embouchure or 'input' to the instrument. The acoustic impedance Z is the ratio of acoustic pressure p to acoustic volume flow U and its extrema identify the frequencies of resonances and antiresonances due to standing waves in the bore. (For example see [1].)

Backus [2] reports measurements of the acoustic impedance of the clarinet for a small number of fingerings. Advances in measurement technology (reviewed by Dalmont [3] and Dickens *et al*. [4]) have allowed improvements since then. Further, there are advantages in having a rather complete set of data that includes all of the standard fingerings. Detailed information about individual fingerings and the notes they can produce is of obvious interest to players and teachers. It is then helpful to provide, for each fingering, a sound sample and sound spectrum. This paper introduces such a database, available via the internet.

A further important application of such a complete data set is possible. An analogous database [5,6] for the flute was used to develop a computer model for the instrument. This in turn led to a web service for flutists that "knows" many of the playing properties of all 40,000 fingerings for the instrument and uses these to advise musicians on unconventional fingerings for rapid passages, multiphonics, and timbral and pitch variations [7,8]. It is our intention to use the clarinet database reported here in a similar way. Finally, a number of personal communications concerning the flute database convince us that such data bases are of

use to researchers studying performance technique either from the acoustical or the musical perspective.

In this paper, we also report measurements of impedance spectra made on a flute, and on purely cylindrical pipes that have dimensions comparable with those of a clarinet or a flute. These are included for didactic reasons. The clarinet and flute are regularly used as iconic examples of a closed-open and an open-open pipe in acoustics and general physics texts (e.g. [9]) and comparison of
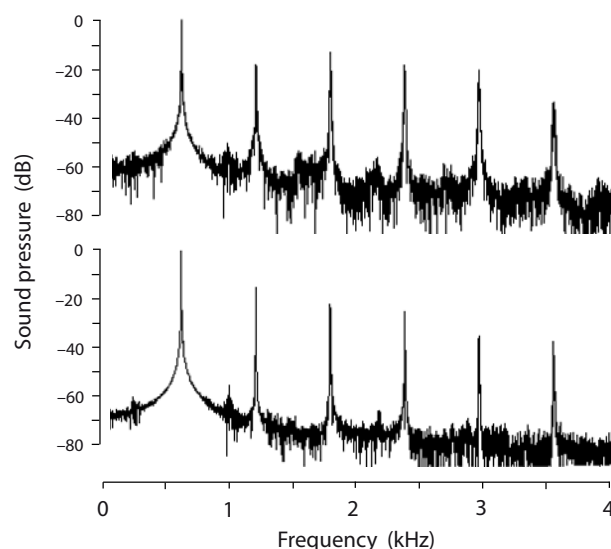

Figure 1. Sound pressure spectra for the note D5 played *mezzoforte* on a flute and on a clarinet, measured near the first open tone hole in each case. Identification of which instrument produced each spectrum is presented at the end of this paper. The 0 dB level was arbitrary and the same for both instruments.

measured acoustic properties can clarify a range of subtle acoustical effects.

For example, the two sound spectra in Fig. 1 are for the note D5 played on a flute and a clarinet under very similar conditions. But which is which? Imagine that, as an acoustician, you are trying to explain this to a musician: Do you find yourself looking for systematically weaker even harmonics?

The clarinet has been used as a model instrument for studying the bore-reed interaction (e.g. [10-15]) and so one might in principle appeal to such studies to begin to answer the question posed by Fig. 1. However, for reasons that we shall see, the spectral envelope of the clarinet varies over the range of the instrument, and even for different fingerings used for the same note. For a simple theory of sound production, or for simple experiments, it is reasonable to connect a clarinet mouthpiece to a simple cylinder with well known acoustical properties. Such a simplification does not, of course, satisfy the clarinettist, for whom the details of each fingering are important.

The database described here has been assembled to provide such details not only as a resource for the musician, but also for the next generation of acoustical models of reed-bore interaction. Further, and for the purposes of this much less detailed article, we select a number of examples so as to answer the general question posed by Fig. 1, and one that might be confronted in the physics or acoustics classroom: When and how can simple arguments about the geometry of these instruments be used to explain their spectra and timbre?

MATERIALS AND METHODS

The clarinet used for most measurements was a Yamaha Custom CX B flat clarinet. A Yamaha Custom CX A clarinet was also measured for some notes, to allow comparisons.

The acoustic impedance was measured using a spectrometer described previously [4]. It uses three microphones, three non-resonant calibration loads and a signal that comprises a sum of sine waves with amplitudes chosen to distribute the errors due to noise and frequency-dependent instrumental sensitivity approximately equally over the frequency range. The three microphones in the impedance heads were located 10, 50 and 250 mm from the reference plane (Fig. 2). With the microphones positioned thus, a singularity occurs at around 4.3 kHz under typical measurement conditions (see [4] for details). At this frequency the smallest microphone separation is equal to half a wavelength, and the impedance cannot be determined. Thus the impedance spectra are measured between 120 Hz and 4 kHz for the clarinet and between 200 Hz and 4 kHz for the flute. These ranges encompass the fundamental frequency of all of the notes on each instrument and include their cut off frequencies.
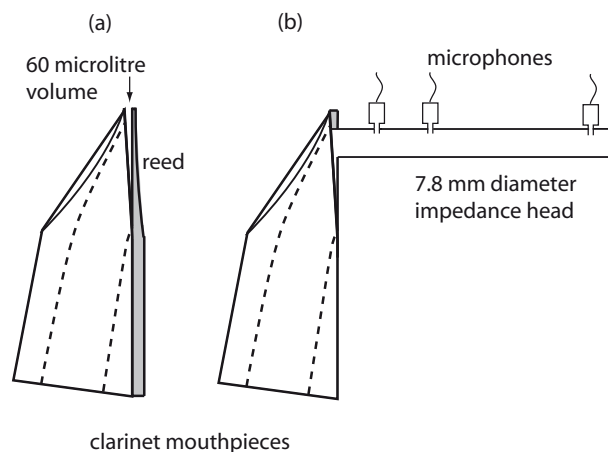

Figure 2. Schematic diagrams of the clarinet mouthpiece and impedance head (not to scale). An arrow in Fig (a) indicates a small volume of about 60 microlitres left between the undeformed reed and the mouthpiece. For measurements, the reed is removed and this volume is enclosed and sealed with a gasket against the impedance head when they are attached – see fig (b). The gasket (shaded in the diagram at right) and the ends of the mouthpiece and impedance head are enclosed in a block of Teflon (not shown) for measurements. The dotted lines show the shape of the bore. During playing, the player's lower lip is pressed against the reed, while the upper teeth and lips touch the slightly curved upper surface of the mouthpiece (top left in the diagram).

The impedance head used for clarinet measurements is shown in Fig. 2. Two geometrical areas are of interest in measurements of the impedance at the clarinet mouthpiece: one is a typical opening between reed and mouthpiece in the mouth and the other is the effective area of the reed upon which pressure variations produce substantial vibrations. To approximate these, a pipe with internal diameter 7.8 mm was used. The same impedance head was used for the few flute measurements reported here. The attachment used to attach the impedance head to the flute was similar to that used previously [5,16]. The flute was a Pearl PF-661 with closed keys and a C foot.

The embouchure apertures of flutes and clarinets are both smaller than their bores, so measurements were also made on cylinders with an internal diameter comparable to the bore. A separate impedance head, with internal diameter 15 mm, was used to measure the impedance spectra of sections of stiff plastic pipe with the same internal diameter. The lengths of such pipe sections were chosen to equal the lengths of the equivalent air column for flute and clarinet resonances, for cases discussed later. The impedance spectra plotted here and in the clarinet database include a compliance corresponding to that of the reed, using the value given by Nederveen [18]. Similarly, an inertance corresponding to the radiation impedance at the embouchure has been added to the impedance curves here and on the flute database [6].

The sound files and the sound spectra available in the database were measured in a recording studio using a condenser

microphone positioned one metre in front of the player, at shoulder height. The recording environment reflected typical practice for solo recording in a studio but allows for acoustical properties of the room to influence the spectra. In contrast, the sound files whose spectra are shown in this paper were recorded in a room treated to reduce reverberation, using a microphone positioned 2 cm from the first open hole (for the higher notes) or 2 cm from the bell (for the lowest notes on each instrument). At this distance, room effects and interference effects are negligible. They were played by one of the authors using the same flute and clarinet described above, under very similar conditions for the two instruments.

RESULTS AND DISCUSSION

Simple geometries and the interpretation of impedance spectra.

Before discussing the impedance spectra of the clarinet, with its complicated geometry and consequent complications, it is instructive to look at the impedance spectrum of a simple open pipe. That shown in Fig. 3 is for an open cylinder with length $L = 650$ mm and internal diameter $a = 15$ mm. These dimensions are comparable with those of a clarinet and a flute (the internal diameters of the cylindrical portions of a clarinet and a flute are 15 mm and 19 mm and the lengths of their bores are 660 mm and 615 mm respectively). The impedance spectrum of this cylindrical pipe should thus be helpful in understanding the behaviour of both clarinets and flutes. This pipe is open at the end remote from the measurement head. Should it be considered therefore an open-open or a closed-open pipe? The answer depends on the conditions that are imposed at the proximal end by the valve or jet mechanism used for excitation.

How would this pipe respond if excited by a clarinet reed? Discussions of the interaction of a reed with a pipe are given by a number of researchers, who agree that the system will resonate at frequencies close to impedance maxima of the pipe. (See [1,10-15,17,18].) Qualitatively, one may also say that the pipe is largely closed off by the reed, so the acoustic flow U would be small. The reed would be excited by large variations in the acoustic pressure p. Consequently, the frequencies of the expected playing regimes fall close to those of the maxima in $Z (\equiv p/U)$. Hence the observation that a clarinet behaves acoustically as a closed-open pipe, with one end almost sealed by the mouthpiece and reed.

The extrema in Fig. 3 fall at frequencies 130, 261, 392, 524, 655, 787, 919, 1052, 1183, 1315, 1449, 1579, 1711, 1844, 1975, 2107, 2238, 2371, 2502, 2636, 2766, 2900, 3031, 3165, 3296, 3429, 3559, 3692, 3824 and 3957 ± 1 Hz. If n is the number of the extremum, then the mean and standard deviation of f_n/n are 131.5 and 0.5 Hz.
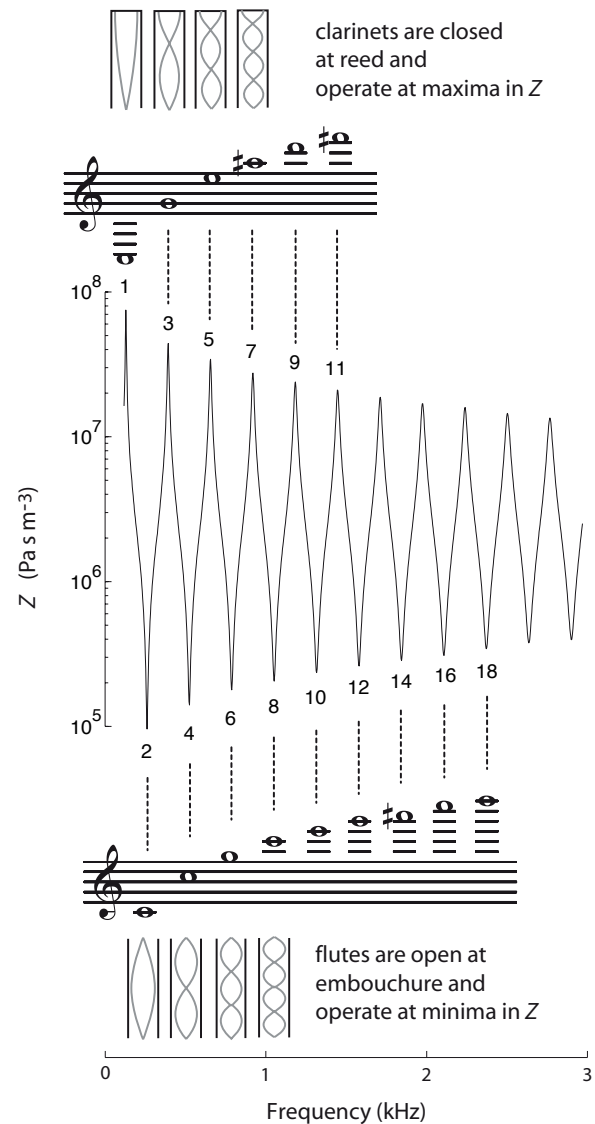

Figure 3. The magnitude of the measured acoustic impedance spectrum of a cylindrical pipe, length 650 mm and diameter 15 mm. The frequencies of several impedance maxima are notated on a musical stave above the spectrum, using the semi-log vertical axis widely used by musicians. The first several notes shown are approximately what would be played by a hypothetical clarinet with this bore, closed at the embouchure by a reed. Similarly, below several impedance minima, are shown the notes that would be played by a hypothetical (end-blown) flute with this bore, open at the embouchure and negligibly baffled by the player's face. The numbers above or below these extrema refer to the harmonics of the lowest note that would be played of the hypothetical cylindrical clarinet. Diagrams indicate the standing waves of pressure associated with some extrema.

The maxima in Fig. 3 thus fall almost exactly at frequencies corresponding to wavelengths of $4L/n$, where n is an odd integer. (Small differences are expected due to the radiation impedance at the open end, which has a small frequency dependence [1].) A clarinet with a purely cylindrical bore of these dimensions and with all tone holes

closed might then be expected to play approximately the pitches indicated in Fig. 3. These correspond to the note C3 (one octave below middle C, with nominal frequency f_1 = 131 Hz) and its odd harmonics with frequencies in the ratio 1:3:5 etc, i.e. frequencies nf_1 Hz, where n is an odd integer. Sketches representing standing waves are included on the figure.
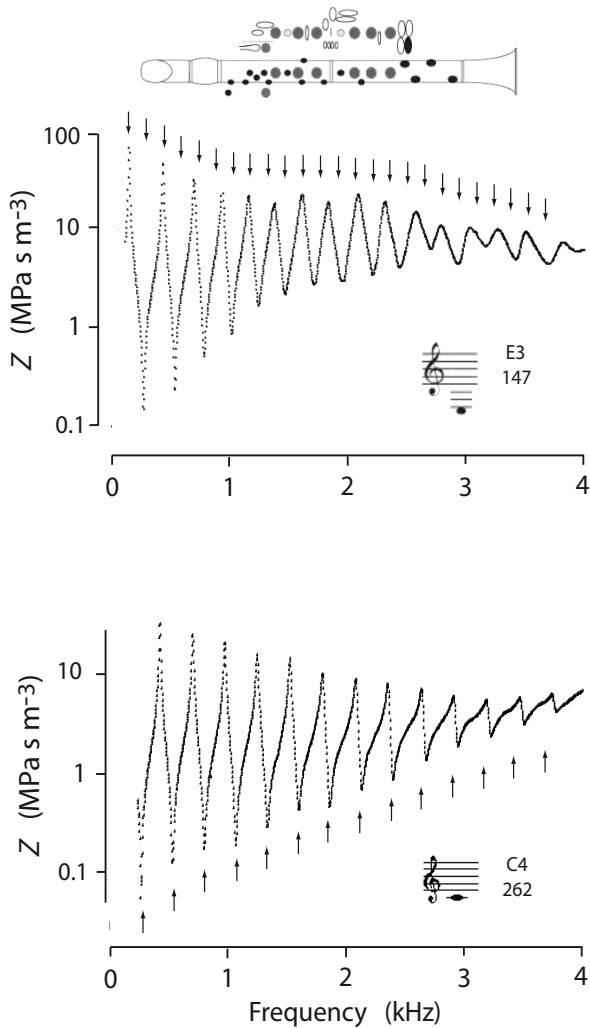

Figure 4. The magnitude of the measured impedance curves for the lowest notes on the clarinet (upper) and flute (lower). The lowest note on the clarinet is D3 (called E3 on the clarinet, a transposing instrument) and can be produced by closing all tone holes, as indicated on the schematic located above the spectrum. There are two other fingerings (not shown) that will also close all tone holes – alternatives are required because the clarinet overblows at a musical twelfth, and twelve tone holes exceeds the number of fingers available to standard players. The lowest note on the flute with a C foot is C4, which is produced by closing all tone holes (see schematic) and is nearly an octave above the lowest note on the clarinet. The vertical arrows indicate the harmonics of the note that would be played.

In contrast, the bore of a flute is open to the air at both ends so, with all tone holes closed, it is acoustically an open-open pipe. A model of the excitation of a pipe by an air jet is given by Fletcher and Rossing [1]. Here however we can make the following simple argument: Because the flute is excited by volume flow at the embouchure, which is open to the outside air, one might expect to find, near the embouchure, a minimum in p and a maximum in U. Consequently, a simple cylindrical pipe played as an end-blown flute is expected to play at frequencies near those of the impedance minima in the curve in Fig. 3. This idealised open-open pipe has resonances with frequencies in the ratio 1:2:3 etc, whose modes are shown in the sketches in Fig. 3. For the pipe shown, this is the note C4 (middle C, nominally 262 Hz) and its harmonics are both odd and even. Animations that represent standing waves in closed and open pipes in the time domain are shown on www.phys.unsw.edu.au/jw/flutes.v.clarinets.html

In practice, the lowest notes on a clarinet are either D3, for a Bb instrument, or C#3, for an A clarinet. (The clarinet is a transposing instrument, meaning that although the aforementioned notes are sounded, they are written as E3 for both instruments in the printed parts, and fingered the same on the two instruments, the A clarinet being just 6% longer and thus one semitone lower.) That the 660 mm B flat clarinet has a lowest note that is a tone higher than the first resonance of the 650 mm cylindrical tube may be explained by the fact that the clarinet is flared in the lower half and has a bell, both of which reduce the effective length. That the 615 mm C flute has a lowest note (C4) at the frequency of the 650 mm cylinder is explained by the constriction at one of its open ends (the embouchure) that lowers the frequency of modes that have a flow antinode at this end [19].

Impedance spectra of a clarinet.

The upper part of Fig. 4 shows the impedance spectrum for the lowest note on a B flat clarinet, measured at room temperature. The first maximum occurs at f_1 = 148 Hz, which is close to the sounding pitch of the lowest note on the clarinet (D3 has a nominal frequency of 147 Hz). The upper part of Fig. 5 shows sound spectra recorded close to the bell of the clarinet for this note.

Fig. 3 shows that, for the cylindrical pipe, the frequencies of the first several maxima occur at odd integral multiples of that of the lowest. In contrast, Fig. 4 shows that, while the second maximum occurs at a frequency only a little less than $3f_1$, the frequencies of the next several subsequent peaks occur at frequencies successively less than odd multiples of this frequency, as is indicated by arrows in the figure. The frequencies of the maxima shown here are 148, 435, 699, 938, 1159, 1380, 1612, 1837, 2088, 2318, 2576, 2795, 3039, 3269, 3513 and 3820 Hz. If n is the number of the maximum, then the mean and standard deviation of $f_n/(2_n-1)$ are 127.7 and 9.0 Hz, so they are rather more

closely spaced, on average, than f_1. The primary cause of this is the bell, which gives the instrument an effective length that increases with frequency: the effective point of reflection for waves travelling down the bore is more distant for higher frequencies.

Sometimes, a maximum will occur with a frequency close to an even harmonic of the lowest maximum, e.g., the fifth maximum coincides with $8f_1$. The effect of a small difference in frequency between an impedance peak and a harmonic is shown in the sound spectrum in Fig. 5: the first and third harmonics are very much stronger than the second and fourth. The fifth to seventh harmonics decrease regularly, however, while the eighth harmonic is stronger than its neighbours.
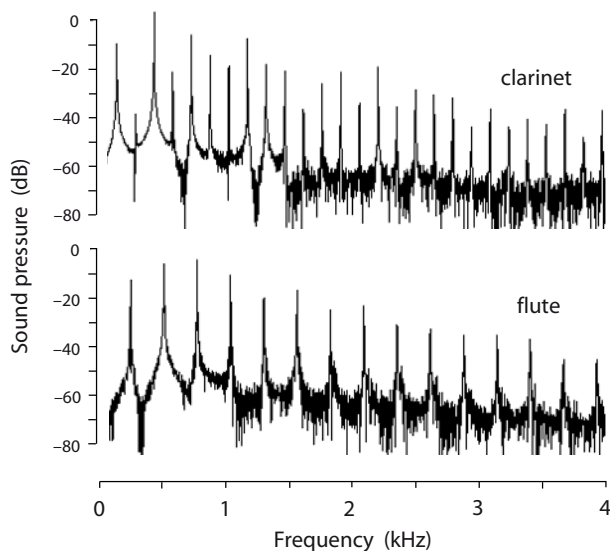

Figure 5. Sound pressure spectra for the notes played *mezzoforte* with all tone holes closed on a B flat clarinet and on a flute. See Fig. 4 for more details. Here, in contrast with Fig 1, the first two even harmonics of the clarinet sound spectrum are weaker than their neighbours. The 0 dB level was arbitrary and the same for both instruments.

Further clear differences between the clarinet impedance spectrum and that shown in Fig. 3 for a cylinder are also due to the clarinet's bell. The amplitude of the maxima and minima decrease with increasing frequency at a rate greater than that for the simple cylinder. One function of the bell is to radiate high frequencies (i.e. wavelengths that are not long compared with its dimensions). Increased radiation means less reflection at high frequencies, and so weaker standing waves or resonances. Second, the geometrical mean of the clarinet impedance increases overall with frequency. This is due to the shape of the mouthpiece: its cross-sectional area, which increases with distance from the embouchure and measuring point, can be considered as an impedance matcher that operates at sufficiently high frequencies. Impedance spectra may be calculated with

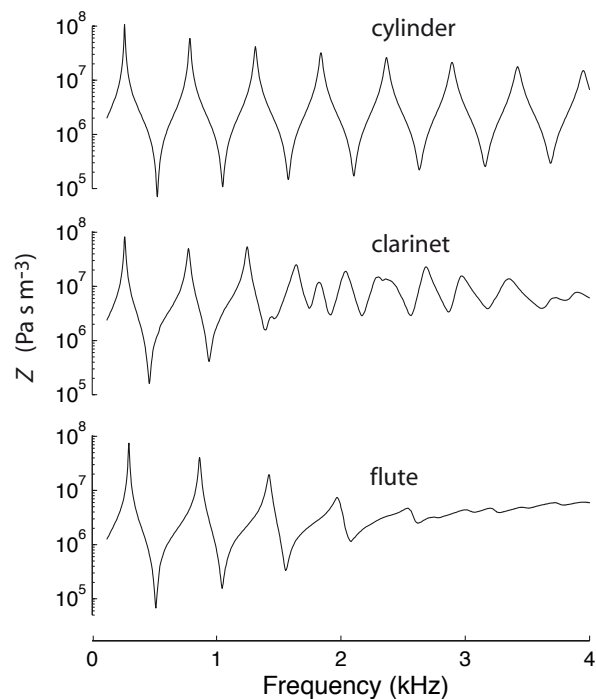a simple waveguide model, and the respective effects of mouthpiece and bell may be illustrated [20].

Figure 6. The magnitude of the measured impedance spectra of a cylinder (top), a B flat clarinet fingered to play the note C4 or 'middle C' and a flute fingered to play the note C5 (bottom), each with the same effective length. (C4 is called D4 on the B flat clarinet, a transposing instrument.)

Impedance spectra of a flute.

Figs. 4 and 5 also show, for comparison, the impedance spectrum and sound spectrum for the lowest note on a flute (C4, nominally 262 Hz). The first *minimum* in Z falls at 259 Hz, close to the frequency at which it plays. As discussed above, the open embouchure of the flute means that it plays at minima in impedance. The minima shown in this figure fall at the frequencies 259, 525, 790, 1059, 1326, 1590, 1856, 2124, 2390, 2671, 2938, 3217, 3500 and 3771 Hz. These are approximately integral multiples (both even and odd) of this frequency: if n is the number of the minimum, then the mean and standard deviation of f_n/n are 265.4 and 2.7 Hz. The flute will play at eight or more of these minima. Sound files illustrating the notes obtained by overblowing the lowest note fingerings of the flute and clarinet are at www.phys.unsw.edu.au/jw/flutes.v.clarinets.html. These files show that, while the first several flute resonances play notes in nearly harmonic ratios, only the first two clarinet notes are in harmonic ratios.

Comparison with the simple cylinder shows several differences. The decrease in the magnitude of the extrema with increasing frequency in this case has a different explanation. The bore is in series with the air in the

downpipe, which increases the impedance over the range shown. Further, the embouchure end of the flute includes a Helmholtz resonator, of which the mass is the air in the small downpipe or chimney into which the player blows, and the 'spring' is the volume of air between the chimney and the cork in one end. The combined effect attenuates the resonances over the high end of the frequency range shown. More detail on these and other effects is given elsewhere [5,21].

Comparing clarinet and flute with all holes closed

Qualitatively, the impedance spectra of both flute and clarinet are somewhat similar to those of the cylindrical pipe for their lowest note (i.e. with all holes closed). Further, the sound spectra of the lowest notes shown in Fig. 5 reflect this similarity: the first few odd harmonics of the clarinet sound are relatively strong, because they excite corresponding resonances in the bore, whereas the low frequency even harmonics do not coincide with resonances.In contrast, the flute's sound spectrum exhibits no systematic difference between odd and even harmonics. For all notes other than the lowest, however, there will be open tone holes and/ or register holes. These are responsible for some of the complications suggested by Fig. 1.

Comparing clarinet and flute with open holes

Fig. 6 shows three impedance spectra. The first is for a cylindrical pipe, 15 mm in diameter and 325 mm in length. Its length was chosen so that its first impedance maximum corresponds to the note C4 and the first minimum to C5. Simplistically, we should expect it to play these notes respectively if excited by a reed (making it an open-closed pipe) or an air jet (making it open-open).

Fig. 6 also shows the impedance spectrum of a clarinet with the fingering to play the note C4. In this configuration, most of the keys on the lower half of the instrument are open, so we could simplistically say that its effective length is the same as that of the cylindrical pipe. At low frequencies, this simplistic picture is adequate: the first three maxima correspond closely to those of the pipe and the instrument will play notes near these frequencies. The minima are displaced in frequency, because of the mouthpiece constriction mentioned earlier.

Above about 1.5 kHz, however, the behaviour is qualitatively different. This is due to the cut-off frequency of the array of open tone holes. At sufficiently high frequency, the force required to accelerate the mass of air in and near the open hole is sufficiently great that there is little radiation from the hole [22, 23]. The array of open tone holes and the short sections of bore connecting them thus behave like inertances and compliances in a finite element transmission line. Benade [23] derives a theoretical expression for the cut off frequency of a continuous waveguide approximating this situation. Wolfe and Smith [21] give an explicit derivation

for an infinite tone hole array. The cut off frequency for the clarinet is about 1.5 kHz. (Because the holes on a clarinet are neither uniformly sized nor uniformly spaced, the cut off frequency varies for different fingerings. For this fingering, using the average value of parameters for the next five tone holes, the calculated value is 1.5 kHz.)

Below 1.5 kHz, the average spacing between impedance maxima is about 500 Hz, as expected for a pipe with effective length 330 mm, i.e. a length roughly equal to the length of bore down to the first open tone hole. Above 1.5 kHz, however, the average spacing is about 280 Hz, corresponding to an effective length equal to that of the *whole* bore of the clarinet. At frequencies above the cut off, the sound waves 'do not notice' the open holes, the air masses in which have sufficiently large inertia that they effectively seal the bore.

Fig. 6 also shows the impedance spectrum for a flute fingered to play the note C5, one octave above its lowest note. For this fingering, most of the tone holes in the downstream half of the instrument are open. At low frequencies, it also behaves somewhat like the simple cylinder that has the same effective length. The frequencies of the three first minima have harmonic ratios, and each of these minima may be played as a note. Above about 3 kHz, very few features are seen, because of the Helmholtz resonator mentioned above. The cut off frequency for the flute is higher than that for the clarinet, because the holes are considerably larger. The Boehm mechanism of the flute also opens a larger proportion of the tone holes, reducing the average distance between open tone holes for this example. However, the effect of the cut off frequency can be seen on the flute at frequencies above that of the Helmholtz resonance, typically above 7 kHz [21]. In this high frequency range, the effective length of the flute is close to that of the complete instrument, in spite of the open tone holes.

Fig. 6 shows that, once a number of tone holes are open, treating the clarinet as a closed-open cylinder and the flute as an open-open one is an appropriate approximation only at low frequencies. This explains the difficulty of the question posed in Fig. 1: at frequencies above the cut off, the resonances do not match the frequencies of the harmonics. For the flute, the Helmholtz resonance also reduces the magnitude of the extrema in the impedance. Thus, in the high frequency range, the spectral envelope depends on features of the reed and air jet operation and less strongly on features of the bore.

Fig. 7 shows the acoustic impedance spectra for the two notes whose sound spectra are shown in Fig. 1, measured on the same instruments. These impedance spectra also show another interesting feature, because both use register keys. A register key is usually a small key, located well away from the open end of the bore, whose purpose is to weaken and/ or to detune the lowest resonance(s), so that the instrument will more easily play a note whose fundamental coincides with

one of the higher resonances. Arrows on the insets of Fig. 7 indicate the register keys, whose effects may be seen by comparing the low frequency extrema in Fig. 7 with those in Figs. 3 and 6.
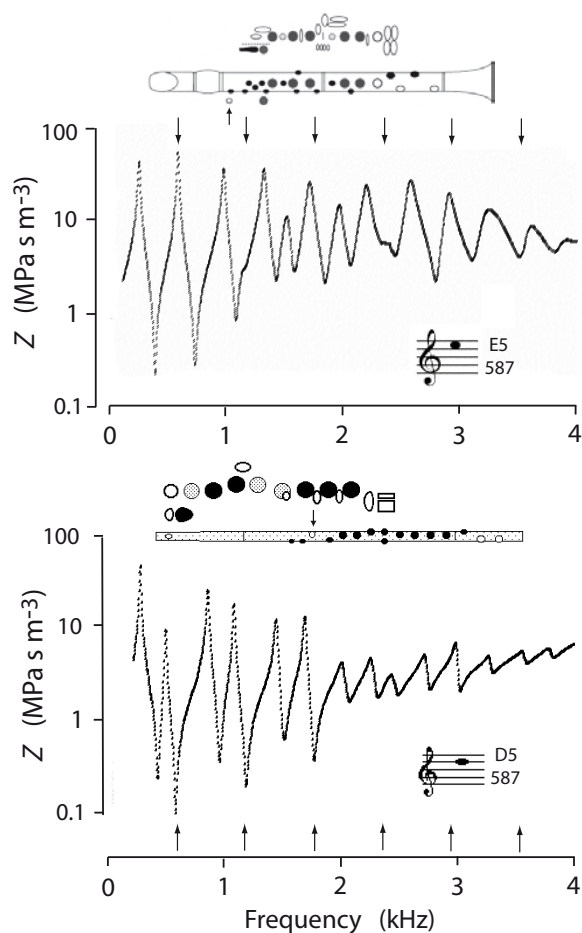

Figure 7. The magnitude of the measured impedance curves for the clarinet (upper) and flute (lower) for the fingerings used to play the note D5, whose spectra are shown in Fig. 1. The fingerings used are indicated by the schematics above each spectrum. The vertical arrows on the spectra indicate the harmonics of the note played. The arrows on the instrument schematics show holes opened to act as register holes, whose function is to weaken the lowest resonance(s), as is evident in the impedance spectra. (D5 is called E5 on the clarinet, a transposing instrument.)

It is tempting to extend this discussion to consider further subtleties of the acoustics of the clarinet. However, these are mainly of interest to clarinet players and researchers. So we have added such discussions to each of the pages in the clarinet acoustics database reported here. (Similarly, such comments appear on each page of the flute acoustics database.) Sound files are also provided.

We return, however, to the question posed by Fig. 1, to which the answer is printed below. Can one, in general, tell a clarinet from a flute simply from looking at the sound spectrum of a sustained note? Sometimes this is possible: in the Western classical and romantic tradition, clarinets are played without vibrato, while flutes are often played with considerable vibrato. In such cases, this difference may be distinguished by the width of the spectral peaks produced by the harmonics. Further, depending on the style of playing and the level of background noise, it may be possible to see, in the spectrum, the broad band noise associated with the jet of the flute, as is the case here.

In the notes of the lowest register of the clarinet (including, of course, the lowest note – Figs. 4 and 5), at least a few harmonics fall below the cut off frequency. Further, the presence of the bell produces only weak frequency dependence of the effective length. Consequently, for the first few harmonics at least, the clarinet's sound spectrum exhibits even harmonics that are weaker than their neighbours. However, this does not extend beyond the second resonance.

Further, one cannot, in general, rely on the spectral envelope, even at low frequencies. In Fig. 7, vertical arrows indicate the harmonics of the played notes. The fundamental of the clarinet note is largely determined by the frequency of the first impedance peak. The next few higher harmonics do not systematically coincide with impedance peaks and so do not systematically benefit from the impedance matching. Their relative amplitudes depend, in part, on the nonlinearity of the reed vibration and thus to some extent on how loudly the instrument is played. At high frequencies, the bell is acting as an efficient radiator of all frequencies.

For the flute, which has a higher cutoff frequency, the first three harmonics all fall close to impedance minima. At higher frequencies, one might argue that there is little need for an impedance matcher, as the impedance of the radiation field doubles with each octave in frequency. So, again, the sound spectrum depends in part on nonlinearities in the behaviour of the jet.

Many further examples are given in the database, which is at www.phys.unsw.edu.au/music/clarinet

ANSWER TO THE INTRODUCTORY QUESTION

The clarinet is the lower spectrum

ACKNOWLEDGMENTS

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