THE SOUND OF MUSIC: ORDER FROM COMPLEXITY

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Musical and biological sounds have the property of being well organised and usually strictly harmonic in spectrum, though there are a few notable exceptions such as the shimmering crash of a cymbal or the cry of the sulphur-crested cockatoo. It turns out, however, that this apparent simplicity is constructed by the interaction of highly nonlinear feedback generators linked to resonators whose vibrational modes are not in simple harmonic frequency ratios. This paper explores the way in which this apparent simplicity emerges from complex interactions in the generation of instrumental sound and in the songs of humans and other animals.

INTRODUCTION

Musical instruments such as violins, flutes and trumpets are designed to produce sounds that are pleasing to our ears, and analysis shows that these sounds, when played steadily, have exact harmonic spectra. This leads to the expectation that we might hear smooth concords between notes whose frequencies are in simple integer ratios, as is indeed found. This seems to imply that everything about these instruments is simple and linear, but this is very far from being the case. Indeed nonlinearity is essential to produce these apparently simple results. But sometimes nonlinearity takes control, as in cymbals and gongs, giving rise to effects such as pitch glide, subharmonics, and transitions to chaotic vibration. Very much the same is true of the sounds produced in human speech and singing and, more noticeably, in the songs of birds, where we encounter almost pure tones, harmonic spectra, and even chaotic screeches. In this short paper we explore the physics and mathematics underlying this behaviour. The subject has been discussed in more detail by Fletcher and Rossing [1] (chapter 5) and by Fletcher [2, 3, 4] and in less technical form by Backus [5] and by Johnston [6].

IMPULSIVELY EXCITED INSTRUMENTS

Musical instruments came in two types: those that produce steady sustained sounds, such as violins and trumpets, and those that are sharply excited and produce transient sounds, such as plucked strings, bells and gongs. In the first case there is a continuous input of energy and some sort of feedback oscillation is generated, while in the second there is an initial impulse after which the energy stored in the system gradually decays because of internal and radiation losses. In both cases the vibrating system is geometrically extended, whether it be a taut string, a column of air, or a carefully shaped plate or shell, so that it has many possible vibrational modes, and it is the interplay of these modes that controls the sound that is produced.

It might be thought that an elastic string held under tension between two rigid supports is an ideally simple system, but is it really? The "standard theory" treats the string as being ideally thin and the vibration amplitude as being planar and infinitesimal, but none of these assumptions holds in practice. Instead, even for planar oscillation, the string motion is described by an equation of the form

$$m\frac{\partial^2 z}{\partial t^2} \approx T\frac{\partial^2 z}{\partial x^2} + AEd^4\frac{\partial^4 z}{\partial x^4} + \frac{BEd^2}{L}\left[\int_0^L \left(\frac{\partial z}{\partial x}\right)^2 dx\right]\frac{\partial^2 z}{\partial x^2} \quad (1)$$

where z is string the displacement at point x, L is the string length, m is the string mass per unit length, and T the string tension. The first term on the right-hand side is that for an ideal string, the second term is the restoring force due to string stiffness, which is proportional to its Young's modulus E and the fourth power of string diameter d, while the third term is an approximation to the extra tension produced by displacement of the string from a straight line. The stiffness term stretches the mode frequencies so that of the nth mode becomes

$$f_n \approx nf_1(1+\alpha n^2) \quad (2)$$

where α is proportional to the ratio of string stiffness to string tension. In the steel strings of the piano this causes a pitch stretch of about half a semitone over the complete keyboard compass because octaves are tuned to match the second harmonic of the octave below.

In a string with inadequate initial tension, plucked with large amplitude, the third term in equation (1) leads to an unpleasant effect in which the tension, and therefore the musical pitch, starts high and then gradually falls. This is avoided by tightening strings to almost their breaking point. The third term also introduces harmonic distortion, so that an initial simple vibration at f_1 generates another vibration at $2f_1$ and so on, or mixes the frequencies of two existing modes. As if this were not enough, the vibrational tension of the third term in equation (1) also excites longitudinal waves in the string, and these couple the transverse modes in the z direction to orthogonal modes in the y direction [7]. And all this is not just mathematics — the effects contribute significantly to the actual sound of a piano! Some piano makers, notably Wayne Stuart in Newcastle Australia, have changed the exact form of the pinning of the strings in their instruments to control some of these effects, but I am not aware of any scientific study of the results, which could well be concealed by other changes in the design anyway. The whole behaviour of piano strings is

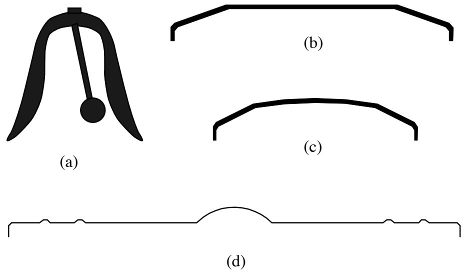

Figure 1. Profiles of (a) a church bell, (b) a flat-centre Chinese Opera gong, (c) a curved-centre Chinese Opera gong, and (d) an orchestral tam-tam

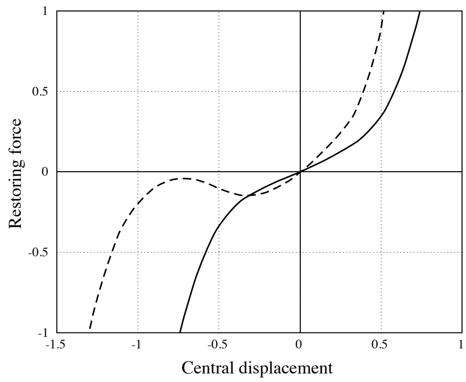

Figure 2. Restoring force as a function of deflection for a flat circular plate pinned at its edges (full curve) and for a slightly curved plate of the same size (broken curve). The units shown are arbitrary

further complicated by the fact that they mostly occur in pairs or triplets for each note, and the strings for a given note interact with each other through the non rigid bridge over which they pass. Weinreich [8] has treated this complication in detail and shown many interesting consequences.

Many musical instruments of the impulsive or percussive variety do not rely primarily on tension, as does the string, but rather on elastic stiffness. The simplest are bells, which have very thick walls, as shown in Figure 1(a), and are almost completely linear in behaviour. The tuning of the modes of such complex shapes is not simple, however. Church and carillon bells are cast to traditional shapes and then adjusted by removing material internally so that the frequencies of their principal modes are in simple integer ratios, giving pleasant musical sounds, one of the characteristics however being a low-pitched musical minor third, with frequency ratio 6/5 relative to the ‘prime’ or dominant mode, and it is this that gives bells their characteristic sound [1]. Many Eastern bells or gongs, such as the Javanese gamelan, are not tuned in this way however, and this gives rise to musical scales that are quite different from the familiar Western ones [9].

Gongs are another form of percussive instrument, but here the metal shell is thin compared to its diameter, as shown in later panels of Figure 1, so that tension effects become noticeable or even dominant. One very impressive effect is that achieved in Chinese opera gongs, of which there are two types. In the larger gong, shown in Figure 1(b), the main vibrating element in the centre of the gong is quite flat so that any vibration stretches it radially and raises both the tension and the vibration frequency, as shown by the full curve in Figure 2. As the vibration decays, the pitch falls towards its small-amplitude value over a time of the order of one second. In the smaller gong shown in Figure 1(c), the central portion is slightly domed to a height of about 1 mm over its 10 cm diameter. The tension forces are thus initially compressive for small downward motion of the dome before becoming tensile when the displacement exceeds twice the dome height, though the stiffness maintains a restoring force as illustrated by the dashed curve in Figure 2. For vibrations of moderate amplitude this causes the frequency to rise as the vibration decays, producing a sound that is complementary to that of the larger gong [10].

Sharp changes in shape are important here, since the tension term in equation (1) then acts at an angle and becomes converted in part to a lateral force, generating modes at two and three times the frequency of the original because of further coupling to the original exciting mode slope [11]. The nonlinearity thus leads to a progressive transfer of vibrational energy from low to high modes, an effect that is particularly noticeable in the large Chinese tam-tam gong often used in orchestras, which has two rings of sharp bumps in its outer profile, as shown in Figure 1(d). It is struck with a soft hammer and the low-frequency initial sound becomes transformed over a period of a second or so into a shimmering high-pitched sound that is actually chaotic [12]. Indeed simple experiments with metal gongs or cymbals of simpler shape show that when vibrated at their centre they can display energy transfer to higher harmonics of the exciting frequency, subharmonic generation at frequencies as low as one-fifth of the exciting frequency, or a transition to chaotic oscillation, depending upon minor variations in the exciting frequency. In normal musical playing, of course, cymbals are supported at their centre and driven by a sharp asymmetrical impact, so that essentially all modes are sharply excited. Mode interaction and transitions of the kind discussed above then lead to a wide-band ‘shimmering’ sound.

SUSTAINED-TONE INSTRUMENTS

Sustained-tone instruments are very different in operation from their impulsive cousins, and each consists of a system of the type shown in Figure 3. The resonant system is normally driven at a sufficiently low level that it is essentially linear in behaviour, with all the nonlinearity being contained in the active driver, the operation of which is controlled by feedback from the resonator. If the driver were linear, then it would simply maintain all the resonant modes of the string (for violin-type instruments) or the air column (for woodwind and brass instruments) at their natural frequencies, which are never in exact integer ratios because of stiffness and end-pinning effects in strings and geometric irregularities and the radiation end-correction in wind instruments. It is clear, therefore, that the nonlinearity of the feedback-driven oscillator must somehow be responsible for producing the exact harmonics observed in the sounds of these instruments. How does this occur?

First let us look at why the driving oscillator is nonlinear. In the case of a string driven by a bow, which consists of

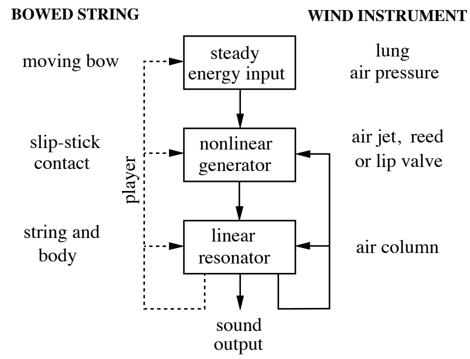

Figure 3. System diagram for a sustained-tone instrument consisting of a feedback-excited nonlinear oscillator coupled to a multi-mode linear resonator. Additional low-frequency control feedback is provided by the performer in the case of a musical instrument

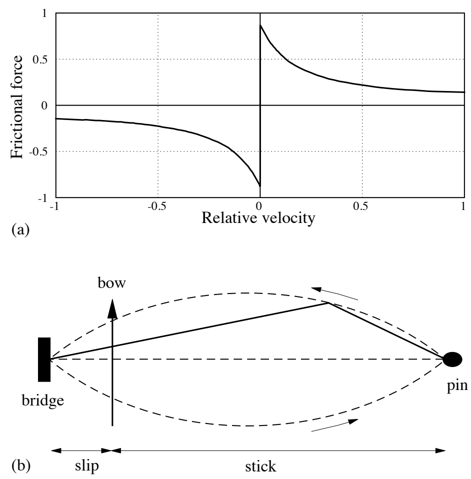

Figure 4. (a) Frictional force of a bow as a function of the velocity of the string relative to that of the bow. Units are arbitrary (b) Motion of the string for bowing in the position and direction shown, the kink moving around the broken curve

hairs coated with rosin, the static friction is very high so that the string essentially sticks to the bow and is drawn sideways. Ultimately, however, the restoring force due to tension exceeds the static frictional force and the string begins to slip. Because sliding friction is much smaller than static friction, as shown in Figure 4(a), the string then slips almost unimpeded until it reaches the farther extreme of its motion, when it is again captured by the bow and the cycle repeats [13, 14]. The envelope of the string motion is shown in Figure 4(b), with the slope discontinuity moving with uniform velocity between two parabolic envelope curves. The string itself does not radiate appreciable sound, but it passes over a bridge support at one end and its varying slope at this point produces an excitation force that is passed on to the body of the instrument. The vibrational response of the instrument body is linear, but is modified by its own resonances, so that the spectrum of the radiated sound depends both upon the string excitation and the instrument body response.

The structure of a woodwind reed instrument such as the clarinet is illustrated in Figure 5(a). For such an instrument, the generator consists of a valve with a fixed aperture covered by a thin cane reed that is partially open in its rest state. Because the

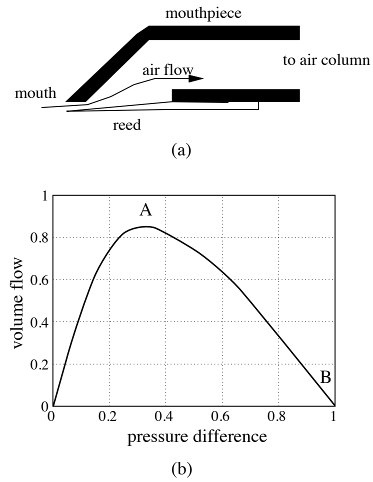

Figure 5. (a) Geometry of the mouthpiece and reed of a clarinet (b) Volume flow through the reed aperture as a function of the pressure difference across the reed The effective resistance is negative in the region AB

natural vibration frequency of the reed is much higher than that of the note being played, a quasi-static analysis shows what is going on. Initially the volume flow through the reed increases as the pressure difference p across it is increased, conservation of energy dictating that the flow velocity is proportional to the square root of the pressure difference (Bernoulli's law). However the pressure difference also tends to progressively close the reed opening, so that the overall volume flow U has the form

$$\begin{aligned} U(p) &= \alpha p^{1/2}(1-\beta p) \quad \text{if} \quad p < \beta^{-1} \\ &= 0 \qquad\qquad\qquad\;\; \text{if} \quad p > \beta^{-1} \end{aligned} \tag{3}$$

where α and β are constants. This relation is shown in Figure 5(b), and it can be seen that the valve impedance dp/dU is very nonlinear and is negative in the range A to B, which is what allows the valve to act as an acoustic generator. This will work, though with somewhat more complex analysis, at all frequencies up to the free resonance frequency of the reed itself. The constricted double reed of an oboe has an even more complicated flow behaviour because of the flow constriction caused by its narrow channel.

When we consider lip excited brass instruments, the situation is rather different because the lip aperture is blown open by the pressure differential, rather than being blown closed. This complicates the behaviour, and valve oscillations can be maintained only at a frequency just above the natural vibration frequency of the lips, the phase shift again making the resistance negative in this region, at least in a simple model [1, 15]. Because trumpets and trombones are often played very loudly, this is the one case in which the resonator can become nonlinear, leading to distortion of the propagating wave in the cylindrical part of the bore and consequent transfer of acoustic energy from low to high-frequency modes and even to shock waves [16].

Finally, consider instruments such as flutes and organ pipes that are excited by a planar air jet blown across an aperture near one end of the pipe as shown in Figure 6(a). The physics of this jet excitation is rather complicated [1, 17], but essentially the jet is acted on by the acoustic flow through the aperture and this generates a displacement wave that travels along the

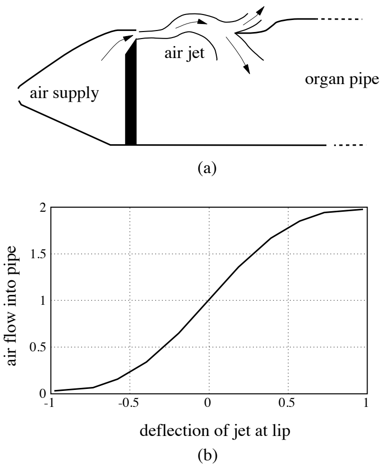

Figure 6. (a) Geometry of the mouth of an organ flue pipe. Waves are excited on the jet by acoustic flow through the mouth of the pipe and grow in amplitude as they propagate towards the pipe lip (b) Flow into the pipe mouth as a function of jet deflection at the lip. Units are arbitrary

jet at about half its airspeed. The jet then blows alternately into and out of the pipe where it meets the other edge of the aperture. The phase shift introduced by the initial excitation and the travel time of the displacement wave along the jet makes the flow resistance negative over a limited frequency range, thus allowing the player to choose which mode of the pipe resonator is being excited. For displacements that are small compared with the jet width, the inflow is linearly related to the excitation, but for larger displacements the flow saturates, as shown in Figure 6(b), generating odd harmonics of the fundamental. If the jet centre-plane is offset with respect to the edge, then the flow waveform becomes asymmetrical and even harmonics are generated as well [18]. These air-jet excited instruments are one of the few cases in which the transition to mode-locked harmonic sound has been studied, as will be discussed in the next section.

FORMAL ANALYSIS

Now that the origins of inharmonicity on the resonator and of nonlinearity in the generator have been explained, it is possible to proceed with a formal analysis of the behaviour of the musical instrument system shown in Figure 3. There are many ways in which this can be done, but one of the simplest is in the frequency domain where one examines the combined response of the resonator modes coupled to the generator using the method of slowly varying parameters [1].

For simplicity, consider first just the case of a single vibrational mode

$$y_n(t) = a_n \sin(\omega_n t + \phi_n) \tag{4}$$

and allow that both the amplitude a_n and the phase ϕ_n may vary with time slowly compared with ω_n. Suppose that, in the system considered, y_n satisfies the equation

$$\ddot{y}_n + \omega_n^2 y_n = g(y_n, \dot{y}_n, t) \tag{5}$$

where the dots signify differentiation with respect to time and the nonlinear function g represents the excitation provided by the generator under the influence this mode. Substitution of equation (4) into equation (5) gives a complicated result, but this can be simplified if we assume that

$$\dot{y}_n = a_n \omega_n \cos(\omega_n t + \phi_n)\,. \tag{6}$$

This assumption requires that

$$\dot{a}_n \sin(\omega_n t + \phi_n) + a_n \dot{\phi}_n \cos(\omega_n t + \phi_n) = 0\,, \tag{7}$$

but substituting equations (4), (6) and (7) into equation (5) then leads to the simple results

$$\dot{a}_n = \frac{g}{\omega_n}\cos(\omega_n t + \phi_n) \tag{8}$$

$$\dot{\phi}_n = -\frac{g}{a_n \omega_n}\sin(\omega_n t + \phi_n)\,, \tag{9}$$

where g is expressed in terms of y_n and $\dot{y}_n$ as given by equations (4) and (6).

These two equations (8) and (9) allow us to calculate the steady mode amplitude a_n for which $\dot{a}_n = 0$ and also the steady vibrational frequency $\omega_n + \dot{\phi}_n$, which will generally be different from the resonant frequency ω_n. The whole analysis can also be extended to treat the realistic case of a multi-mode system in which the modes interact because of the nonlinearity of the generator. It can be shown that this interaction leads to the locking of all the mode oscillations into an exactly harmonic (integer frequency ratio) distribution provided the natural mode frequencies are not too distant from this relationship initially [19]. If two prominent modes ω_i and ω_j are very far from integrally related in frequency, as can happen with peculiar fingerings of woodwind instruments, then the resulting oscillation may involve both of them, and the system nonlinearity will produce a “multiphonic” sound containing all frequencies $m\omega_i \pm n\omega_j$ where m and n are integers. These sounds are exploited in certain modern musical compositions for woodwinds [20].

This analysis can also be applied to transient sounds in which the generating function g depends upon time. An obvious example is the case of impulsively excited instruments, but the initial transient is also of great importance in defining the character of sustained-tone instruments, psychoacoustical studies showing that it is largely the initial transients that characterise the identity of a musical instrument, since identity is lost to the hearer if these are removed. This feature is also of great importance in the electronic synthesis of realistic musical sounds.

If the excitation begins abruptly, as is the case at the beginning of a musical note, then this stepwise or even impulsive excitation will excite all the modes of the instrument to vibrate at their natural frequencies, which are never in exact harmonic relationship, and it is only after a significant time, typically ten or more cycles of the fundamental, that the mode frequencies are captured by the nonlinearity in the way discussed above and locked together to produce an exactly harmonic sound. This transition has been examined in detail in the case of organ pipes [21] and an example of the calculated results is shown in Figure 7. The pipe modes, which are not in exactly harmonic relationship, are initially all excited in-phase by the jet impulse. Interaction between the modes

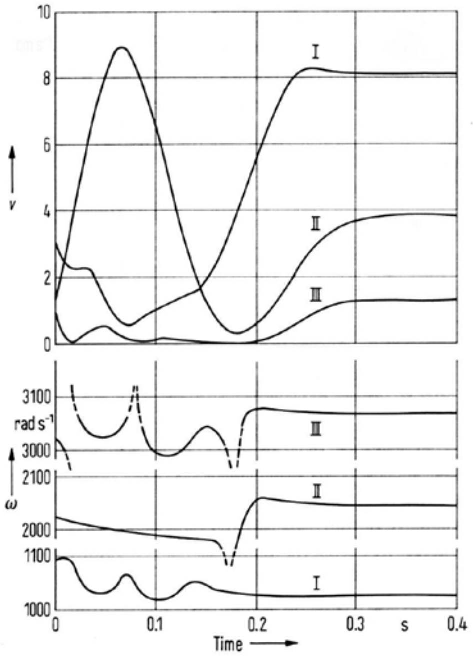

Figure 7. Evolution of the velocity amplitudes and frequencies of the first three modes of an organ pipe as calculated from equations (8) and (9) and a detailed model of wave propagation on the air jet and its interaction with the pipe air-column [21]

through the nonlinear behaviour of the jet then causes both mode amplitudes and frequencies to evolve to a stable harmonic relationship — a typical example of the emergence of order through complex interactions.

ANIMAL SOUNDS

As in musical instruments, the sounds produced by animals can be divided into two classes — those in which the excitation is essentially impulsive and those in which it is continuous — but the division is not so clear since many quasi-continuous insect sounds are really prolonged sequences of impulsive sounds. Once this is recognised, much of the analysis applied to musical systems can also be applied to biological systems [2, 3].

The simplest systems, which are employed largely by water-dwelling crustaceans such as crabs, produce sounds that are just sequences of clicks. This is done by snapping together or flicking apart two body parts that are covered by a stiff elastic shell. Because the shell generally covers some sort tissue, the sound is abruptly damped and has little if any tonal component. In air-dwelling insects such as crickets this system has evolved so that a file containing many teeth, and generally located on a leg, is drawn rapidly across a resonant structure such as a wing. While this bears some resemblance to a bowed string, the damping is again such that the oscillation produced by the impact of each tooth has almost dissipated before the next impact occurs. Since the pic on the structure must slip off the tooth each time, however, there is some chance that these releases will be phase-locked to the vibration, perhaps giving harmonic relations. The loudest insect of all, the cicada, makes its sounds by the rhythmic collapse and release of thin ribbed plates, or tymbals, that cover a large resonant abdominal cavity, giving a rapidly pulsating song near its characteristic frequency, which ranges from about 600 Hz to above 3 kHz depending upon the species. From the point of view of the present chapter, there is nothing very interesting about these sounds.

The more interesting sounds from the present viewpoint are those produced by air-breathing animals, including birds and humans. These are generally sustained sounds, like the vowels in human speech, punctuated by impulsive or chaotic sounds like the consonants. The vowel-like sounds are produced by a vibrating valve, rather like a tiny pair of human lips, located between the lungs and the mouth. In the case of most mammals, including humans, this valve is near the top of the trachea or wind-pipe near its junction with the mouth, while in birds it is near the base of the trachea. In the case of songbirds there are actually two vocal valves, one in each of the bronchi or tubes leading from the lungs just below their junction with the trachea, allowing them to sing two notes at once if they choose to do so. This structure is known as the syrinx.

In humans, other mammals and many birds the vibration frequency of the vocal valve is below the frequency of the first resonance of the vocal tract, so that there is not the same sort of coupling between the source and the resonant filter provided by the upper vocal tract. Indeed it is reasonable to model these systems as an autonomous vibrating valve producing a pulsating airflow that is rich in harmonics because the valve ordinarily closes once in each cycle, coupled to the upper vocal tract which then provides an adjustable filter that modifies the spectral envelope of the sound to produce distinctive patterns that we know as vowels, each being characterised by maxima in its spectra, known as formants, close to the tract resonance frequencies. This is called the ‘source-filter model’. Only in high soprano singing does the fundamental pitch approach the frequency of one of the higher vocal tract resonances, and there is then a coupling between the two which gives a comparatively pure tone and reduces the distinction between different vowels. Something similar happens in the case of pure-tone songbirds as is discussed below.

While humans can make a pure-tone sound by whistling, involving an air jet and the mouth as a Helmholtz resonator, some birds can produce similar nearly pure-tone songs that are swept over a large frequency range, but they do not do it by whistling. Their vocal tract is connected to an elastic part of the upper esophagus, leading to the stomach, and this can be expanded to produce a quite large vocal cavity of adjustable size. Once again the pitch of the song is adjusted largely by changing the volume of this cavity [22]. Some animals, such as doves and frogs, can even produce pure-tone songs with their beak or mouth closed, a feat which they perform by inflating a vocal sac in the throat with very thin skin surrounding it and exhaling into this through their vibrating vocal valve, which they tune to the resonant frequency of the sac volume loaded by its vibrating walls. In the second or so that the call lasts, the volume of the inflated sac does not change greatly, and the small change is compensated for by the thinning of the walls.

Another interesting case is the very loud cry of the sulphur-crested cockatoo (*Cacatua galerita*). This harsh sound is at odds with the beauty of the bird, but accords well with its destructive behaviour! The interesting thing is that, when the waveform of the recorded cry is examined closely, it is found to be truly chaotic with a Lyapunov exponent of about 0.3 [23]. The great loudness of the screech is partly explained by its frequency, which covers a broad band from about 2 to 4 kHz where human hearing is most sensitive, but the bird also invests a considerable effort in producing it. The anatomical and physiological basis of this cry (one hesitates to call it a song!) has not yet been established, but it seems likely that the vocal valve, or syrinx, has an extended flap-like structure that can move chaotically like a flag flapping in the wind, even though it is probably secured all around its periphery.

Finally we should mention the interesting combination of vocal and instrumental sounds that is sometimes used in the Australian didjeridu, or yidaki as it is called by the Yolngu people of Arnhem Land who perhaps originated the instrument [24]. The didjeridu consists of a simple tube, about 1.5 m long and 30 to 50 mm in diameter, cut from a small tree trunk that has been hollowed out by termites. The drone sound, typically at about 60 Hz, is made by buzzing the lips as in a brass instrument, and this is accompanied by a full range of upper harmonics. The interesting tonal sounds characteristic of the didjeridu are then made by varying the geometry of the vocal tract, again largely by moving the tongue, so as to produce spectral peaks or formants not unlike those in speech, the emphasised frequencies being those lying close to a minimum in the vocal tract impedance as seen from the lips [25, 26]. Even more interesting in the present context are the sounds that can be made by simultaneously vibrating the folds of the vocal valve, as when singing a note of frequency ω, and also the lips, under the control of the fundamental didjeridu resonance at frequency ω_0. Since the two vibrating valves are in series, their effect is multiplicative rather than additive, and the resulting flow contains frequencies $m\omega_0 \pm n\omega$ [27]. Thus if the player vocalises a note a musical fifth above the drone fundamental, so that $\omega = (3/2)\omega_0$, then the nonlinearity of the combined valves will produce a subharmonic at frequency $\omega_0/2$, together with all its harmonics. For the singing of more complex sounds, there will be a large variety of frequencies produced.

CONCLUSIONS

In this short paper there has been time only to glance briefly at some of the interesting features of sound production in musical and biological systems. While the systems themselves consist of complex interacting nonlinear elements, the interesting outcome is that these often act together to produce a deceptively simple outcome, with strictly harmonic waveforms and well controlled behaviour. There are a few interesting exceptions, however, and these will doubtless repay detailed study some time in the future.

The work described in this paper has been carried out by many researchers around the world, and the literature documenting it is voluminous. The references I have cited are largely to work with which I have been personally associated, a fault for which I should perhaps apologise.

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